

1. Consider the “rectified cosine function” defined by

$$f(x) = \cos(\pi x/2L), \quad -L \leq x \leq L, \quad (1)$$

and continued periodically so that $f(x + 2L) = f(x)$. [2+3+5+5=15 pts.]

- (a) Sketch the function $f(x)$ over several periods.
 - (b) Use the symmetry to explain why the Fourier coefficients b_n vanish.
 - (c) Find the non-vanishing Fourier coefficients. (*Hints:* (i) To clean things up, change variables to $\theta = \pi x/L$. (ii) You’ll need to do a probably unfamiliar integral, which you can look up or work out for yourself.)
 - (d) Using a computer program (Mathematica, Maple, Matlab, or something else) plot the sum of the first few terms in the Fourier series, together with (1), for $\theta \in (-2\pi, 2\pi)$. Show the result with 1 (just the constant part), 2, 5, and 20 terms included. With 5 terms the sum should already be quite close to (1), except near the zeros where the slope is discontinuous.
2. Find the Fourier transform of $f(t) = A \sin(\omega_0 t + \varphi)$. [10 pts.]
3. Problems 15.6 g,h (Fourier transform of correlation and Parseval’s theorem) [10 pts.]
(*Note:* The conventions (15.42), (15.43) are used here.)