Homework #2 — Phys622 — Fall 1998
Deadline: Wednesday, September 16, 1998.
Turn in homework in the class or put
in the box on the door of Phys 2314.

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Do not forget to write your name and the homework number!
Equation numbers with the period, like (3.25), refer to the equations of Schwabl.
Equation numbers without period, like (5), refer to the equations of this homework.

Harmonic Oscillator (Chapter 3)

1. **(a)** [**2 points**] Find the wave function of a coherent state $\varphi_\alpha(x)$ by solving Eq. (3.36) as a differential equation with the differential operator $\hat{a}$ given by Eq. (3.7a). Compare with Problem 7 of Homework 1.

(b) [**2 points**] What is the physical meaning of the real and imaginary parts of α? What is the value of the uncertainty $\Delta p \Delta x$ for a coherent state?

2. [**3 points each part**] Schwabl's Problem 3.2. In part (a) show only that the wave function of a coherent state can be written as

$$\psi_\alpha(x) = e^{\alpha\hat{a}^+ - \alpha^*\hat{a}}\varphi_0(x) = e^{-|\alpha|^2/2}e^{\alpha\hat{a}^+}\varphi_0(x). \tag{1}$$

Hint: Use Eq. (3.37) in parts (a)–(c). Additional part:

(d) Do the states $\varphi_\alpha(x)$ with all possible values of α form a complete basis? An orthogonal basis? Is it possible to represent any function $\psi(x)$ as a superposition of the coherent states:

$$\psi(x) = \int \frac{d^2\alpha}{\pi}\, c_\alpha \varphi_\alpha(x) \quad ? \tag{2}$$

What are the coefficients c_α?

3. [**3 points each part**] Schwabl's Problem 3.3. In part (d) show that $\langle \hat{H} \rangle = \hbar\omega(|\alpha|^2 + 1/2)$.

4. *Adapted from Physics Qualifier, Spring 1980, Problem II-2.*

The oscillator is in its ground state ψ_0. Suddenly, the origin (the equilibrium point) of the oscillator is displaced by a distance l. The wave function does not change during the displacement.

(a) [**3 points**] Calculate the probability to find the oscillator in an energy eigenstate ψ_n defined with respect to the new origin. Hint: Use the results of Problems 1 and 2b.

(b) [**3 points**] Calculate the change of the energy of the system. Hint: Use the result of Problem 3d.

5. A one-dimensional harmonic oscillator is subject to a time-dependent force $F(t)$.

(a) [**2 points**] Write down the Hamiltonian of the system classically in terms of x and p and quantum-mechanically in terms of $\hat{a}$ and $\hat{a}^+$.

(b) [**3 points**] In classical mechanics, let us introduce the complex variable $\alpha(t)$ (compare with Eq. (3.4a)):

$$\alpha(t) = \frac{1}{\sqrt{2}}\left(\frac{x(t)}{x_0} + \frac{i x_0 p(t)}{\hbar}\right), \tag{3}$$

where x_0 is given by Eq. (3.3), and $x(t)$ and $p(t)$ are the (real) classical coordinate and momentum. Show that $\alpha(t)$ obeys the following classical equation of motion:

$$\frac{d\alpha(t)}{dt} = -i\omega\alpha(t) + i x_0 \frac{F(t)}{\hbar\sqrt{2}}. \tag{4}$$

(c) [**3 points**] In quantum mechanics, using the Ehrenfest theorem (Section 2.6), show that $\langle \hat{a} \rangle$ satisfies Eq. (4) with $\langle \hat{a} \rangle$ instead of $\alpha(t)$.

(d) [**5 points**] In quantum mechanics, suppose the oscillator is in a coherent state $\psi_{\alpha_0}(x)$ at $t = 0$. Using the the time-dependent Schrödinger equation, show that at a time t the oscillator will be in the coherent state $\psi_{\alpha(t)}(x)$, where the parameter $\alpha(t)$ satisfies Eq. (4) with the initial condition $\alpha(0) = \alpha_0$. Remember that Eq. (3.36) defines the wave function of a coherent state, $\psi_\alpha(x)$, only up to an overall phase factor $e^{-i\delta}$, which may be time-dependent (**see Hints**).

6. [**2 points**] Schwabl's Problem 3.11, part (a)

7. [**3 points**] Schwabl's Problem 3.12.

8. [**3 points**] Schwabl's Problem 3.15.

Hints

5d Show that the wave function $e^{-i\delta(t)}\psi_{\alpha(t)}(x)$, where $\delta(t)$ is a time-dependent phase, and $\psi_\alpha(x)$ is given by Eq. (1), satisfies the time-dependent Schrödinger equation:

$$i\hbar\frac{\partial}{\partial t}\, e^{-i\delta(t)}\psi_{\alpha(t)}(x) = \hat{H}\, e^{-i\delta(t)}\psi_{\alpha(t)}(x), \tag{5}$$

provided $\alpha(t)$ satisfies Eq. (4).