2011 International Conference on Multimedia Technology (ICMT 2011)

Volume 7 . Part II

July 26-28, 2011

Hangzhou, China

IEEE Catalog Number: CFP1153K-PRT

ISBN: 978-1-61284-772-6

PROCEEDINGS

2011 International Conference on Multimedia Technology

IEEE Catalog Number: CFP1153K-PRT
ISBN : 978-1-61284-772-6

Additional copies of this publication are available from:
Curran Associates, Inc.
57 Morehouse Lane
Red Hook, NY 12571 USA
Phone: (845) 758-0400
Fax: (845) 758-2633
E-mail: curran@proceedings.com

Einstein's Hydrogen Atom

Y. S. Kim
Maryland Center for Fundamental Physics,
University of Maryland, College Park, Maryland 20742, USA
Email: yskim@umd.edu

Abstract—**In 1905, Einstein formulated his special relativity for point particles. For those particles, his Lorentz covariance and energy-momentum relation are by now firmly established. How about the hydrogen atom? It is possible to perform Lorentz boosts on the proton assuming that it is a point particle. Then what happens to the electron orbit? The orbit could go through an elliptic deformation, but it is not possible to understand this problem without quantum mechanics, where the orbit is a standing wave leading to a localized probability distribution. Is this concept consistent with Einstein's Lorentz covariance? Dirac, Wigner, and Feynman contributed important building blocks for understanding this problem. The remaining problem is to assemble those blocks to construct a Lorentz-covariant picture of quantum bound states based on standing waves. It is shown possible to assemble those building blocks using harmonic oscillators.**

KEYWORDS: **Quantum bound states, Lorentz covariance.**

I. Introduction

Niels Bohr had a great respect for Einstein, and he adds "time" whenever he mentions "space" in his philosophical writings. However, for his hydrogen atom, the proton was sitting at the center of the absolute frame. Einstein presumably thought about how the hydrogen atom would look to a moving observer, but he never raised the issue. The reason is that the hydrogen atom moving with a relativistic speed was not conceivable for them.

Things are different these days. Protons can move with a speed close to the light speed. In addition, like the hydrogen atom, the proton is a bound state of the more fundamental particles called the "quarks." The proton thus has the same quantum mechanical ingredients as the hydrogen atom has. We can therefore study the hydrogen atom in Einstein's world by studying the proton in high-energy physics. This historical trend is illustrated in Fig. 1.

Without the quark model, Paul A. M. Dirac devoted much of research life to the problem of constructing Lorentz-covariant wave functions. He published four papers on this problem from 1927 to 1963 [1], [2], [3], [4]. We shall construct the bound-state model by combining these four papers.

In order to do this, we have to understand the symmetry problems for bound-state problems. In 1939, Eugene Wigner worked out the internal space-time symmetries of relativistic particles [5]. In so doing he worked out the symmetries of bound states in the Lorentz-covariant world [6].

Richard Feynman invented Feynman diagrams, but he said in 1970 that we should use harmonic oscillators, instead of Feynman diagrams, for understanding bound state problems in

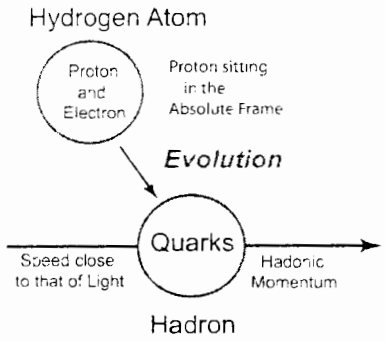

Fig. 1. Evolution of the hydrogen atom. It is still not possible to accelerate the hydrogen atom to a relativistic speed. It is however possible these days to produce protons moving with a speed close to the light speed. Also these days, the proton is thought to be a bound state of quarks. It is thus sufficient to study high-energy protons to study the hydrogen atom in Einstein's world.

the Lorentz-covariant world cite7. He then published a paper saying the same with his students in 1971 [8]. The difficulty of using the S-matrix for boundstates had been noted before [9].

In Sec. II, we list Dirac's four papers, and point out what he did and what he could have don in these papers. We do the same for Feynman's three papers in Sec. III. In Sec IV, it was noted first that space-time symmetry of quantum bound states is simpler than the full-fledged Lorentz group. Unlike Klein-Gordon waves, the symmetry of standing wave is that of the three-dimensional rotation group [5]. This point is missing in Dirac's papers and Feynman's 1971 paper [8]. It is noted also that that the covariant harmonic oscillators satisfy all the required symmetries.

We then discuss the essential features of the oscillator formalism which describes the effect of the proton wave function under Lorentz boot. It is shown that the wave function becomes "squeezed" when it is boosted.

It is then shown in Sec. V that this squeeze effect manifests itself in Feynman's parton picture for the proton moving with a speed close to that of light. We establish that the quark model and the parton model are two different manifestations of one Lorentz-covariant model of quantum bound states. This is what Einstein's hydrogen atom is about.

II. Dirac's Four Papers

Paul A. M. Dirac devoted much of his research efforts to making quantum mechanics consistent with special relativity.

978-1-61284-774-0/11/$26.00 ©2011 IEEE

- In his 1927 paper on time-energy uncertainty relation [1], Dirac noted that there are no quantum excitations along the time variable, unlike Heisenberg's position-momentum relation. He said this space-time asymmetry makes the problem difficult.
- In 1945 [2], Dirac attempted to construct harmonic oscillator wave functions which can be Lorentz-boosted. He wrote down the Gaussian form

$$\exp\left[-\frac{1}{2}\left(x^2 + y^2 + z^2 - t^2\right)\right], \tag{1}$$

but did not explain the physics of the Gaussian distribution in the time variable.
- In 1949 [3], he started with the Lorentz transformation

$$\begin{pmatrix} z' \\ t' \end{pmatrix} = \begin{pmatrix} \cosh\eta & \sinh\eta \\ \sinh\eta & \cosh\eta \end{pmatrix} \begin{pmatrix} z \\ t \end{pmatrix}. \tag{2}$$

He then introduced the light-cone variables

$$u = \frac{z - t}{\sqrt{2}}, \qquad v = \frac{z + t}{\sqrt{2}}. \tag{3}$$

In terms of these variables, the Lorentz boost takes the form He then diagonalize this equation to

$$u' = e^{\eta}u, \qquad v' = e^{-\eta}v. \tag{4}$$

These light-cone variables serve very useful purposes. Here one coordinate expands and the other contracts. Thus, the Lorentz boost is a squeeze transformation [10]. In the same paper, Dirac stated that the problem of constructing relativistic dynamics is the same as that of constructing a suitable representation of the Poincaré group. In his earlier paper [2], Dirac started this work using harmonic oscillators, but he did not elaborate on this in his 1949 paper.
- In 1963 [4], Dirac used two harmonic oscillators to construct the $O(3,2)$ deSitter group, which is a Lorentz group applicable to thee space-like and two time-like coordinates. This representation later became the mathematical basis for two-mode squeezed states in quantum optics [11], [12], and became a bridge between special relativity and optical sciences.

In the present paper, we address these soft spots in these papers according to Dirac's own suggestion: to construct the representation of the Poincaré group using harmonic oscillators [2], [3]. Dirac missed this point again in his 1963 paper [4] while he was constructing the representation of the $O(3,2)$ group which contains the Lorentz group $O(3,1)$ as a subgroup.

We can remove these soft spots by constructing Wigner's little groups [5] of the Poincaré group using harmonic oscillators [6], [13].

III. Feynman's Three Papers

Richard Feynman made important contributions in many different branches of physics. In the following three papers, he left some important questions as home work problems for younger generations.

- In 1969 [14], [15], Feynman introduced the concept of partons. If the proton moves with a velocity close to that of light, it appears like a collection of partons whose properties are quite different from the quarks which are constituent particles inside the proton at rest. The question then is whether the quarks and partons are two different manifestation of one Lorentz-covariant entity.
- In 1970, Feynman gave a talk at the spring meeting of the American physical Society held in Washington. He started with hadrons which are bound states of quark [16]. He noted that the hadronic spectra could best be understood in terms of the three-dimensional harmonic oscillators. As for the Lorentz covariant aspect of his oscillator formalism, he pointed out that there is the time separation between the quarks. However, since he did not know what to do with it, he chose to ignore the variable. He then published the content of this talk with his students in 1971 [8]. He did not justify what he did on this time separation variable.
- In his book on statistical mechanics published in 1972 [17], Feynman discussed density matrices and measurement problems. He stated *When we solve a quantum-mechanical problem, what we really do is divide the universe into two parts - the system in which we are interested and the rest of the universe. We then usually act as if the system in which we are interested comprised the entire universe. To motivate the use of density matrices, let us see what happens when we include the part of the universe outside the system.*

  Feynman then used one harmonic oscillator to illustrate his rest of the universe. The question is how one oscillator can explain both the real world and the rest of the universe. He could have used two coupled oscillators to illustrate his his rest of the universe, but he left this problem as a homework problem for us [18].

In these three papers, Feynman raised very fundamental issues in physics, but did not provide complete solutions. The issue on his rest of the universe has been discussed in the literature in terms of the coupled oscillators [18], and also in terms of the time-separation variable in the Lorentz-covariant world [19].

In the present paper, we are interested in addressing the soft spots in Feynman's 1969 papers on the parton picture and those in his 1971 paper on harmonic oscillators. As in the case of Dirac, it is possible to transform Feynman's oscillator formalism into the representation of Wigner's little group using harmonic oscillators [6], [13].

IV. Covariant Harmonic Oscillators

In Sec. II and Sec. III, we stated that it is possible to remove the soft spots in Dirac's four papers and Feynman's three papers by constructing Wigner's little groups. These little groups are the subgroups of the Poincaré group whose transformations leave the four-momentum of a given particle invariant [5], [6]. For a massive particle, we can consider the

Lorentz frame where this particle is at rest. In this frame, the space-time symmetry is the three-dimensional rotation group.

In dealing with plane waves, we start with the Klein-Gordon equation. The solutions of this equation are Lorentz-invariant. The running waves in the Lorentz-covariant world share the same symmetry property as that of the Klein-Gordon waves. It contains the full symmetry of the Poincaré group with ten interdependent paperers. These aspects of the space-time symmetry is illustrated in Fig. 2. This figure describes the space-time symmetry of Einstein's hydrogen atom given in Fig. 1.

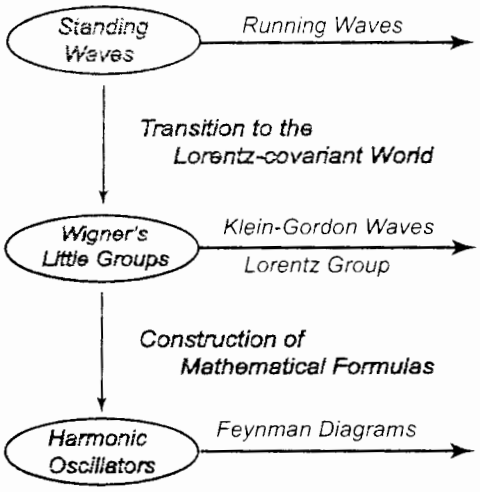

Fig. 2. Running waves and standing waves in quantum theory. If a particle is allowed to travel from infinity to infinity, it corresponds to a running wave according to the wave picture of quantum mechanics. If, on the other hand, it is trapped in a localized region, we have to use standing waves to interpret its location in terms of probability distribution.

Since the internal space-time symmetry is like the three-dimensional rotation group, the standing waves trapped within a quantum bound state should also satisfy this symmetry. It is important to note that we are dealing here with space-time separations. For instance, the Bohr radius is the separation between the proton and electron. One of the soft spots in Dirac's four papers is that Dirac did not clarify this separation issue. The soft spots in both Dirac's papers and Feynman's 1971 paper [8] is that the time-like direction is not required in Wigner's three-dimensional space.

Thus, Dirac's concern about the space-time asymmetry is not necessary. Feynman *et al.* said they wanted to drop the time-like variable because they do not know what to do with it. They did not know they were right. They did not have to do anything about what does not exist.

Then, our next problem is to build a model of bound states satisfying Wigner's $O(3)$-like symmetry, which is consistent with Einstein's Lorentz covariance. As was noted by Feynman [7], the easiest way is to start with harmonic oscillators. The oscillator system does not require additional boundary conditions. Indeed, before the paper of Feynman *et al.*, a number of authors published their papers on this subject [20], [22], [23], [24].

According to Gell-Mann [16], the proton is a bound state of two quarks, but we consider here for simplicity a bound state of two quarks. As is the case of Feynman *et al.*, we start with the two quarks whose space-time positions are x_a and x_b. Then the standard procedure is to use the variables

$$X = (x_a + x_b)/2, \qquad x = (x_a - x_b)/2\sqrt{2}. \tag{5}$$

The four-vector X specifies where the proton is located in space and time, while the variable x measures the space-time separation between the quarks. This x variable has four components, but it has only three degrees of freedom according to Wigner's symmetry. This will appear as the lack of excitations along the time-like direction as noted by Dirac [1], [3].

Does this time-separation variable exist when the proton is at rest? Yes, according to Einstein. In the present form of quantum mechanics, we pretend not to know anything about this variable. Indeed, this variable belongs to Feynman's rest of the universe [19].

Also in the present form of quantum mechanics, there is an uncertainty relation between the time and energy variables. However, there are no known time-like excitations. Unlike the position or momentum variable, the time-separation variable is c-number, and its uncertainty with the energy separation a c-number uncertainty relation [1]. With this point in mind, let us go to the oscillator formalism proposed by Feynman [7], [8].

Feynman *et al.* start with the Lorentz-invariant differential equation [8]

$$\frac{1}{2}\left\{x_\mu^2 - \frac{\partial^2}{\partial x_\mu^2}\right\}\psi(x) = \lambda\psi(x). \tag{6}$$

This partial differential equation has many different solutions depending on the choice of separable variables and boundary conditions. Feynman *et al.* insist on Lorentz-invariant solutions which are not normalizable. On the other hand, if we insist on normalization, the ground-state wave function takes the form of Eq.(1), which now can be written as

$$\psi(z,t) = \exp\left[-\frac{1}{2}(z^2 - t^2)\right]. \tag{7}$$

where we dropped the transverse components of x and y. As in the case of Eq.(2), we make Lorentz boosts along the z. We dropped also the normalization constant for simplicity. In terms of the light-cone variables, this wave function becomes

$$\psi(u,v) = \exp\left[-\frac{1}{2}(u^2 - v^2)\right]. \tag{8}$$

If the system is boosted, the u and v variables are replaced by $u\,e^{-\eta}$ and $v\,e^{\eta}$ respectively. The wave function then

becomes

$$\exp\left[-\frac{1}{2}\left(e^{-2\eta}u^2 \quad e^{2\eta}v^2\right)\right],$$

$$\exp\left[-\frac{1}{4}\left(e^{-2\eta}(z \quad t)^2 \quad e^{2\eta}(z-t)^2\right)\right]. \qquad (9)$$

The wave function satisfied the Lorentz-invariant differential equation of Eq.(6). This wave function is expanded along the u direction, while it becomes contracted along the v direction. This aspect of the Lorentz-squeeze is illustrated in Fig. 3.

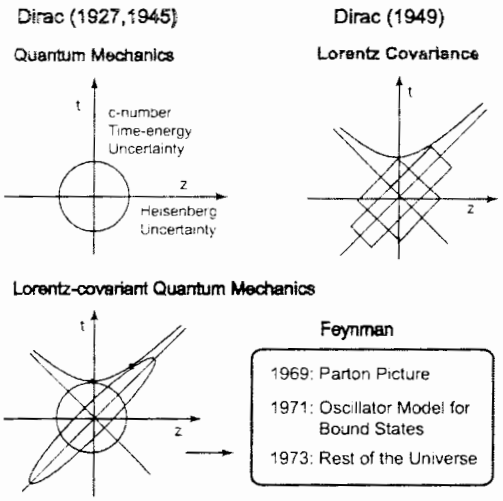

Fig. 3. Space-time picture of quantum mechanics. There are quantum excitations along the space-like longitudinal direction, but there are no excitations along the time-like direction. The time-energy relation is a c-number uncertainty relation.

Let us go back to the Gaussian form of Eq.(7). If we allow excitations along the z direction while keeping the t component in its ground state, the wave function takes the form

$$\psi_n(z,t) \quad \exp\left[-\frac{1}{2}\left(z^2 \quad t^2\right)\right] H_n(z). \qquad (10)$$

where H_n is a Hermite polynomial. This wave function satisfies Dirac's c-number time-energy uncertainty relation. It can also be Lorentz-boosted in the same manner as the ground-sate wave function of Eq.(7) becomes its squeezed form of Eq.(9). This aspect of the Lorentz-covariant c-number time-energy uncertainty relation is discussed in the literature [31], [32].

Since the oscillator system is separable in the Cartesian coordinate system, the Gaussian form of Eq.(7) can be restored to its dimensional form of Eq.(1). This form can allow excitations along the transverse directions of x and y. We we add the Hermite polynomials in along these components, this wave function can possess the symmetry under rotations in the three-dimensional space. This is the content of Wigner's $O(3)$-like little group applicable to this system. These transverse excitation remain invariant when the system is boosted. This aspect has also been discussed in the literature [13].

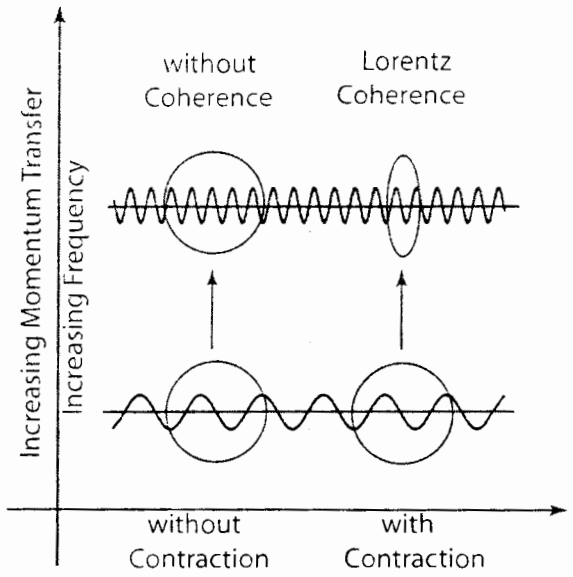

Fig. 4. Coherence between the wavelength and the proton size. As the momentum transfer increases, the external signal sees Lorentz-contracting proton distribution. On the other hand, the wavelength of the signal also decreases. Thus, the cutoff is not as severe as the case where the proton distribution is not contracted.

This elliptic squeeze is consistent with the Lorentz contraction along the longintudinal direction according to Einstein's special relativity. However, it raises a new wuestion of how to deal with the the time-separation variable which becomes more prominet as the proton picks up the speed [10], [19].

As for the experimental side of this Lorentz squeeze, this problem was studied in connection with the proton form factor for high momentum transfer in electron-proton scattering. The early authors attempted to explain the dipole cut-off behavior using the oscillator formalism presented in this section [22], [23], [24].

There is still more work to be done. For instance, the effect of the quark spin should be addressed [25], [26]. Also there are reports of deviations from the exact dipole cut-off [27]. There have been attempts to study the form factors based on the four-dimensional rotation group [28], and also on the lattice QCD [29]. p Indeed, this form factor behavior is one of the centrail issues in high-energy physics.

Yet, it is gratifying to note that the effect of Lorentz squeeze leads to the polynomial decrease in the momentum transfer, thanks to the Lorentz coherence illustrated in Fig. 4. This aspect of coherence probem has been discussed in the literature [6], [10], [30].

V. Feynman's Parton Picture

It is a widely accepted view that the hadrons are quantum bound states of quarks with localized probability distributions. As in all bound-state cases, this localization condition is responsible for the existence of discrete mass spectra. The most convincing evidence for this bound-state picture is the hadronic mass spectra which are observed in high-energy laboratories [6], [8]. The proton is one of those hadrons.

In 1969, Feynman observed that a fast-moving proton can be regarded as a collection of many "partons" whose properties appear to be quite different from those of the quarks [15]. For example, the number of quarks inside a static proton is three, while the number of partons in a rapidly moving proton appears to be infinite. The question then is how the proton looking like a bound state of quarks to one observer can appear different to an observer in a different Lorentz frame? Feynman made the following systematic observations.

a. The picture is valid only for protons moving with velocity close to that of light.
b. The interaction time between the quarks becomes dilated, and partons behave as free independent particles.
c. The momentum distribution of partons becomes widespread as the proton moves fast.
d. The number of partons appear to be infinite or much larger than that of quarks.

Because the proton is believed to be a bound state of two or three quarks, each of the above phenomena appears as a paradox, particularly b) and c) together.

In order to resolve this paradox, let us consider the momentum-energy wave function for this two-quark system. If we let the quarks have the four-momenta p_a and p_b, it is possible to construct two independent four-momentum variables [8]

$$P \quad p_a \quad p_b, \qquad q \quad \sqrt{2}(p_a - p_b). \tag{11}$$

where P is the total four-momentum. It is the proton four-momentum.

The variable q measures the four-momentum separation between the quarks. Their light-cone variables are

$$q_u \quad (q_0 \quad q_z)/\sqrt{2}, \qquad q_v \quad (q_0 - q_z)/\sqrt{2}. \tag{12}$$

The resulting momentum-energy wave function is

$$\phi_\eta(q_z, q_0) \quad \exp\left[-\frac{1}{2}\left(\epsilon^{-2\eta} q_u^2 \quad \epsilon^{2\eta} q_v^2\right)\right]. \tag{13}$$

Because we are using here the harmonic oscillator, the mathematical form of the above momentum-energy wave function is identical to that of the space-time wave function. The Lorentz squeeze properties of these wave functions are also the same. This aspect of the squeeze has been exhaustively discussed in the literature [6], [33], [34].

When the proton is at rest with $\eta \quad 0$, both wave functions behave like those for the static bound state of quarks. As η increases, the wave functions become continuously squeezed until they become concentrated along their respective positive light-cone axes. Let us look at the z-axis projection of the space-time wave function. Indeed, the width of the quark distribution increases as the proton's speed approaches that of the speed of light. The position of each quark appears widespread to the observer in the laboratory frame, and the quarks appear like free particles.

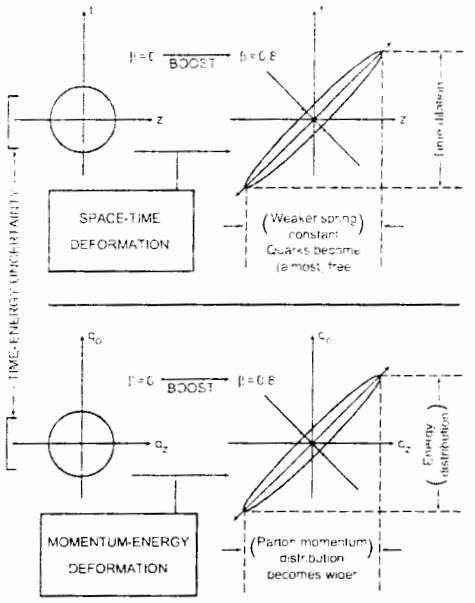

Fig. 5. Lorentz-squeezed space-time and momentum-energy wave functions. As the proton's speed approaches that of light, both wave functions become concentrated along their respective positive light-cone axes. These light-cone concentrations lead to Feynman's parton picture.

The momentum-energy wave function is just like the space-time wave function, as is shown in Fig. 5. The longitudinal momentum distribution becomes wide-spread as the proton's speed approaches the velocity of light. This is in contradiction with our expectation from non-relativistic quantum mechanics that the width of the momentum distribution is inversely proportional to that of the position wave function. Our expectation is that if the quarks are free, they must have their sharply defined momenta, not a wide-spread distribution.

However, according to our Lorentz-squeezed space-time and momentum-energy wave functions, the space-time width and the momentum-energy width increase in the same direction as the proton is boosted. This is of course an effect of Lorentz covariance. This indeed is the key to the resolution of the quark-parton paradox [6], [33], [34].

Feynman's parton picture is one of the most controversial physical models proposed in the 20th century. The original model is valid only in Lorentz frames where the initial proton moves with infinite momentum. It is gratifying to note that this model can be produced as a limiting case of one covariant

model which produces the quark model in the frame where the proton is at rest. We need Feynman's parton model to complete the third row of Table I.

TABLE I

MASSIVE AND MASSLESS PARTICLES IN ONE PACKAGE. EINSTEIN UNIFIED THE ENERGY-MOMENTUM RELATION FOR SLOW (MASSIVE) AND FAST (MASSLESS) PARTICLES WITH ONE LORENTZ-COVARIANT FORMULA. LIKEWISE, CAN THE QUARK MODEL AND THE PARTON MODEL CAN BE COMBINED INTO ONE LORENTZ-COVARIANT? THE ANSWER IS YES.

	Massive Slow	Lorentz Covariance	Massless Fast
Energy-Momentum	$E = p^2/2m$	Einstein's $E = (p^2 + m^2)^{1/2}$	$E = p$
Relativistic Extended Particles	Quark Model	One Covariant Theory	Parton Model

CONCLUDING REMARKS

Since 1973 [30], mostly with Marilyn Noz, I have been publishing papers on constructing a model of quantum bound states in Einstein's Lorentz-covariant world. In 1986 [6], we published a book on this subject. Of course, we were not the first ones to study this problem.

It was noted first that Dirac and Feynman made pivotal contributions. However, they looked at the same problem differently in their papers. It was seen in the present report that their results can become much stronger if they are combined into one paper. During this process, Wigner's 1939 paper [5] plays the essential role.

Here, the key word is "harmony." The works of those great physicists can be put together in harmony. I am very happy to mention this point in China, where the concept of harmony was formulated many centuries ago through the philosophy of "Taoism."

As for Einstein, let us go to Table I. This is a table on harmony. Observers in different Lorentz frames see things differently, but they are in harmony. Then, did Einstein study the oriental philosophy of Taoism? I do not know.

However, it is well known that he studied the philosophy of Immanuel Kant in his early years. It is also known that his formulation of relativity was influenced by Kant's view of the world. Different observers can see differently one thing which is called "Ding an Sich" by Kant.

What is Ding an Sich?. A Coca-Cola can looks like a rectable to an observer who looks at its side. It is a circle viewed from the top. Here, the Coke can is the Ding an Sich.

Thus, according to Kant, Einstein's special relativity requires an absolute frame (Ding an Sich). This is not what Einstein wanted. In Table I, there are no places for Kant's Ding an Sich.

If not Kantianism, where is Einstein's philosophical base? How can the observers in two different Lorentz frames reconcile their differences? The answer to this question seems to lie within the framework of Taoism. We have to study more along this direction [35].

So far, the question of harmony has been restricted to Einstein's world of relativity. Our ultimate question is how quantum mechanics and relativity will be combined together in harmony.

REFERENCES

[1] P. A. M. Dirac, Proc. Roy. Soc. (London) **A114**, 284 (1927).
[2] P. A. M. Dirac, Proc. Roy. Soc. (London) **A183**, 284 (1945).
[3] P. A. M. Dirac, Rev. Mod. Phys. **21**, 392 (1949).
[4] P. A. M. Dirac, J. Math. Phys. **4**, 901 (1963).
[5] E. Wigner, Ann. Math. **40**, 149 (1939).
[6] Y. S. Kim and M. E. Noz, *Theory and Applications of the Poincaré Group* (Reidel, Dordrecht, 1986).
[7] R. P. Feynman, Invited talk presented at the *1970 Washington meeting of the American Physical Society* (Washington, DC, U.S.A.).
[8] R. P. Feynman, M. Kislinger, and F. Ravndal, Phys. Rev. D **3**, 2706 (1971).
[9] The basic difficulty with using plane waves for bound-state problems had been noted earlier in connection with the the calculation of the neutron-proton mass difference by Dashen and Frautschi. See R. F. Dashen and S. C. Frautschi, Phys. Rev. **135**, B1190 and B1196 (1964); F. J. Dyson, Physic Today **18**, No. 6, 21 (1965); Y. S. Kim, Phys. Rev. **142**, 1150 (1966).
[10] Y. S. Kim and M. E. Noz, Symmetry **3**, 16 (2011).
[11] H. P. Yuen, Phys. Rev. A **13**, 2226 (1976).
[12] J. R. Klauder, S. L. McCall, and B. Yurke, Phys. Rev. A **33**, 3204 (1986).
[13] Y. S. Kim, M. E. Noz, S. H. Oh, J. Math. Phys. **20**, 1341 (1979).
[14] R. P. Feynman, Phys. Rev. Lett. **23**, 1415 (1969).
[15] R. P. Feynman, *The Behavior of Hadron Collisions at Extreme Energies*, in *High Energy Collisions*, Proceedings of the Third International Conference, Stony Brook, New York, edited by C. N. Yang *et al.*, Pages 237-249 (Gordon and Breach, New York, 1969).
[16] M. Gell-Mann, Phys. Lett. **13**, 598 (1964).
[17] R. P. Feynman, *Statistical Mechanics* (Benjamin/Cummings, Reading, MA, (1972).
[18] D. Han, Y. S. Kim, and M. E. Noz, Am. J. Phys. **67** 61 (1999).
[19] Y. S. Kim and E. P. Wigner, Phys. Lett. A **147**, 343 (1990).
[20] H. Yukawa, Phys. Rev. **91**, 415 (1953).
[21] M. Markov, Suppl. Nuovo Cimento **3**, 760 (1956).
[22] V. L. Ginzburg, V. I. Man'ko, Nucl. Phys. **74**, 577 (1965).
[23] K. Fujimura, T. Kobayashi, M. Namiki, Prog. Theor. Phys. **1970**, 73 (1970)
[24] A. L. Licht, A. Pagnamenta, Phys. Rev. D **2**, 1150 (1970).
[25] R. Lipes, Phys Rev. D **5**, 2849 (1972).
[26] A. B. Henriques, B.H. Keller, R. G. Moorhouse, Ann. Phys. (NY) **93**, 125 (1975).
[27] V. Punjabi, C. F. Perdrisat, *et al.* Phys. Rev. C **71**, 055202-27 (2005). Erratum-*ibid.*, Phys. Rev. C **71**, 069902 (2005)
[28] R. Alkofer, A. Holl, M. Kloker, A. Karssnigg; C. D. Roberts, On Nucleon Electromagnetic Form Factors, Few-Body Systems **37**, 1 (2005).
[29] H. H. Matevosyan, A. W. Thomas, G. A. Miller, Phys.Rev. C **72**, 065204 (2005).
[30] Y. S. Kim and M. E. Noz, Phys. Rev. D **8**, 3521 (1973).
[31] D. Han, Y. S. Kim, M. E. Noz, and D. Son, Phys. Rev. D **27**, 3032 (1983).
[32] Y. S. kim and M. E. Noz, Am. J. Phys. **53**, 142 (1985).
[33] Y. S. Kim and M. E. Noz, Phys. Rev. D **15**, 335 (1977).
[34] Y. S. Kim, Phys. Rev. Lett. **63**, 348 (1989).
[35] Y. S. Kim, *Einstein, Kant, and Taoism* ArXiv http://arxiv.org/abs/physics/0604027 (2006).