

# The Question of Simultaneity in Relativity and Quantum Mechanics

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**Abstract.** In relativity, two simultaneous events at two different places are not simultaneous for observers in different Lorentz frames. In the Einstein-Podolsky-Rosen experiment, two simultaneous measurements are taken at two different places. Would they still be simultaneous to observers in moving frames? It is a difficult question, but it is still possible to study this problem in the microscopic world. In the hydrogen atom, the uncertainty can be considered to be entirely associated with the ground-state. However, is there an uncertainty associated with the time-separation variable between the proton and electron? This time-separation variable is a forgotten, if not hidden, variable in the present form of quantum mechanics. The first step toward the simultaneity problem is to study the role of this time-separation variable in the Lorentz-covariant world. It is shown possible to study this problem using harmonic oscillators applicable to hadrons which are bound states of quarks. It is also possible to derive consequences that can be tested experimentally.

**Keywords:** simultaneity, uncertainty, time separation variable, partons, Feynman's "rest of the universe"

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## INTRODUCTION

In his book entitled "Encounters with Einstein" [1], Heisenberg states that the mathematics of Lorentz transformations was easy to understand and appreciate, but the concept of simultaneity in Einstein's relativity was difficult to grasp. Heisenberg had this problem before he formulated his uncertainty relation, and the concept of simultaneity plays a pivotal role in the interpretation there. Is Heisenberg's simultaneity consistent with Einstein's simultaneity?

When we talk about simultaneous measurements, we are uncritical about whether those measurements are taken at the same place or different places. In the EPR-type experiments [2], two simultaneous measurements are taken at two different places. Would these two measurements appear simultaneous to an observer on a bicycle? We do not know where the story stands on this issue, because the problem includes both macroscopic and microscopic scales. This involves localization problems, in addition to the simultaneity issue. We are not able to provide a resolution to this problem in the present report.

On the other hand, we can study the problem in the microscopic scale. The radius of the ground-state hydrogen atom can be regarded as an uncertain quantity. There is a spacial separation between between the proton and electron. These two particles are located at different places. We have to measure the position of the proton and that of the electron to take the difference. In the present form of quantum mechanics, we assume that they are taken simultaneously because we never worry about the time separation between them. If we believe in Einstein's relativity, there is necessarily a time-separation variable between these two particles, and it will play a prominent role for observers in different Lorentz frames.

In order to approach the problem, let us go back to Heisenberg's problem. It is easier to understand the mathematics of the Lorentz transformation than the concept of simultaneity. It would thus be easier if we build the mathematical framework first. We may then be able to give physical interpretations, and also derive consequences derivable from the mathematical formalism. The modern version of the hydrogen atom is a bound-state of quarks, called the hadron. While there are no experimental data on hydrogen atoms moving with relativistic speed, the physics of hadrons involves bound states in the Lorentz-covariant world.

According to our experience, the present form of quantum mechanics is largely a physics of harmonic oscillators. Since the group consisting of two-by-two unimodular matrices, or  $SL(2, C)$ , forms the universal covering group of the Lorentz group, special relativity is a physics of two-by-two matrices. Therefore, the coupled harmonic oscillator can

provide a concrete model for relativistic quantum mechanics.

With this point in mind, Dirac and Feynman used harmonic oscillators to test their physical ideas. In this paper, we first examine Dirac's attempts to combine quantum mechanics with relativity in his own style: to construct mathematically appealing models. We then examine how Feynman approached this problem. He insisted on his own style: observe the experimental world, tell the story of the real world, and then write down mathematical formulas as needed.

In this paper, we use coupled harmonic oscillators to build a bridge between the two different attempts made by Dirac and Feynman. The coupled oscillator system not only connects the ideas of these two great physicists, but also serves as an illustrative tool for some of the current ideas in physics, such as entanglement and decoherence.

As for observable consequences of the oscillator formalism which connects Dirac and Feynman, we would like to discuss in this report Feynman's parton picture which is valid in the Lorentz frame in which hadronic speed is close to that of light. It is widely believed that hadrons are bound states of quarks like the hydrogen atom when they are at rest. Then why are partons so different from the quarks inside the static hadron? We shall discuss how the time-separation variable plays the crucial role in resolving this quark-parton puzzle.

In Sec. , we start with the classical Hamiltonian for two coupled oscillators. It is possible to obtain an explicit solution for the Schrödinger equation in terms of the normal coordinates. We then derive a convenient form of this solution from which the concept of entanglement can be studied thoroughly. Section examines Dirac's life-long attempt to combine quantum mechanics with special relativity. In Sec. , we study some of the problems which Dirac left us to solve. In Sec. , we construct a covariant model of relativistic extended particles by combining Dirac's oscillators with Feynman's phenomenological approach to relativistic quark model. It is shown that Feynman's parton model can be interpreted as a limiting case of one covariant model for a covariant bound-state model.

## COUPLED OSCILLATORS AND ENTANGLED OSCILLATORS

Two coupled harmonic oscillators serve many different purposes in physics. It is well known that this oscillator problem can be formulated into a problem of a quadratic equation in two variables. The diagonalization of the quadratic form includes a rotation of the coordinate system. However, the diagonalization process requires additional transformations involving the scales of the coordinate variables [3, 4]. Indeed, it was found that the mathematics of this procedure can be as complicated as the group theory of Lorentz transformations in a six dimensional space with three spatial and three time coordinates [5].

However, in this paper, we start with a simple problem of two oscillators with equal mass. This contains enough physics for our present purpose. Then the Hamiltonian takes the form

$$H = \frac{1}{2} \left\{ \frac{1}{m} p_1^2 + \frac{1}{m} p_2^2 + Ax_1^2 + Ax_2^2 + 2Cx_1x_2 \right\}. \quad (1)$$

If we choose coordinate variables

$$\begin{aligned} y_1 &= \frac{1}{\sqrt{2}}(x_1 + x_2), \\ y_2 &= \frac{1}{\sqrt{2}}(x_1 - x_2), \end{aligned} \quad (2)$$

the Hamiltonian can be written as

$$H = \frac{1}{2m} \{p_1^2 + p_2^2\} + \frac{K}{2} \{e^{-2\eta}y_1^2 + e^{2\eta}y_2^2\}, \quad (3)$$

where

$$\begin{aligned} K &= \sqrt{A^2 - C^2}, \\ \exp(2\eta) &= \sqrt{\frac{A-C}{A+C}}, \end{aligned} \quad (4)$$

The classical eigenfrequencies are  $\omega_{\pm} = \omega e^{\pm}$  with

$$\omega = \sqrt{\frac{K}{m}}. \quad (5)$$

If  $y_1$  and  $y_2$  are measured in units of  $(mK)^{1/4}$ , the ground-state wave function of this oscillator system is

$$\psi_{\eta}(x_1, x_2) = \frac{1}{\sqrt{\pi}} \exp \left\{ -\frac{1}{2} (e^{-\eta} y_1^2 + e^{\eta} y_2^2) \right\}, \quad (6)$$

The wave function is separable in the  $y_1$  and  $y_2$  variables. However, for the variables  $x_1$  and  $x_2$ , the story is quite different, and can be extended to the issue of entanglement.

There are three ways to excite this ground-state oscillator system. One way is to multiply Hermite polynomials for the usual quantum excitations. The second way is to construct coherent states for each of the  $y$  variables. Yet, another way is to construct thermal excitations. This requires density matrices and Wigner functions [4].

The key question is how the quantum mechanics in the world of the  $x_1$  variable is affected by the  $x_2$  variable. If the  $x_2$  space is not observed, it corresponds to Feynman's rest of the universe. If we use two separate measurement processes for these two variables, these two oscillators are entangled.

Let us write the wave function of Eq.(6) in terms of  $x_1$  and  $x_2$ , then

$$\psi_{\eta}(x_1, x_2) = \frac{1}{\sqrt{\pi}} \exp \left\{ -\frac{1}{4} [e^{-\eta} (x_1 + x_2)^2 + e^{\eta} (x_1 - x_2)^2] \right\}. \quad (7)$$

When the system is decoupled with  $\eta = 0$ , this wave function becomes

$$\psi_0(x_1, x_2) = \frac{1}{\sqrt{\pi}} \exp \left\{ -\frac{1}{2} (x_1^2 + x_2^2) \right\}. \quad (8)$$

The system becomes separable and becomes disentangled.

As was discussed in the literature for several different purposes [6, 7, 8], this wave function can be expanded as

$$\psi_{\eta}(x_1, x_2) = \frac{1}{\cosh \eta} \sum_k \left( \tanh \frac{\eta}{2} \right)^k \phi_k(x_1) \phi_k(x_2), \quad (9)$$

where  $\phi_k(x)$  is the harmonic oscillator wave function for the  $k$ -th excited state. This expansion serves as the mathematical basis for squeezed states of light in quantum optics [8], among other applications.

In addition, this expression clearly demonstrates that coupled oscillators are entangled oscillators. Let us look at the expression of Eq.(9). If the variable  $x_1$  and  $x_2$  are measured separately.

In Sec , we shall see that the mathematics of coupled oscillators can serve as the basis for the covariant harmonic oscillator formalism where the  $x_1$  and  $x_2$  variables are replaced by the longitudinal and time-like variables, respectively. This mathematical identity will lead to the concept of space-time entanglement in special relativity.

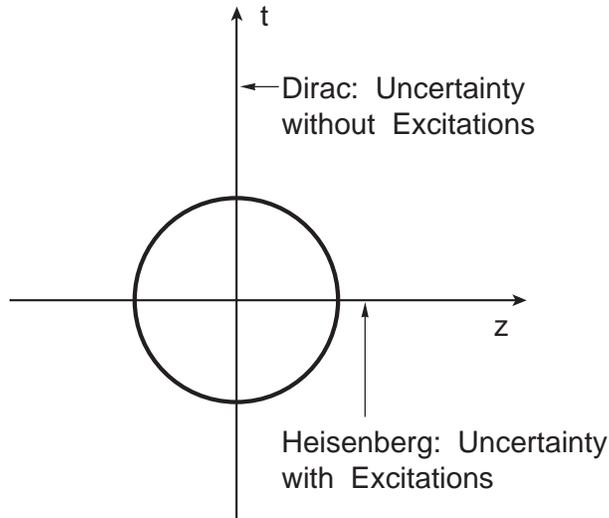
## DIRAC'S HARMONIC OSCILLATORS

Paul A. M. Dirac is known to us through the Dirac equation for spin-1/2 particles. But his main interest was in foundational problems. First, Dirac was never satisfied with the probabilistic formulation of quantum mechanics. This is still one of the hotly debated subjects in physics. Second, if we tentatively accept the present form of quantum mechanics, Dirac insisted that it had to be consistent with special relativity. He wrote several important papers on this subject. Let us look at some of his papers on this subject.

During World War II, Dirac was looking into the possibility of constructing representations of the Lorentz group using harmonic oscillator wave functions [9]. The Lorentz group is the language of special relativity, and the present form of quantum mechanics starts with harmonic oscillators. Presumably, therefore, he was interested in making quantum mechanics Lorentz-covariant by constructing representations of the Lorentz group using harmonic oscillators.

In his 1945 paper [9], Dirac considers the Gaussian form

$$\exp \left\{ -\frac{1}{2} (x^2 + y^2 + z^2 + t^2) \right\}. \quad (10)$$



**FIGURE 1.** Space-time picture of quantum mechanics. There are quantum excitations along the space-like longitudinal direction, but there are no excitations along the time-like direction. The time-energy relation is a c-number uncertainty relation.

We note that this Gaussian form is in the  $(x, y, z, t)$  coordinate variables. Thus, if we consider a Lorentz boost along the  $z$  direction, we can drop the  $x$  and  $y$  variables, and write the above equation as

$$\exp \left\{ -\frac{1}{2} (z^2 + t^2) \right\}. \quad (11)$$

This is a strange expression for those who believe in Lorentz invariance. The expression

$$\exp \left\{ -\frac{1}{2} (z^2 - t^2) \right\}. \quad (12)$$

is invariant, but Dirac's Gaussian form of Eq.(11) is not.

On the other hand, this expression is consistent with his earlier papers on the time-energy uncertainty relation [10]. In those papers, Dirac observes that there is a time-energy uncertainty relation, while there are no excitations along the time axis. He called this the "c-number time-energy uncertainty" relation. When one of us (YSK) was talking with Dirac in 1978, he clearly mentioned this word again. He said further that this is one of the stumbling block in combining quantum mechanics with relativity. This situation is illustrated in Fig. 1.

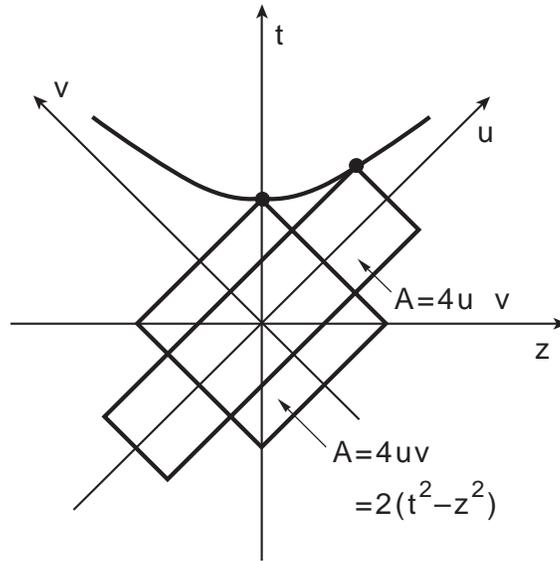
Let us look at Fig. 1 carefully. This figure is a pictorial representation of Dirac's Eq.(11), with localization in both space and time coordinates. Then Dirac's fundamental question would be how to make this figure covariant? This is where Dirac stops. However, this is not the end of the Dirac story.

Dirac's interest in harmonic oscillators did not stop with his 1945 paper on the representations of the Lorentz group. In his 1963 [11] paper, he constructed a representation of the  $O(3,2)$  deSitter group using two coupled harmonic oscillators. This paper contains not only the mathematics of combining special relativity with the quantum mechanics of quarks inside hadrons, but also forms the foundations of two-mode squeezed states which are so essential to modern quantum optics [8]. Dirac did not know this when he was writing his 1963 paper.

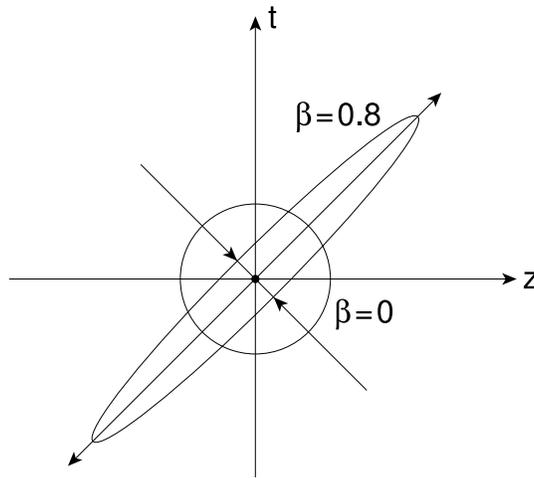
Furthermore, the  $O(3,2)$  deSitter group contains the Lorentz group  $O(3,1)$  as a subgroup. Thus, Dirac's oscillator representation of the deSitter group essentially contains all the mathematical ingredient of what we are doing in this paper.

## ADDENDUM TO DIRAC'S OSCILLATORS

In 1949, the Reviews of Modern Physics published a special issue to celebrate Einstein's 70th birthday. This issue contains Dirac paper entitled "Forms of Relativistic Dynamics" [12]. In this paper, he introduced his light-cone coordinate system, in which a Lorentz boost becomes a squeeze transformation.



**FIGURE 2.** Lorentz boost in the light-cone coordinate system.



**FIGURE 3.** Effect of the Lorentz boost on the space-time wave function. The circular space-time distribution in the rest frame becomes Lorentz-squeezed to become an elliptic distribution.

When the system is boosted along the  $z$  direction, the transformation takes the form

$$\begin{pmatrix} z' \\ t' \end{pmatrix} = \begin{pmatrix} \cosh(\eta/2) & \sinh(\eta/2) \\ \sinh(\eta/2) & \cosh(\eta/2) \end{pmatrix} \begin{pmatrix} z \\ t \end{pmatrix}. \quad (13)$$

This is not a rotation, and people still feel strange about this form of transformation. In 1949 [12], Dirac introduced his light-cone variables defined as [12]

$$u = (z+t)/\sqrt{2}, \quad v = (z-t)/\sqrt{2}, \quad (14)$$

the boost transformation of Eq.(13) takes the form

$$u' = e^{\eta/2}u, \quad v' = e^{-\eta/2}v. \quad (15)$$

The  $u$  variable becomes expanded while the  $v$  variable becomes contracted, as is illustrated in Fig. 2. Their product

$$uv = \frac{1}{2}(z+t)(z-t) = \frac{1}{2}(z^2 - t^2) \quad (16)$$

remains invariant. In Dirac's picture, the Lorentz boost is a squeeze transformation.

If we combine Fig. 1 and Fig. 2, then we end up with Fig. 3. In mathematical formulae, this transformation changes the Gaussian form of Eq.(11) into

$$\psi_\eta(z, t) = \left(\frac{1}{\pi}\right)^{1/2} \exp\left\{-\frac{1}{2}(e^{-\eta}u^2 + e^\eta v^2)\right\}. \quad (17)$$

Let us go back to Sec. on the coupled oscillators. The above expression is the same as Eq.(6). The  $x_1$  variable now became the longitudinal variable  $z$ , and the  $x_2$  variable became the time like variable  $t$ .

We can use coupled harmonic oscillators as the starting point of relativistic quantum mechanics. This allows us to translate the quantum mechanics of two coupled oscillators defined over the space of  $x_1$  and  $x_2$  into the quantum mechanics defined over the space time region of  $z$  and  $t$ .

This form becomes (11) when  $\eta$  becomes zero. The transition from Eq.(11) to Eq.(17) is a squeeze transformation. It is now possible to combine what Dirac observed into a covariant formulation of the harmonic oscillator system. First, we can combine his c-number time-energy uncertainty relation described in Fig. 1 and his light-cone coordinate system of Fig. 2 into a picture of covariant space-time localization given in Fig. 3.

In addition, there are two more homework problems which Dirac left us to solve. First, in defining the  $t$  variable for the Gaussian form of Eq.(11), Dirac did not specify the physics of this variable. If it is going to be the calendar time, this form vanishes in the remote past and remote future. We are not dealing with this kind of object in physics. What is then the physics of this time-like  $t$  variable?

The Schrödinger quantum mechanics of the hydrogen atom deals with localized probability distributions. Indeed, the localization condition leads to the discrete energy spectrum. Here, the uncertainty relation is stated in terms of the spatial separation between the proton and the electron. If we believe in Lorentz covariance, there must also be a time-separation between the two constituent particles, and an uncertainty relation applicable to this separation variable. Dirac did not say in his papers of 1927 and 1945, but Dirac's "t" variable is applicable to this time-separation variable. This time-separation variable will be discussed in detail in Sec. for the case of relativistic extended particles.

Second, as for the time-energy uncertainty relation, Dirac's concern was how the c-number time-energy uncertainty relation without excitations can be combined with uncertainties in the position space with excitations. Dirac's 1927 paper was written before Wigner's 1939 paper on the internal space-time symmetries of relativistic particles.

Both of these questions can be answered in terms of the space-time symmetry of bound states in the Lorentz-covariant regime. In his 1939 paper, Wigner worked out internal space-time symmetries of relativistic particles. He approached the problem by constructing the maximal subgroup of the Lorentz group whose transformations leave the given four-momentum invariant. As a consequence, the internal symmetry of a massive particle is like the three-dimensional rotation group.

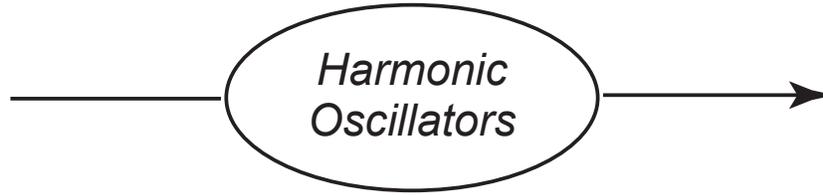
If we extend this concept to relativistic bound states, the space-time asymmetry which Dirac observed in 1927 is quite consistent with Einstein's Lorentz covariance. The time variable can be treated separately. Furthermore, it is possible to construct a representations of Wigner's little group for massive particles [7]. As for the time-separation, it is also a variable governing internal space-time symmetry which can be linearly mixed when the system is Lorentz-boosted.

## FEYNMAN'S OSCILLATORS

Quantum field theory has been quite successful in terms of Feynman diagrams based on the S-matrix formalism, but is useful only for physical processes where a set of free particles becomes another set of free particles after interaction. Quantum field theory does not address the question of localized probability distributions and their covariance under Lorentz transformations. In order to address this question, Feynman *et al.* suggested harmonic oscillators to tackle the problem [13]. Their idea is indicated in Fig. 4.

Before 1964 [14], the hydrogen atom was used for illustrating bound states. These days, we use hadrons which are bound states of quarks. Let us use the simplest hadron consisting of two quarks bound together with an attractive force,

## Feynman Diagrams



## Feynman Diagrams

**FIGURE 4.** Feynman’s roadmap for combining quantum mechanics with special relativity. Feynman diagrams work for running waves, and they provide a satisfactory resolution for scattering states in Einstein’s world. For standing waves trapped inside an extended hadron, Feynman suggested harmonic oscillators as the first step.

and consider their space-time positions  $x_a$  and  $x_b$ , and use the variables

$$X = (x_a + x_b)/2, \quad x = (x_a - x_b)/2\sqrt{2}. \quad (18)$$

The four-vector  $X$  specifies where the hadron is located in space and time, while the variable  $x$  measures the space-time separation between the quarks. According to Einstein, this space-time separation contains a time-like component which actively participates as in Eq.(13), if the hadron is boosted along the  $z$  direction. This boost can be conveniently described by the light-cone variables defined in Eq.(14). Does this time-separation variable exist when the hadron is at rest? Yes, according to Einstein. In the present form of quantum mechanics, we pretend not to know anything about this variable. Indeed, this variable belongs to Feynman’s rest of the universe.

What do Feynman *et al.* say about this oscillator wave function? In their classic 1971 paper [13], Feynman *et al.* start with the following Lorentz-invariant differential equation.

$$\frac{1}{2} \left\{ x_\mu^2 - \frac{\partial^2}{\partial x_\mu^2} \right\} \psi(x) = \lambda \psi(x). \quad (19)$$

This partial differential equation has many different solutions depending on the choice of separable variables and boundary conditions. Feynman *et al.* insist on Lorentz-invariant solutions which are not normalizable. On the other hand, if we insist on normalization, the ground-state wave function takes the form of Eq.(11). It is then possible to construct a representation of the Poincaré group from the solutions of the above differential equation [7]. If the system is boosted, the wave function becomes given in Eq.(17).

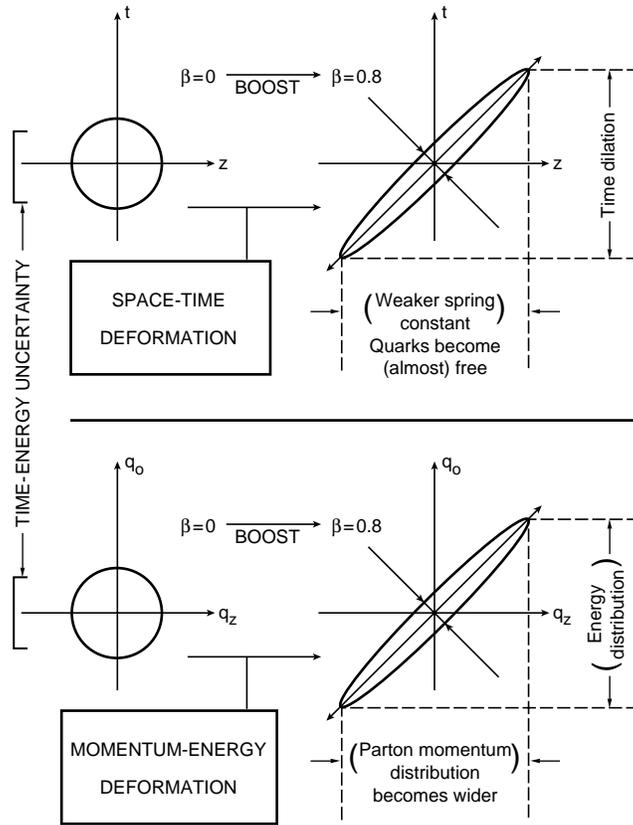
This wave function becomes Eq.(11) if  $\eta$  becomes zero. The transition from Eq.(11) to Eq.(17) is a squeeze transformation. The wave function of Eq.(11) is distributed within a circular region in the  $uv$  plane, and thus in the  $zt$  plane. On the other hand, the wave function of Eq.(17) is distributed in an elliptic region with the light-cone axes as the major and minor axes respectively. If  $\eta$  becomes very large, the wave function becomes concentrated along one of the light-cone axes. Indeed, the form given in Eq.(17) is a Lorentz-squeezed wave function. This squeeze mechanism is illustrated in Fig. 3.

There are many different solutions of the Lorentz invariant differential equation of Eq.(19). The solution given in Eq.(17) is not Lorentz invariant but is covariant. It is normalizable in the  $t$  variable, as well as in the space-separation variable  $z$ . It is indeed possible to construct Wigner’s  $O(3)$ -like little group for massive particles [15], and thus the representation of the Poincaré group [7]. Our next question is whether this formalism has anything to do with the real world.

In 1969, Feynman observed that a fast-moving hadron can be regarded as a collection of many “partons” whose properties appear to be quite different from those of the quarks [16]. For example, the number of quarks inside a static proton is three, while the number of partons in a rapidly moving proton appears to be infinite. The question then is how the proton looking like a bound state of quarks to one observer can appear different to an observer in a different Lorentz frame? Feynman made the following systematic observations.

- a. The picture is valid only for hadrons moving with velocity close to that of light.

# QUARKS → PARTONS



**FIGURE 5.** Lorentz-squeezed space-time and momentum-energy wave functions. As the hadron's speed approaches that of light, both wave functions become concentrated along their respective positive light-cone axes. These light-cone concentrations lead to Feynman's parton picture.

- b. The interaction time between the quarks becomes dilated, and partons behave as free independent particles.
- c. The momentum distribution of partons becomes widespread as the hadron moves fast.
- d. The number of partons seems to be infinite or much larger than that of quarks.

Because the hadron is believed to be a bound state of two or three quarks, each of the above phenomena appears as a paradox, particularly b) and c) together.

In order to resolve this paradox, let us write down the momentum-energy wave function corresponding to Eq.(17). If we let the quarks have the four-momenta  $p_a$  and  $p_b$ , it is possible to construct two independent four-momentum variables [13]

$$P = p_a + p_b, \quad q = \sqrt{2}(p_a - p_b), \quad (20)$$

where  $P$  is the total four-momentum. It is thus the hadronic four-momentum.

The variable  $q$  measures the four-momentum separation between the quarks. Their light-cone variables are

$$q_u = (q_0 - q_z)/\sqrt{2}, \quad q_v = (q_0 + q_z)/\sqrt{2}. \quad (21)$$

The resulting momentum-energy wave function is

$$\phi_\eta(q_z, q_0) = \left(\frac{1}{\pi}\right)^{1/2} \exp\left\{-\frac{1}{2}(e^\eta q_u^2 + e^{-\eta} q_v^2)\right\}. \quad (22)$$

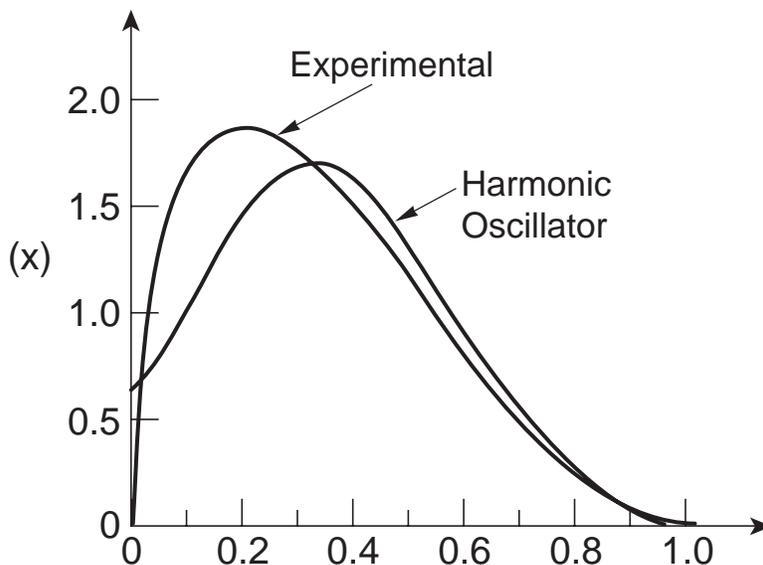


FIGURE 6. Parton distribution function. Theory and experiment.

Because we are using here the harmonic oscillator, the mathematical form of the above momentum-energy wave function is identical to that of the space-time wave function. The Lorentz squeeze properties of these wave functions are also the same. This aspect of the squeeze has been exhaustively discussed in the literature [7, 17, 18].

When the hadron is at rest with  $\eta = 0$ , both wave functions behave like those for the static bound state of quarks. As  $\eta$  increases, the wave functions become continuously squeezed until they become concentrated along their respective positive light-cone axes. Let us look at the z-axis projection of the space-time wave function. Indeed, the width of the quark distribution increases as the hadronic speed approaches that of the speed of light. The position of each quark appears widespread to the observer in the laboratory frame, and the quarks appear like free particles.

The momentum-energy wave function is just like the space-time wave function, as is shown in Fig. 5. The longitudinal momentum distribution becomes wide-spread as the hadronic speed approaches the velocity of light. This is in contradiction with our expectation from non-relativistic quantum mechanics that the width of the momentum distribution is inversely proportional to that of the position wave function. Our expectation is that if the quarks are free, they must have sharply defined momenta, not a wide-spread distribution.

However, according to our Lorentz-squeezed space-time and momentum-energy wave functions, the space-time width and the momentum-energy width increase in the same direction as the hadron is boosted. This is of course an effect of Lorentz covariance. This indeed is the key to the resolution of the quark-parton paradox [7, 17].

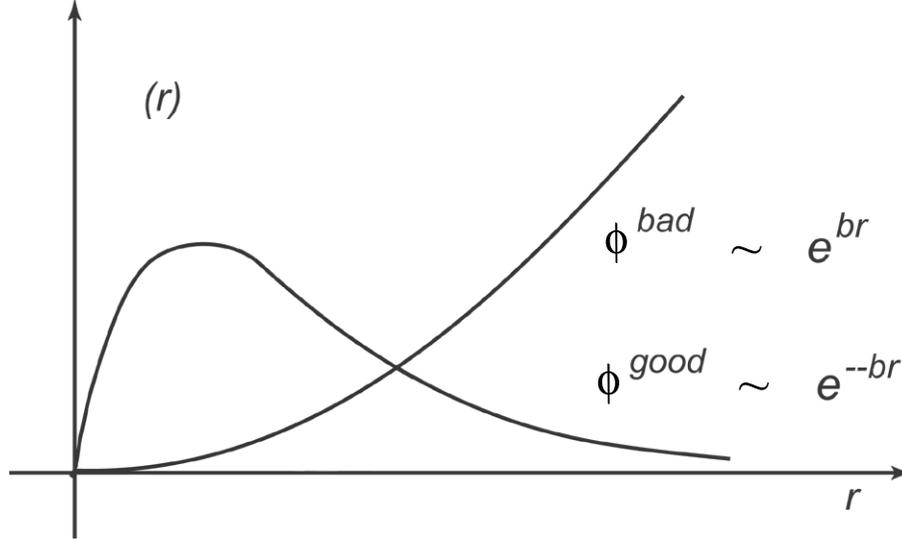
After these qualitative arguments, we are interested in whether Lorentz-boosted bound-state wave functions in the hadronic rest frame could lead to parton distribution functions. If we start with the ground-state Gaussian wave function for the three-quark wave function for the proton, the parton distribution function appears as Gaussian as is indicated in Fig. 6. This Gaussian form is compared with experimental distribution also in Fig. 6.

For large  $x$  region, the agreement is excellent, but the agreement is not satisfactory for small values of  $x$ . In this region, there is a complication called the “sea quarks.” However, good sea-quark physics starts from good valence-quark physics. Figure 6 indicates that the boosted ground-state wave function provides a good valence-quark physics.

Feynman’s parton picture is one of the most controversial models proposed in the 20th century. The original model is valid only in Lorentz frames where the initial proton moves with infinite momentum. It is gratifying to note that this model can be produced as a limiting case of one covariant model which produces the quark model in the frame where the proton is at rest.

## CONCLUDING REMARKS

In this report, we considered Einstein's relative simultaneity working in the wave-function picture of quantum mechanics. It was shown that the covariant harmonic oscillator applicable to hadrons in the quark model gives an illustration of how this problem can be approached. A non-zero spacial-separation in the hadronic rest frame with zero time-separation gives a measurable effect in the Lorentz frame which moves with a velocity close that of light.



**FIGURE 7.** Good and bad wave functions contained in the S-matrix. Bound-state wave functions satisfy the localization condition and are good wave functions. Analytic continuations of plane waves do not satisfy the localization boundary condition, and become bad wave functions at the bound-state energy.

It is widely understood that the present form of quantum field theory, with the S-matrix and Feynman diagrams, does the job of combining quantum mechanics and relativity. The question then is why we did not use field theory to deal with the simultaneity problem. The answer is very simple. The present form of field theory can deal only with scattering problems. There have been many attempts in the past to extend the field theory algorithm to bound state problems. This requires analytic continuation of incoming and outgoing waves to negative energy regions. The outgoing waves become localized bound-state wave functions, but the incoming waves increase exponentially for large distances. We do not know how to control this localization problem in quantum field theory, as we can see from the Dashen-Frautschi fiasco [19, 20]. Feynman was right. We should start bound-state problems with localized harmonic oscillator wave functions.

Indeed, the once-celebrated calculation of the neutron-proton mass difference by Dashen and Frautschi illustrates difficulties of using the present form of field theory for bound state problems [19]. In order to calculate the mass difference as an electromagnetic perturbation, they developed a perturbation formula solely based on the S-matrix quantities, but they ended up with a first-order energy shift corresponding to [20]

$$\delta E = \left( \phi^{good}, \delta V \phi^{bad} \right), \quad (23)$$

where the good and bad bound-state wave functions are like

$$\phi^{good} \sim e^{-br}, \quad \phi^{bad} \sim e^{br}, \quad (24)$$

for large values of  $r$ , as illustrated in Fig. 7. We are not aware of any S-matrix or field theoretic method which guarantees the localization of bound-state wave functions.

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