Phys 623, Introduction to Quantum Mechanics II, Spring 1999, Dr. Yakovenko

Midterm Examination

Friday, March 12, 1999, 12–12:50 p.m.

1. **[10 points]** In the WKB approximation, find discrete energy levels of the attractive Coulomb potential $V(r) = -e^2/r$. Consider a general case where the angular momentum l is not equal to zero. Apply the WKB approximation to the radial motion with an effective potential containing the centrifugal term. Since the potential is singular at $r = 0$, apply the Bohr-Sommerfeld quantization condition in a generalized form $\int p(r)\,dr = (N+\gamma)\hbar\pi$ with some unknown term $0 \le \gamma < 1$ that may depend on l. Compare your result with the spectrum of the hydrogen atom in the limit $N,\ l \gg 1$.

The following formula may be useful:

$$\int_a^b \frac{dr}{r}\sqrt{(b-r)(r-a)} = \int_a^b \frac{dr}{r}\sqrt{-r^2+(a+b)r-ab} = \frac{\pi}{2}(a+b-2\sqrt{ab}). \qquad (1)$$

(Verify it *at home, not at exam* by contour integration in the complex plane.)

2. Consider a spin-1/2 particle of mass m bound in a three-dimensional harmonic oscillator of frequency ω. The particle is subject to a small perturbation described by the Hamiltonian

$$\hat{H}' = \lambda\,\mathbf{r}\cdot\boldsymbol{\sigma}, \qquad (2)$$

where $\mathbf{r}$ is the 3D vector coordinate of the particle, and $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices acting on the spin of the particle.

(a) **[3 points]** Using perturbation theory, calculate the change in the ground state energy of the particle to order λ^2. Does this correction remove the Kramers degeneracy of the ground state (of the up and down spin states)?

(b) **[3 points]** Is the Hamiltonian (2) invariant under the parity operation? Under the time reversal? Under combined parity and time reversal?

Would the perturbation (2) remove the Kramers degeneracy in any order of λ?

(c) **[3 points]** Is Hamiltonian (2) invariant under the rotation of $\mathbf{r}$? Under the rotation of spin? Under the combined rotation of $\mathbf{r}$ and spin? Is the total angular momentum j ($\hat{\mathbf{J}} = \hat{\mathbf{S}} + \hat{\mathbf{L}}$) a good quantum number?

What are the possible values of j for the first excited energy level? Describe qualitatively what kind of the energy splitting of the six states of the first excited energy level is compatible with the symmetry of the problem.

(d) **[3 points]** Does the perturbation (2) split the initially degenerate first excited energy level to order λ?

(e) **[3 points]** Describe qualitatively whether the perturbation (2) would split the initially degenerate first excited energy level to order λ^2, and how this problem should be addressed mathematically.