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Physics 731 HOMEWORK ASSIGNMENT #5 Due: Tuesday, Oct. 16, 2001

No class on Thursday, Oct. 18; make-up class Wednesday evening, Oct. 17, 7:15p.m.

Hour test: late October

Finish reading about phonons: A&M chapters 23, 24, 25. Chap. 24 is straightforward and rather descriptive. Chap. 23 will be covered thoroughly in class. After reading pp. 464–465, read pp. 143–145, substituting $\omega_s(k)$ for $\varepsilon_n(k)$, s^{th} branch for n^{th} band, and removing the factor of 2 from spin degeneracy. [Thus, for phonons there is no factor of 2 in eqns. (8.53), (8.54), and (8.58), the $1/4\pi^3$ should be $1/8\pi^3$ in eqns. (8.57), (8.59), (8.60), and (8.63).] In chap. 25 we will only have time to cover lattice thermal conductivity (pp. 495–505) with any care. The rest of that chapter can be skimmed very casually. The objective should be to get a sense of what results are known. Finally, review Appendix L and study Appendix M (pp. 784–787).

Problems to turn in (read the rest):

1. 23-1 (parts a and c only)
   Hint: Use $\Sigma_s\lambda_s(\mathbf{k}) = \Sigma_\mu D_{\mu\mu}(\mathbf{k})$, and note that the trace is independent of the representation.
2. 23-2
3. 23-3 Hint for part b: assume $\omega(\mathbf{k}) = \omega(\mathbf{k}_0) - \alpha(\mathbf{k} - \mathbf{k}_0)^2$
4. 24-3 (parts a and b only; you can simply accept eqn. (N.17) as reasonable or read Appendix N).
5. 25-5

old 6. [Do NOT hand in; solution will be provided.] Calculate the eigenfrequency of a mass defect $M_0 \neq M$ in a linear chain at the position n=0 by invoking the ansatz $u(na) = u_0 \exp[-\kappa(\omega)|n|a - i\omega t]$ for displacements (and then eliminating κ from the coupled equations that result). For what range of M_0 do localized vibrations exist (i.e. for what range is $\omega^2 > 0$)? (Warning: this problem, drawn from Ibach and Lüth, is not well posed: there is the following inconsistency. You can show that $\omega^2/2k_0 = 1 - \exp(-\kappa a)$, which is problematic for negative ω^2.)

6. a) Find the power of ω for the phonon density of states of the Debye model in 1 and in 2 dimensions, i.e. for $g(\omega) \sim \omega^\alpha$, what is α?
   b) Consider a dispersion relation with ω = const times k^m. What is the value of α in 1, 2, and 3 dimensions? (E.g., m=2 for spin waves (magnons).)