**Exam 2** —Phys374—Fall 2013 Prof. Alessandra Buonanno
Tuesday, Nov. 19, 2013 Room 4212, (301)405-1440
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1. Consider the vector field $\mathbf{A} = 3y^2\,(\hat{\mathbf{x}} + \hat{\mathbf{z}})$. *[2+4+4=10 pts]*
   (a) Compute $\mathbf{B} = \nabla \times \mathbf{A}$.
   (b) Compute the flux of $\mathbf{B}$ outward through the five square faces of the cube $0 < x, y, z < L$, other than the $z = 0$ face. (*Hint*: To do it quickly, use Stokes' theorem.)
   (c) (i) Give a simple argument showing that $\mathbf{A}$ be expressed as the curl of another vector field $\mathbf{F}$, and (ii) give one such $\mathbf{F}$.

2. Find (i) the Cartesian form and (ii) a polar form of $\frac{1+i}{(1-i)^2}$. *[2+3=5 pts]*

3. Find all complex numbers $\beta$ for which $\cosh(z+\beta) = -\cosh(z)$ for all complex $z$. (*Hint*: Express cosh in terms of exponentials.) *[3 pts]*

4. State whether each of the following functions is analytic, and how you know. (i) $1/z$, (ii) $|z|$, (iii) $Re(z)$. (Exclude the point $z = 0$.) (iv) For what values of the complex number $\gamma$ is $x^2 - 2\gamma xy - y^2$ an analytic function of $z = x + iy$, and why? *[3+3+3+3=12 pts]*

5. Consider a velocity potential given by the real part of $h(z) = A/z$, where $A$ is a real positive constant. *[3+3+4=10 pts]*
   (a) Find the components $(v_x, v_y)$ of the flow velocity as functions of $x$ and $y$.
   (b) Find the equation for a general flow line in this flow.
   (c) Sketch the rough shape and location of the flow lines that go through the points $(x, y) = (\pm 1, 0)$ and $(x, y) = (0, \pm 1)$. Include arrows showing the direction of the flow. (The origin is a singular point of this flow.)

6. Evaluate the integral of $1/(1 + z^4)$ on the following two contours: *[4+6=10 pts]* (a) a counterclockwise circle of radius 1/2 centered on the origin, (b) the positive real axis from 0 to $\infty$. Be sure to fully justify all your steps and results. For part (b), show explicitly that your result is a positive real number.