Phys 622, Introduction to Quantum Mechanics I, Fall 1998, Prof. Yakovenko

Midterm Examination

Monday, October 19, 1998, 12:00–12:50 p.m.

1. *Adapted from Physics Qualifier, January 1996, Problem II-1.*

Consider a quantum particle of mass m moving in the following potential:

$$V(x) = \begin{cases} \lambda\delta(x), & |x| < a, \\ \infty, & |x| > a. \end{cases} \tag{1}$$

(a) [**4 points**] Show that the energy eigenstates of the particle can be divided into two classes by parity, so that the eigenfunctions and eigenvalues of one class do not depend at all on the parameter λ. Write these eigenfunctions and eigenvalues.

Now, let us discuss the wave functions that do depend on λ, which may be positive or negative. First, consider the eigenstates with a positive energy.

(b) [**8 points**] Write the energy eigenfunctions (don't bother to normalize) and derive a transcendental equation for the eigenvalues of energy.

Explain how to find solutions of this equation graphically.

Qualitatively sketch the energy eigenvalues as functions of the parameter

$$\xi = \lambda m a/\hbar^2, \tag{2}$$

when it changes from $-\infty$ to ∞.

Do the eigenenergies $E(\lambda)$ ever cross over the eigenenergies from the other class that are λ-independent?

Now, let us consider the eigenstate with negative energy.

(c) [**8 points**] Answer question (b) when the energy is negative.

What is the range of ξ where a negative energy eigenvalue exists?