

## Lorentz boosts as squeeze transformations and the parton picture†

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**Abstract.** It was shown by Gribov, Ioffe, Pomeranchuk in 1966 and by Ioffe in 1969 that a space-time picture is needed for the Lorentz deformation of hadronic interaction region. It is shown that this deformation is a squeeze transformation. It is shown also that Feynman's parton picture emerges as a consequence of Lorentz-squeezed hadrons in the quark model.

### 1. Introduction

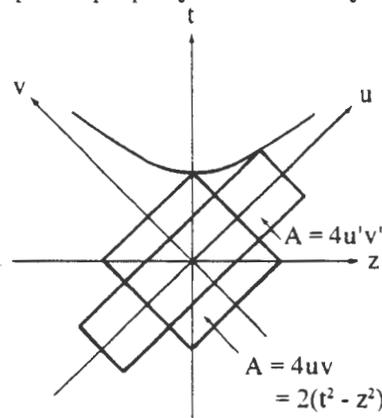
According to special relativity, the longitudinal length of a moving object becomes contracted while the transverse components remain invariant. There is therefore a tendency to assume that hadrons from an accelerator look like “pancakes” with a contracted longitudinal dimension [1]. Yes, an extended hadron should have the three-dimensional rotational symmetry when it is on the table [2]. If it moves with a speed close to that of light, it should look different. For static or slow hadrons, we use the quark model to understand what we observe in laboratories. For fast-moving hadrons, we use the parton picture to interpret the experimental data. In both the quark and parton models, quarks or partons interact directly with the external signal. In the quark model, we add those interaction amplitudes before calculating the cross section. On the other hand, in the parton model, we calculate the cross section for each parton before summing up the cross sections for all the partons. Does this mean that the Lorentz boost destroys the superposition principle?

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In order to answer this question, let us examine the "pancake effect" more carefully. This effort was started by Gribov, Ioffe, and Pomeranchuk in 1966 [3]. According to the old-fashioned picture of Lorentz pancakes, only the longitudinal component becomes contracted. In their 1966 paper [3], Gribov *et al* showed that combinations of space and time variables are needed in measuring the dimension of the interaction region as well as the interaction time. They showed further that the interaction time is proportional to the contracted component of the space-time variables which are known today as the light-cone variables. In his 1969 paper [4], Ioffe essentially completed the Lorentz-squeeze picture of the interaction region. This squeeze picture was shown to be convenient in explaining the high-energy data by Drell and Yan [5]. This picture is illustrated in one of the figures in the Drell-Yan paper. Figure 1 of the present paper reproduces the Lorentz squeeze property formulated by these authors.



**Figure 1.** Space-time picture of the Lorentz boost. The invariant quantity  $(z^2 - t^2)$  can be written as  $(z+t)(z-t)$ . This is proportional to the product of the light-cone variables  $u=(z+t)/\sqrt{2}$  and  $v=(z-t)/\sqrt{2}$ . The most appropriate name for this area-preserving deformation is SQUEEZE.

For a hadron with its space-time extension, the interaction region is essentially the region in which the quarks are distributed. Thus, the problem is reduced to the study of space-time distribution of the quarks inside the hadron. This means that we have to learn how to boost wave functions in quantum mechanics. This problem is then reduced to that of constructing Lorentz-covariant wave functions. While this is not a trivial problem, it is possible to construct a model based within the framework of Wigner's little groups which dictate the internal space-time symmetries of relativistic particles [2].

If we are to construct covariant wave functions, they should possess the symmetry of the little groups. After constructing such a set of wave functions, we should be able to take both the low-speed and high-speed limits of the wave functions to generate the quark and parton models respectively. Indeed, there is in the literature a formalism of covariant bound-state wave functions which can be Lorentz-boosted. It is called the covariant harmonic oscillator formalism [6, 7]. The formalism meets the following three basic requirements.

- (1) The formalism is consistent with the established physical principles including the uncertainty principle in quantum mechanics and the transformation laws of special relativity [6].

- (2) The formalism is consistent with the basic hadronic features observed in high-energy laboratories, including hadronic mass spectra, the proton form factor, and the parton phenomena [6].
- (3) The formalism constitutes a representation of the Poincaré group for relativistic extended hadrons [8], and a representation of Wigner's little group.

In addition, this oscillator system provides the mathematical basis for a certain set of coherent photon states commonly known as the squeezed state of light [9]. Through this formalism, we are able to see clearly that Lorentz boosts are squeeze transformations.

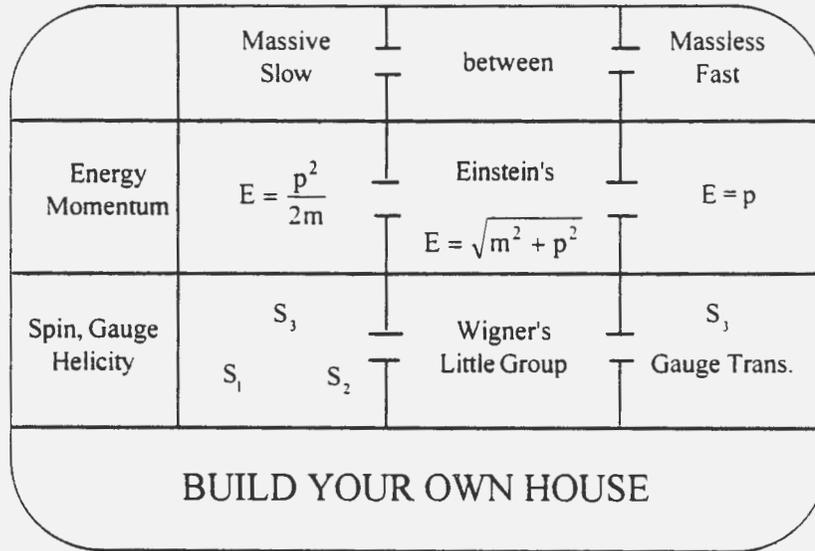
In this paper, we use this covariant oscillator formalism to see that the quark model and the parton models are two different manifestations of one covariant formalism. We shall see how the parton picture emerges from the Lorentz-squeezed hadronic wave function. This squeeze effect will also explain why the partons appear as incoherent particles, within the present framework of quantum mechanics based on the superposition principle.

Since we are going to use the language of little groups in this paper, we give in Section [2] a historical review of Wigner's little groups. In Section [3], we use the light-cone coordinate system to show that Lorentz boosts are squeeze transformations. Section [4] contains an outline of the covariant oscillator formalism which will exhibit the squeeze property of Lorentz boosts in quantum mechanics. Finally, in Section [5], it is shown that the covariant oscillator wave function gives a static wave function for the hadron at rest and the parton distribution function for the hadron moving with a speed very close to the speed of light. It is shown also that the time interval for the quark to interact with the external signal becomes contracted while the interval for the quark-quark interaction becomes dilated.

## 2. Wigner's little groups

From the principles of special relativity, Einstein derived the relation  $E = mc^2$  in 1905. This formula unifies the momentum-energy relations for both massive and massless particles, which are  $E = p^2/2m$  and  $E = cp$  respectively. In addition to the energy-momentum variables, relativistic particles have internal space-time degrees of freedom. A massive particle at rest has three rotational degrees of freedom, and they appear in the real world as the spin of the particle. Massless particles have only one rotational degree of freedom which appears as the helicity in the real world. In addition, they have gauge degrees of freedom which are not shared by massive particles. Why are these two symmetries different from each other? Is it possible to unify the symmetries for both cases as Einstein did for the energy-momentum relation? This problem is summarized in Figure 2.

In his 1939 paper [2], Wigner took the first step toward the resolution of this problem. He observed that the internal space-time symmetries of relativistic particles are dictated by their respective little groups. The little group is the maximal subgroup of the Lorentz group which leaves the four-momentum of the particle invariant. He showed that the little groups for massive and massless particles are isomorphic to  $O(3)$  (three-dimensional rotation group) and  $E(2)$  (two-dimensional Euclidean group) respectively. Wigner's 1939 paper indeed gives a covariant picture massive



**Figure 2.** Further implications of Einstein's  $E=mc^2$ . Massive and massless particles have different energy-momentum relations. Einstein's special relativity gives one relation for both. Wigner's little group unifies the internal space-time symmetries for massive and massless particles which are locally isomorphic to  $O(3)$  and  $E(2)$  respectively. It is a great challenge for us to find another unification. In this note, we present a unified picture of the quark and parton models which are applicable to slow and ultra-fast hadrons respectively.

particles with spins, and connects the helicity of massless particle with the rotational degree of freedom in the group  $E(2)$ . This paper also gives many homework problems, including the following four pressing problems in particle physics.

- (1) Like the three-dimensional rotation group,  $E(2)$  is a three-parameter group. It contains two translational degrees of freedom in addition to the rotation. What physics is associated with the translational-like degrees of freedom for the case of the  $E(2)$ -like little group?
- (2) As is shown by Inonu and Wigner [10], the rotation group  $O(3)$  can be contracted to  $E(2)$ . Does this mean that the  $O(3)$ -like little group can become the  $E(2)$ -like little group in a certain limit?
- (3) It is possible to interpret the Dirac equation in terms of Wigner's representation theory [11]. Then, why is it not possible to find a place for Maxwell's equations in the same theory?
- (4) The proton was found to have a finite space-time extension in 1955 [12], and the quark model has been established in 1964 [13]. The concept of relativistic extended particles has now been firmly established. Is it then possible to construct a representation of the Poincaré group for particles with space-time extensions?

Indeed, there are many papers written in the literature on the above-mentioned problems [6, 14], and the present situation is summarized in Figure 2. In this report,

we are interested only in the fourth question. It is about whether Wigner's little groups are applicable to high-energy hadrons fresh from particle accelerators. The question is whether it is possible to construct a representation of the little group for hadrons which are believed to be quantum-bound states of quarks [6, 15]. This representation should describe Lorentz-boosted hadrons. The next question is whether those boosted wave functions generate Feynman's parton picture [16] in the large-momentum limit.

Within the framework of Wigner's little groups, the ultimate question is whether the quark model and the parton model can be framed into the  $O(3)$ -like little group for massive particles and the  $E(2)$ -like little group for massless particles [17]. This mathematical question is beyond the scope of the present paper.

### 3. Lorentz boosts as squeeze transformations

The boost matrix for the longitudinal and time-like variables takes the form

$$B(\eta) = \begin{pmatrix} \cosh \eta & \sinh \eta \\ \sinh \eta & \cosh \eta \end{pmatrix} \quad (1)$$

applicable to the column matrix of  $(z, t)$ , with  $\tanh \eta = \beta$  where  $\beta$  is the velocity parameter of the hadron. In 1949, Dirac chose the coordinate variables [18]

$$u = \frac{z+t}{\sqrt{2}} \quad v = \frac{z-t}{\sqrt{2}} \quad (2)$$

in order to simplify the formula for Lorentz boosts. The boost matrix applicable to the column vector  $(u, v)$  now becomes diagonal and takes the form

$$B(\eta) = \begin{pmatrix} \exp \eta & 0 \\ 0 & \exp(-\eta) \end{pmatrix}. \quad (3)$$

The  $u$  and  $v$  variables are called the light-cone variables. Under this transformation, the  $u$  and  $v$  variables become expanded and contracted by the factors  $\exp \eta$  and  $\exp(-\eta)$  respectively. The product  $uv$  remains invariant. From Figure 1, it is quite clear that the Lorentz boost is an area-preserving "squeeze" transformation.

The transformation matrix of equation (1) is applicable also to the momentum-energy column matrix  $(P, E)$ , where  $P$  and  $E$  are the longitudinal momentum and the total energy respectively. As for the light-cone variables

$$P_+ = \frac{P+E}{\sqrt{2}} \quad P_- = \frac{P-E}{\sqrt{2}} \quad (4)$$

the transformation matrix is equation (3). This is also a squeeze transformation.

The word "squeeze" is commonly used these days in quantum optics for a certain class of two-photon coherent states [9], but the concept is squeeze transformations is applicable to many different branches of physics, including the Lorentz boost so fundamental in high-energy physics.

#### 4. Covariant harmonic oscillators

If we construct a representation of the Lorentz group using normalizable harmonic oscillator wave functions, the result is the covariant harmonic oscillator formalism [6, 7]. The formalism constitutes a representation of Wigner's  $O(3)$ -like little group for a massive particle with internal space-time structure. This oscillator formalism has been shown to be effective in explaining the basic phenomenological features of relativistic extended hadrons observed in high-energy laboratories. In particular, the formalism shows that the quark model and Feynman's parton picture are two different manifestations of one covariant entity [6, 17].

The covariant harmonic oscillator formalism has been discussed exhaustively in the literature, and it is not necessary to give another full-fledged treatment in the present paper. We shall discuss here only the squeeze property of the oscillator wave functions. Let us consider a bound state of two particles. For convenience, we shall call the bound state the hadron, and call its constituents quarks. Then there is a Bohr-like radius measuring the space-like separation between the quarks. There is also a time-like separation between the quarks, and this variable becomes mixed with the longitudinal spatial separation as the hadron moves with a relativistic speed. There are no quantum excitations along the time-like direction. On the other hand, there is the time-energy uncertainty relation which allows quantum transitions. It is possible to accommodate these aspect within the framework of the present form of quantum mechanics. The uncertainty relation between the time and energy variables is the  $c$ -number relation [20], which does not allow excitations along the time-like coordinate. We shall see that the covariant harmonic oscillator formalism accommodates this narrow window in the present form of quantum mechanics.

Let us consider now a hadron consisting of two quarks. If the space-time position of two quarks are specified by  $x_a$  and  $x_b$  respectively, the system can be described by the variables

$$X = \frac{x_a + x_b}{2} \quad x = \frac{x_a - x_b}{2\sqrt{2}}. \quad (5)$$

The four-vector  $X$  specifies where the hadron is located in space and time, while the variable  $x$  measures the space-time separation between the quarks. In the convention of Feynman *et al* [15], the internal motion of the quarks bound by a harmonic oscillator potential of unit strength can be described by the Lorentz-invariant equation

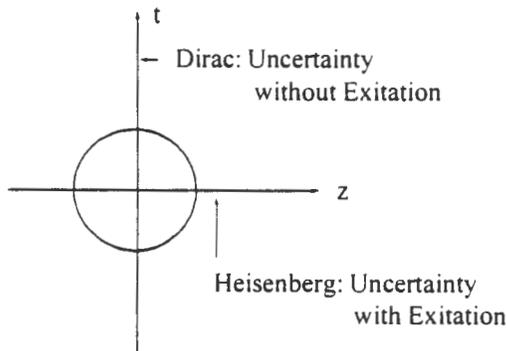
$$\frac{1}{2} \left( x_\mu^2 - \frac{\partial^2}{\partial x_\mu^2} \right) \psi(x) = \lambda \psi(x). \quad (6)$$

We use here the space-favored metric:  $x^\mu = (x, y, z, t)$ .

It is possible to construct a representation of the Poincaré group from the solutions of the above differential equation [6]. If the hadron is at rest, the solution should take the form

$$\psi(x, y, z, t) = \psi(x, y, z) \left( \frac{1}{\pi} \right)^{1/4} \exp \left( -\frac{t^2}{2} \right) \quad (7)$$

where  $\psi(x, y, z)$  is the wave function for the three-dimensional oscillator with appropriate angular momentum quantum numbers. Indeed, the above wave function constitutes a representation of Wigner's  $O(3)$ -like little group for a massive particle [6]. In the above expression, there are no time-like excitations, and this is consistent with what we see in the real world. It was Dirac who first noted this space-time asymmetry in quantum mechanics [20]. However, this asymmetry is quite consistent with the  $O(3)$  symmetry of the little group for hadrons. Figure 3 illustrates the uncertainty relations along the space-like and time-like directions.



**Figure 3.** Quantum mechanics with the  $c$ -number time-energy uncertainty relation. The present form of quantum mechanics allows quantum excitations along the space-like directions, but does not allow excitations along the time-like direction even though there is an uncertainty relation between the time and energy variables.

Since the three-dimensional oscillator differential equation is separable in both spherical and Cartesian coordinate systems,  $\psi(x, y, z)$  consists of Hermite polynomials of  $x$ ,  $y$ , and  $z$ . If the Lorentz boost is made along the  $z$  direction, the  $x$  and  $y$  coordinates are not affected, and can be dropped from the wave function. The wave function of interest can be written as

$$\psi^n(z, t) = \left(\frac{1}{\pi}\right)^{1/4} \exp\left(-\frac{t^2}{2}\right) \psi_n(z) \quad (8)$$

with

$$\psi^n(z) = \left(\frac{1}{\pi n! 2^n}\right)^{1/2} H_n(z) \exp\left(-\frac{z^2}{2}\right) \quad (9)$$

where  $\psi^n(z)$  is for the  $n$ th excited oscillator state. The full wave function  $\psi^n(z, t)$  is

$$\psi_0^n(z, t) = \left(\frac{1}{\pi n! 2^n}\right)^{1/2} H_n(z) \exp\left[-\frac{1}{2}(z^2 + t^2)\right]. \quad (10)$$

The subscript 0 means that the wave function is for the hadron at rest. The above expression is not Lorentz-invariant, and its localization undergoes a Lorentz squeeze as the hadron moves along the  $z$  direction [6]. This is a Lorentz-covariant expression!

Let us write the above wave functions in terms of the light-cone variables defined in equation (2). The wave function of equation (10) can be written as

$$\psi_0^n(z, t) = \psi_0^n(z, t) = \left( \frac{1}{\pi n! 2^n} \right)^{1/2} H_n \left( \frac{u+v}{\sqrt{2}} \right) \exp \left[ -\frac{1}{2} (u^2 + v^2) \right]. \quad (11)$$

If the system is boosted, the wave function becomes

$$\psi_\eta^n(z, t) = \left( \frac{1}{\pi n! 2^n} \right)^{1/2} H_n \left( \frac{e^{-\eta} u + e^\eta v}{\sqrt{2}} \right) \exp \left[ -\frac{1}{2} (e^{-2\eta} u^2 + e^{2\eta} v^2) \right]. \quad (12)$$

Indeed, in the light-cone coordinate system, the Lorentz-boosted wave function takes a very simple form.

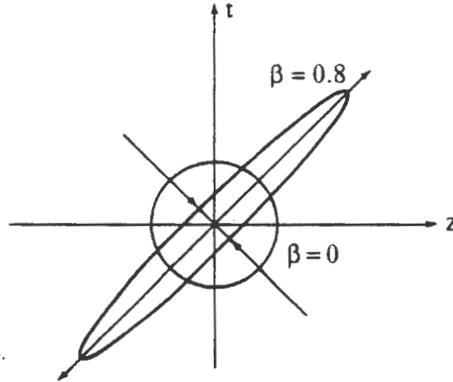
In both equations (11) and (12), the localization property of the wave function in the  $uv$  plane is determined by the Gaussian factor, and it is sufficient to study the ground state only for the essential feature of the boundary condition. The wave functions in equation (11) and equation (12) then respectively become

$$\psi_0(z, t) = \left( \frac{1}{\pi} \right)^{1/2} \exp \left[ -\frac{1}{2} (u^2 + v^2) \right]. \quad (13)$$

If the system is boosted, the wave function becomes

$$\psi_\eta(z, t) = \left( \frac{1}{\pi} \right)^{1/2} \exp \left[ -\frac{1}{2} (e^{-2\eta} u^2 + e^{2\eta} v^2) \right]. \quad (14)$$

The transition from equation (13) to equation (14) is a squeeze transformation. The wave function of equation (13) is distributed within a circular region in the  $uv$  plane, and thus in the  $zt$  plane. On the other hand, the wave function of equation (14) is distributed in an elliptic region. This ellipse is a "squeezed" circle with the same area as the circle, as is illustrated in Figure 4. The Lorentz boost squeezes the oscillator wave function.



**Figure 4.** Relativistic quantum mechanics. If quantum mechanics described in Figure 3 is combined with special relativity in Figure 1, the result will be the circle being squeezed into an ellipse.

## 5. Feynman's parton picture

It is safe to believe that hadrons are quantum-bound states of quarks having localized probability distribution. As in all bound-state cases, this localization condition is responsible for the existence of discrete mass spectra. The most convincing evidence for this bound-state picture is the hadronic mass spectra which are observed in high-energy laboratories [6, 15]. However, this picture of bound states is applicable only to observers in the Lorentz frame in which the hadron is at rest. How would the hadrons appear to observers in other Lorentz frames? More specifically, can we use the picture of Lorentz-squeezed hadrons discussed in Section [4] ?

It was Hofstadter's experiment which showed that the proton charge is spread out. In this experiment, an electron emits a virtual photon, which then interacts with the proton. If the proton consists of quarks distributed within a finite space-time region, the virtual photon will interact with quarks which carry fractional charges. The scattering amplitude will depend on the way in which quarks are distributed within the proton. The portion of the scattering amplitude which describes the interaction between the virtual photon and the proton is called the form factor.

Although there have been many attempts to explain the behavior of form factors within the framework of quantum field theory, it is quite natural to expect that the wave function in the quark model will determine the charge distribution. In high-energy experiments, we are dealing with the situation in which the momentum transfer in the scattering process is large. Indeed, the Lorentz-squeezed wave functions lead to the correct behavior of the hadronic form factor for large values of the momentum transfer [21].

While the form factor is the quantity which can be extracted from the elastic scattering, it is important to realize that in high-energy processes, many particles are produced in the final state. They are called inelastic processes. While the elastic process is described by the total energy and momentum transfer in the center-of-mass coordinate system, there is, in addition, the energy transfer in inelastic scattering. Therefore, we would expect that the scattering cross section would depend on the energy, momentum transfer, and energy transfer. However, one prominent feature in inelastic scattering is that the cross section remains nearly constant for a fixed value of the momentum-transfer/energy-transfer ratio. This phenomenon is called "scaling" [1].

In order to explain the scaling behavior in inelastic scattering, Feynman in 1969 observed that a fast-moving hadron can be regarded as a collection of many "partons" whose properties do not appear to be identical with those of the quarks [16]. For example, the number of quarks inside a static proton is three, while the number of partons in a rapidly moving proton appears to be infinite. The question then is how the proton looking like a bound state of quarks to one observer can appear different to an observer in a different Lorentz frame? Feynman formulated his parton picture based on the following observations:

- (1) The picture is valid only for hadrons moving with velocity close to that of light.
- (2) The interaction time between the quarks becomes dilated, and partons behave as free independent particles.
- (3) The momentum distribution of partons becomes widespread as the hadron moves fast.
- (4) The number of partons seems to be infinite or much larger than that of quarks.

Because the hadron is believed to be a bound state of two or three quarks, each of the above phenomena appears as a paradox, particularly (2) and (3) together. We would like to resolve this paradox using the covariant harmonic oscillator formalism.

For this purpose, we need a momentum-energy wave function. If the quarks have the four-momenta  $p_a$  and  $p_b$ , we can construct two independent four-momentum variables [15]

$$P = p_a + p_b \quad q = \sqrt{2}(p_a - p_b). \quad (15)$$

The four-momentum  $P$  is the total four-momentum and is thus the hadronic four-momentum.  $q$  measures the four-momentum separation between the quarks.

We expect to get the momentum-energy wave function by taking the Fourier transformation of equation (14):

$$\phi_\eta(q_z, q_0) = \left(\frac{1}{2\pi}\right) \int \psi_\eta(z, t) \exp[-i(q_z z - q_0 t)] dx dt. \quad (16)$$

Let us now define the momentum-energy variables in the light-cone coordinate system as

$$q_u = \frac{q_0 - q_z}{\sqrt{2}} \quad q_v = \frac{q_0 + q_z}{\sqrt{2}}. \quad (17)$$

In terms of these variables, the Fourier transformation of equation (16) can be written as

$$\phi_\eta(q_z, q_0) = \left(\frac{1}{2\pi}\right) \int \psi_\eta(z, t) \exp[-i(q_u u - q_v v)] dudv. \quad (18)$$

The resulting momentum-energy wave function is

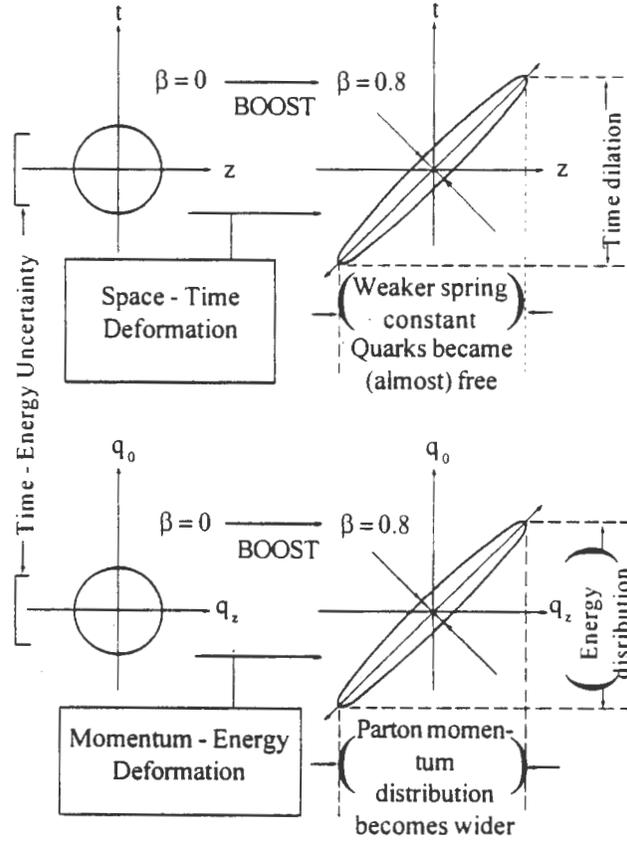
$$\phi_\eta(q_z, q_0) = \left(\frac{1}{\pi}\right)^{1/2} \exp\left[-\frac{1}{2}(e^{-2\eta} q_u^2 + e^{2\eta} q_v^2)\right]. \quad (19)$$

Because we are using here the harmonic oscillator, the mathematical form of the above momentum-energy wave function is identical to that of the space-time wave function. The Lorentz squeeze properties of these wave functions are also the same, as is indicated in Figure 5.

When the hadron is at rest with  $\eta = 0$ , both wave functions behave like those for the static bound state of quarks. As  $\eta$  increases, the wave functions become continuously squeezed until they become concentrated along their respective positive light-cone axes. Let us look at the  $z$ -axis projection of the space-time wave function. Indeed, the width of the quark distribution increases as the hadronic speed approaches that of the speed of light. The position of each quark appears widespread to the observer in the laboratory frame, and the quarks appear like free particles.

Furthermore, interaction time of the quarks among themselves become dilated. Because the wave function becomes widespread, the distance between one end of the harmonic oscillator well and the other end increases as is indicated in Figure 4. This effect, first noted by Feynman [16], is universally observed in high-energy hadronic experiments. Let us look at the time ratio more carefully. The period of oscillation increases like  $e^\eta$  as was predicted by Feynman [16].

On the other hand, the quark's interaction time with the external signal decreases



**Figure 5.** Lorentz-squeezed space-time and momentum-energy wave functions. As the hadron's speed approaches that of light, both wave function become concentrated along their respective positive light-cone axes. These light-cone concentrations lead to Feynman's parton picture.

as  $e^{-\eta}$  as was predicted by Gribov *et al* [3]. In the picture of the Lorentz squeezed hadron given in Figure 4, the hadron moves along the  $u$  (positive light-cone) axis, while the external signal moves in the direction opposite to the hadronic momentum, which corresponds to the  $v$  (negative light-cone) axis. This time interval is proportional to the minor axis of the ellipse given in Figure 4.

If we use  $T_{\text{ext}}$  and  $T_{\text{osc}}$  for the quark's interaction time with external signal and the interaction time among the quarks, their ratio becomes

$$\frac{T_{\text{ext}}}{T_{\text{osc}}} = \frac{\exp(-\eta)}{\exp \eta} = \exp(-2\eta). \quad (20)$$

The ratio of the interaction time to the oscillator period becomes  $e^{-2\eta}$ . The energy of each proton coming out of the Fermilab accelerator is 900 GeV. This leads the ratio to  $10^{-6}$ . This is indeed a small number. The external signal is not able to sense the interaction of the quarks among themselves inside the hadron. Thus, the quarks are

free particles for the external signal. This is the cause of incoherence in the parton interaction amplitudes.

The momentum-energy wave function is just like the space-time wave function in the oscillator formalism. The longitudinal momentum distribution becomes widespread as the hadronic speed approaches the velocity of light. This is in contradiction with our expectation from nonrelativistic quantum mechanics that the width of the momentum distribution is inversely proportional to that of the position wave function. Our expectation is that if the quarks are free, they must have their sharply defined momenta, not a widespread distribution. This apparent contradiction presents to us the following two fundamental questions:

- (1) If both the spatial and momentum distributions become widespread as the hadron moves, and if we insist on Heisenberg's uncertainty relation, is Planck's constant dependent on the hadronic velocity?
- (2) Is this apparent contradiction related to another apparent contradiction that the number of partons is infinite while there are only two or three quarks inside the hadron?

The answer to the first question is "No", and that for the second question is "Yes". Let us answer the first question which is related to the Lorentz invariance of Planck's constant. If we take the product of the width of the longitudinal momentum distribution and that of the spatial distribution, we end up with the relation

$$\langle z^2 \rangle \langle q_z^2 \rangle = \frac{1}{4} [\cosh(2\eta)]^2. \quad (21)$$

The right-hand side increases as the velocity parameter increases. This could lead us to an erroneous conclusion that Planck's constant becomes dependent on velocity. This is not correct, because the longitudinal momentum variable  $q_z$  is no longer conjugate to the longitudinal position variable when the hadron moves.

In order to maintain the Lorentz-invariance of the uncertainty product, we have to work with a conjugate pair of variables whose product does not depend on the velocity parameter. Let us go back to equation (17) and equation (18). It is quite clear that the light-cone variable  $u$  and  $v$  are conjugate to  $q_u$  and  $q_v$  respectively. It is also clear that the distribution along the  $q_u$  axis shrinks as the  $u$ -axis distribution expands. The exact calculation leads to

$$\langle u^2 \rangle \langle q_u^2 \rangle = \frac{1}{4} \quad \langle v^2 \rangle \langle q_v^2 \rangle = \frac{1}{4}. \quad (22)$$

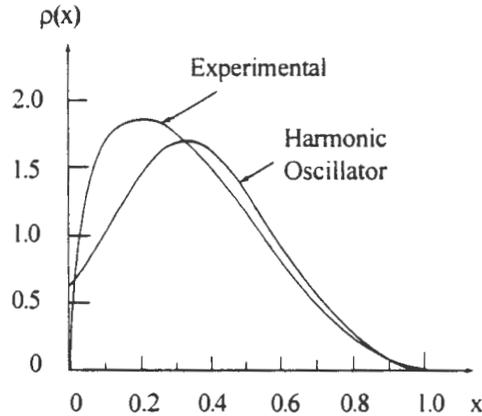
Planck's constant is indeed Lorentz-invariant.

Let us next resolve the puzzle of why the number of partons appears to be infinite while there are only a finite number of quarks inside the hadron. As the hadronic speed approaches the speed of light, both the  $x$  and  $q$  distributions become concentrated along the positive light-cone axis. This means that the quarks also move with velocity very close to that of light. Quarks in this case behave like massless particles.

We then know from statistical mechanics that the number of massless particles is not a conserved quantity. For instance, in black-body radiation, free light-like particles have a widespread momentum distribution. However, this does not contradict the known principles of quantum mechanics, because the massless photons can be divided into infinitely many massless particles with a continuous momentum distribution.

Likewise, in the parton picture, massless free quarks have a widespread momentum

distribution. They can appear as a distribution of an infinite number of free particles. These free massless particles are the partons. It is possible to measure this distribution in high-energy laboratories, and it is also possible to calculate it using the covariant harmonic oscillator formalism. We are thus forced to compare these two results [22]. Figure 6 shows the result.



**Figure 6.** Calculation of the parton distribution based on the harmonic oscillator wave function. It is possible to construct the covariant harmonic oscillator wave functions for the three-quark system, and compare the parton distribution function with experiment. This graph shows a good agreement between the oscillator-based theory and the observed experimental data.

## 6. Concluding remarks

This is largely a review paper, but it contains the following new observation. Let us go to the time ratio given in equation (20). It is a product of two identical numbers. The factor given by Feynman's time dilation effect is  $e^{-\eta}$ . The ratio given by the time contraction effect of Gribov *et al* is also  $e^{-\eta}$ . Thus the combined effect is  $e^{-2\eta}$ . This combined effect makes the parton amplitudes to lose coherence even at moderate hadronic speed.

Another noteworthy point is that Wigner's little group is not only an abstract concept, but also serves as a computational tool in high-energy physics. The covariant harmonic oscillator is one of the tools derivable from the concept of little groups. It is interesting to note that the covariant oscillator formalism gives both Feynman's time dilation and the time contraction of Gribov *et al*.

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