Density Matrix from the Entangled Space and time

Let us start with the ground-state wave function

$$\psi_0(z,t) = \frac{1}{\sqrt{\pi}} \exp\left(-\frac{1}{2}\left[z^2 + t^2\right]\right).$$

This form is separable in the z and t variables.

When boosted, this wave function becomes squeezed to

$$\psi_\eta(z,t) = \frac{1}{\sqrt{\pi}} \exp\left(-\frac{1}{4}\left[e^{-\eta}(z+t)^2 + e^{\eta}(z-t)^2\right]\right),$$

where $\tanh(\eta) = v/c$, and the space and time variables become entangled.

This exponential form can be expanded in terms of the oscillator function, and the result is

$$\psi_\eta(z,t) = \frac{1}{\sqrt{\pi}} = \frac{1}{\cosh\eta} \sum_n (\tanh\eta)^n \phi_n(z)\phi_n(t),$$

where $\phi_n z$ is the k-th excited state wave function. Indeed, this form is identical with that for the two-mode squeezed state in quantum optics, where the two photons are entangled with each other.

If the z and t variables are both measurable, we can construct the density matrix

$$\rho_\eta(z,t;z',t') = \psi_\eta(z,t)\left(\psi_\eta(z',t')\right)^*,$$

However, there are at present no measurement theories which accommodate the time-separation variable t. This time separation variable belongs to Feynman's rest of the universe, and is hidden in the present form of quantuem mechanics.

Thus, we can take the trace of the ρ matrix with respect to the t variable. Then the resulting density matrix is

$$\rho_\eta(z,z') = \int \psi_\eta^n(z,t)\psi_\eta^n(z',t)dt = \left(\frac{1}{\cosh\eta}\right)^2 \sum_n (\tanh\eta)^{2n} \phi_n(z)\phi_n(z').$$

In terms of the hadronic velocity, this density matrix can be written as

$$\rho_\eta(z,z') = \left(1 - (v/c)^2\right) \sum_n (v/c)^{2n} \phi_n(z)\phi_n(z').$$

The standard way to measure this ignorance is to calculate the entropy defined defined as

$$S = -Tr\left(\rho \ln(\rho)\right).$$

With the density matrix $\rho_\eta(z,z')$ given above, the entropy becoms

$$S = \left(\cosh^2\eta\right)\ln\left(\cosh^2\eta\right) - \left(\sinh^2\eta\right)\ln\left(\sinh^2\eta\right).$$

Hadronic Temperature

The ground-state oscillator can be excited in various ways. It can be thermally excited, and the density function takes the form

$$\rho_T(z, z') = \left(1 - e^{-\hbar\omega/kT}\right) \sum_n e^{-n\hbar\omega/kT} \phi_n(z)\phi_n^*(z'),$$

where $\hbar\omega$ and k are the oscillator energy separation and Boltzmann's constant respectively. This form of the density matrix is well known.

If the temperature is measured in units of $\hbar\omega/k$, the above density matrix can be written as

$$\rho_T(z, z') = \left(1 - e^{-1/T}\right) \sum_n e^{-n/T} \phi_n(z)\phi_n^*(z').$$

If we compare this expression with the density matrix coming from the entangled space and time, we are led to

$$\tanh^2\eta = \exp\left(-1/T\right),$$

and to

$$T = \frac{-1}{2\ln(v/c)}.$$

The temperature can be calculated as a function of the hadronic velocity.

Let us look at the velocity dependence of the temperature again. It is almost proportional to the velocity from $\tanh(\eta) = 0$ to 0.7, and again from $\tanh(\eta) = 0.9$ to 1 with different slopes.