

We write the generators of the rotation group as J_i . They satisfy the commutation relations

$$[J_i, J_j] = i\epsilon_{ijk}J_k. \quad (1)$$

The smallest matrices satisfying these three commutation relations are

$$J_i = \frac{1}{2}\sigma_i, \quad (2)$$

where σ_i are the Pauli spin matrices. The Pauli matrices are Hermitian and thus the commutation relations of Eq.(1) are invariant under Hermitian conjugation.

Let us introduce another set of matrices defined as

$$K_i = \frac{i}{2}\sigma_i. \quad (3)$$

These three matrices are anti-Hermitian, and satisfy the commutation relations

$$[K_i, K_j] = -i\epsilon_{ijk}J_k, \quad [J_i, K_j] = -i\epsilon_{ijk}K_k, \quad (4)$$

The three sets of the commutation relations given Eq.(1) and Eq.(4) are like those of the Lorentz group generated by three rotation and three boost generators. K_i generate the Lorentz boost along the i^{th} axis.

These commutation relations are not invariant under the sign change in the rotation generators, but they are invariant under the sign change in the K_i matrices. Thus, one set of generators can accommodate two boost operations in the two opposite directions. Thus, in the four-by-four representation, we can write the boost generators as

$$K_{44i} = \begin{pmatrix} K_i & 0 \\ 0 & -K_i \end{pmatrix}, \quad (5)$$

and

$$J_{44i} = \begin{pmatrix} J_i & 0 \\ 0 & J_i \end{pmatrix} \quad (6)$$

These four-by-four matrices satisfy the commutation relations of Eq.(1) and Eq.(4).

Indeed, the Dirac matrices in the Weyl representation are consistent with this form of the generators of the Lorentz group.