Knowledge activation and organization in physics problem-solving

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Conceptual knowledge is only one aspect of a good knowledge structure: how and when knowledge is activated and used are also important. In this paper, we explore knowledge organization in the context of the resources model of student thinking through observations of student problem-solving behavior on a mechanics task that integrates the concepts of force, motion, and energy. We document in detail that both introductory and advanced students may have knowledge structures with local coherences that may inhibit their access to additional useful knowledge. These results suggest that instructors and researchers need to pay increased attention to how and when students use what they know as well as to what they know.

Conceptual knowledge is an essential part of what students studying physics need to learn in order to solve problems. They need to be able to make sense of what the problem is about, what the relevant physics is, and how to interpret their results. In the past twenty years, physics education researchers have learned a great deal about students' conceptual knowledge[1] and have developed many curricular environments to improve it.[2] However, conceptual knowledge is only one part of what students need to know in order to solve physics problems. They also need to know how and when to use that knowledge.

In this paper, we look at how students access and use their knowledge in the context of solving a mechanics problem that can be solved using either concepts of force or energy or by integrating the two. We report on a study of six advanced physics students (seniors and graduate students) working individually on the problem in an interview context. Then we report on a controlled experiment with approximately 100 introductory engineering physics students using two versions of the problem presented as an ungraded quiz.

Together these studies demonstrate that for both advanced and introductory students, local coherences in their conceptual knowledge can inhibit access to other parts of their knowledge that could have been useful. (By "local coherence" we mean that students see certain elements of their knowledge as closely related and working effectively together.) We suggest that a lack of a broader coherence affects students' ability to integrate concepts of force and energy. Our results suggest that both physics instructors and physics education researchers need to pay attention to issues of the access and use of student knowledge structures as well as to concepts.

In order to describe the motivation for and the implications of these studies, we work in a particular theoretical framework: the resource model of student thinking. This model is based on mutually supporting evidence from the neuro-, cognitive, and behavioral sciences and it provides a structure for discussing the phenomenology of student behavior. In addition, the observations described here give triangulating support that the structures of the model hypothesized from abstract neural and cognitive studies can be useful in making sense of the observations of real students functioning in authentic learning situations.

We begin by giving a brief outline of the resource model together with a description of how it relates to earlier work. We then describe the contexts of the two studies we report on. We first present the results of the observational interviews with the advanced students and consider the implications of the observations. We then discuss the experiment with the introductory engineering physics students and discuss the implications of those results. In conclusion, we consider the broader implications for instruction and research suggested by our studies.

A Model of Thinking

In order to make sense of how students think about topics in physics it is useful to have a model of the way they think in general. Researchers in a variety of fields including neuroscience, cognitive science, and the various behavioral sciences attempt to model human thought. Much has been learned, but caution is needed in applying these research results to real-world situations. Results from neuroscience can be very fine grained and might not carry over to higher-level thinking. Results from cognitive science are often, even in the best cases, "zero-friction" experiments. They inform us about fundamental mechanisms but can be overwhelmed by other mental phenomena when imbedded in actual situations. For this reason, in creating our model of how students think about physics, we rely heavily on "triangulation" — convergent support for the elements of our model from neuroscience (to guarantee the plausibility of mechanism), cognitive

science (to demonstrate the fundamental phenomenon), and behavioral sciences including education research (to prove that the principle is relevant in actual educational contexts). This paper provides some of the educational research supporting the conjecture that the elements of the model are "ecological" — relevant for real students in real learning environments — and demonstrates the explanatory power of the model in the context of physics learning at the university level.

The Neuro/Cognitive Basis of Cognition

A model of cognition is emerging from results in neuro- and cognitive science.[3,4,5,6] In this model, networks of connected neurons represent cognitive elements of knowledge and memory. When someone recalls or uses the knowledge represented by a particular network, the neurons of the network are *activated* (increase their firing rate).[7] Activation can be highly dynamic, "flickering" on and off in response to changing contexts, both external and internal. Particular knowledge elements tend to be multi-modal (i.e., to involve activation and interpretation of multiple sensory and interpretive structures) and involve neurons in many parts of the brain. The term "activate" here plays the role of the term more commonly used in physics education research, "elicit."

These networks arise from the building of associations among neurons through synapse growth. The association of neurons can vary in strength and increases with repeated associational activations. A network corresponding to an element of knowledge becomes robust through practice and experience.

Because cognitive networks are extended in the brain and because neurons have large numbers of synapses with other neurons, an individual neuron may be a part of multiple mutually-linked knowledge structures. As a result, activation of one network may result in the associated activation of other networks. Learning occurs as the result of the growth of new synapses that result in changing the topology of existing networks. Fuster says, "Learning takes place by the formation of new [networks] from old ones, by composition and decomposition of preexistent [networks]."[8] This is the neural analog of the fundamental idea of constructivism that plays such an important role in education research.

Cognitive scientists have studied the formation of associations in a number of different contexts and carried out extensive studies of simple associations. For example, it has been demonstrated that in recalling lists of related words, subjects tend to remember words that were not on the list as being there if the words are canonical members of the group (such as the word "doctor" in a list of health and medical related terms).[9] Collins and Loftus model these kinds of experiments with "spreading activation" — the activation of related elements in a network through a chain of links.[10] These experiments are classic "zero-friction": isolating the fundamental mechanism through a series of carefully controlled and constrained situations and cues. To see that these models from neuro- and cognitive science have relevance for real students in real situations, we have to turn to ecological observations. This is provided by behavioral studies and educational phenomenology.

The Resource Model of Cognition for Physics Education

The *resource model* of thinking provides a bridge between the neuro-cognitive model described above and results from education research to provide an appropriate language for discussing and analyzing the phenomenology of student problem solving in physics.[11,12,13] We refer to the basic elements of knowledge available to students thinking about a physics problem as *resources*.[14] A resource is a basic cognitive network that represents an element of student knowledge or a set of knowledge elements that the student tends to consistently activate together. Since different individuals may associate their knowledge in different ways, different individuals may use different levels of structure as resources. We use the term "resource" when we want to emphasize that a particular bit of knowledge is something a student can call on to solve a problem or draw a conclusion.

An example of a resource is the knowledge that what you see in a mirror is an image of yourself. Another is that the image you see is reversed. Other examples of resources are diSessa's *phenomenological primitives*.[15] These are basic statements about the functioning of the physical world that a user considers obvious and irreducible.[16] One example is "more cause leads to more effect" or "force causes motion."

Note that these resources are not right or wrong. To be applied to a particular situation, they have to be mapped onto physical variables. Thus "force causes acceleration" is a correctly applied resource, but "an unbalanced force is needed for there to be a velocity" is not. Note also that resources do not necessarily represent a full-fledged "conception." A conception (or misconception) may arise from the activation of a collection of resources.

The critical issue in understanding student thinking is how students' resources are activated in a particular context: by what stimuli and in what combinations. We use the term *pattern of association* to represent any set of connections where activation of one or more resources leads to the automatic activation (with some probability and in some contexts) of other resources. We refer to a frequently or easily activated pattern of

association as a *knowledge structure*. We use "pattern of association" and "knowledge structure" as our general terms rather than the more traditional term "schema"[17] because our terms seem more fluid and dynamic, in keeping with our overall neural-based model. We hope that these terms will encourage our readers to interpret them as dynamic rather than as fixed objects. There have been explicit proposals for how some knowledge structures in physics may be organized.[18] We do not consider these details of structure here.

Results from Behavioral Studies and Educational Phenomenology

There is a large literature on network models and the associational analysis of thinking in behavioral science.[19] Particularly important sources for the theory of patterns of association and knowledge structures are the expert-novice literature and the work of Rumelhart[20] and Marshall.[21] (Rumelhart and Marshall both use the term "schema" in a general sense approximately equivalent to our term knowledge structure.) Rumelhart points out that one of the main activities associated with a schema is determining whether it provides the appropriate knowledge for dealing with a presented context.[22]

Recognizing and Using Knowledge

The particular knowledge structure that an individual activates depends on the cues that the individual perceives and how those cues are interpreted. In the studies presented here, the cues come from a problem-solving task. Problem solvers should be able to use the relevant characteristics of a problem to activate knowledge structures that will help them solve the problem.

The issue is not whether an instructor can provide an explicit cue to get the student to give the right answer. The issue is how to help the student learn to knowledge structures that respond to appropriate cues. For example, we might include the cues "action and reaction" in a problem and get a correct Newton's third law (N3) response from most of our students. What we really want is for the students to have a more expert response of activating N3 to the cue of any two interacting objects even when the code words are not given.

Depending on how information is interpreted by the individual, cueing appropriate knowledge structures may be easy or difficult, and cues that activate the appropriate knowledge for an expert may not do so for a novice. For a student with fragile knowledge structures and weak links between distinct parts, cueing on a narrow specific bit of information may not activate the appropriate knowledge. But even if it does, our study suggests that focusing on a local knowledge structure may make it harder for the student to also activate other useful knowledge structures that they know well.

A number of studies provide evidence that experts and novices encode (interpret) information differently.[23] This may cause the expert and novice to activate different knowledge structures when presented with identical cues. In order for a knowledge structure to be useful to a problem solver, he or she must be able to map information from a new situation or problem to an existing knowledge structure.

When instructors solve problems at the board, they want their students to develop powerful general knowledge structures for solving a range of similar problems. When students see a problem solved at the board, they often develop specific narrow knowledge structures that they can only apply to a particular problem and not to a class of problems. We refer to this as *surface pattern matching.*

If the knowledge structures a student develops are not flexible enough to adapt to different problem-solving situations, he or she may attempt to solve a new problem based on how a sample problem has been solved, even though the procedure may be inappropriate. Although surface pattern matching is a type of knowledge structure, it tends to be applicable only to very specific situations and not productive for further learning, unlike more dynamic and effective knowledge structures. In order to succeed, students may attempt to memorize a large set of surface pattern-matching structures. This is can be both more difficult and more volatile than learning a more flexible and powerful knowledge structure that is adaptable to many problems.

Linked Knowledge

For a knowledge structure to be useful in problem-solving, its components must be linked together and not just exist as isolated facts and pieces of knowledge. Marshall represents knowledge structures with *graphs* containing nodes and links from node to node.[24] The nodes represent knowledge resources — declarative facts and procedural rules. Lines connecting nodes represent links or associations among facts and rules. Implicit in the representation is a cueing context and probability weightings on each of the links. Figure 1 shows three sample knowledge structures. The one marked (a) is partially linked, the one marked (b) is completely linked, and the one marked (c) contains weakly linked substructures. Novices often have partially linked knowledge structures, resulting in their using a variety of distinct, surface (and perhaps contradictory) principles instead of a deeper and more consistent set. This is documented in a number of papers in the physics education research literature. Loverude, Kautz, and Heron's paper on work and the First Law of Thermodynamics provides a particularly good example.[25]

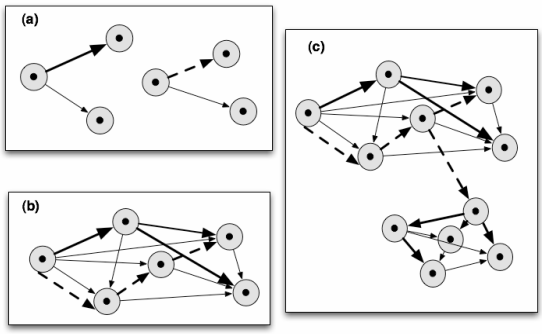


Fig. 1: Representations of knowledge structures. The nodes represent knowledge. The lines represent relations between different nodes.

Some links in an individual's knowledge structures are stronger than others. When a node is activated, it links to nodes with stronger connections more often.[24] This makes it possible for the individual to select appropriate associations beginning from a particular node and not simply activate everything associated with it. In some cases, experts may not have more knowledge than novices (though experts usually do tend to have more correct knowledge than novices[26]) but they make better use of their knowledge because it is better organized.[27]

It is important to note that knowledge structures are a classification that we apply to describe an individual's thought processes, not a rigid structure in the brain. Individuals make many associations; for resources that have a reasonably high probability of activating each other we identify the set as a knowledge structure. To be useful for problem-solving, an individual's knowledge structures must be coherent, reasonably complete, and accurate.

In our interviews, we use the ease with which one bit of knowledge is produced following on another bit as evidence of an association. This makes common sense, since what we mean by association is that one idea leads easily to another. It is also consistent with the spreading activation model[10] and the use of time delays to measure associational strength used in cognitive science.[28]

Local and Global Coherence

The way in which knowledge is encoded and linked leads to different types of coherences among knowledge elements and structures. To describe this, Marshall introduces the ideas of *internal* and *external consistency*. By internal consistency, she means that a particular knowledge structure is self-consistent, i.e., running it in different ways does not lead to contradictions. By external consistency, she means that different knowledge structures (schemas or models, in her terminology) when run do not lead to contradictions. Experts' knowledge structures are more often composed of bundles of knowledge about the physical world that are both internally and externally consistent. In contrast, the novice may have knowledge structures composed of pieces of inconsistent knowledge, as viewed by the expert.

For our study, we want to focus on the access and activation of patterns of association rather than on correctness as evaluated by an expert. We, therefore, introduce the terms *local* and *global coherence*. By saying that a set of knowledge is *locally coherent,* we mean that the user sees the nodes of that knowledge as closely related and appropriate for use with each other. By saying that a set of knowledge is *globally coherent,* we mean that a user considers sets of locally coherent knowledge structures that he or she sees as distinct as appropriate to the problem and useful together. For example, in figure 1(c) the upper and lower clusters of nodes represent locally coherent knowledge structures that are distantly related. If both were appropriate to the problem at hand and the user used both, we would call this an example of global coherence.

This is a dynamic viewpoint and can evolve as the user learns more. Thus, if users can use their various bits of knowledge of force and dynamics of motion together in a related way, we say that their knowledge is locally coherent. Similarly, their knowledge of the concepts of energy and work may be locally coherent. If they can use these two sets of knowledge together in a consistent way but still see them as distinct knowledge elements, we say their knowledge of forces and energy are globally coherent. As a user becomes more expert, what is seen as local or global may change. An expert may have a fully integrated knowledge structure in which energy and dynamics are tightly intertwined and together form a locally coherent knowledge structure.

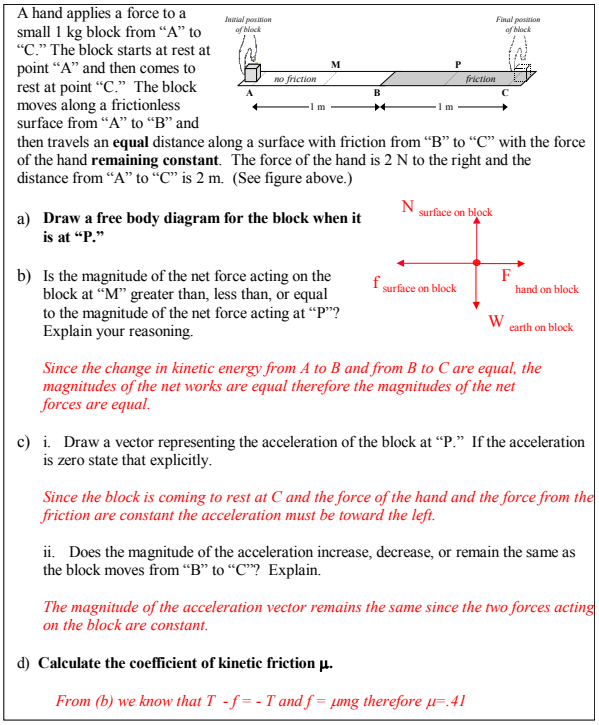
Background and Context of the Studies

We began our investigations by creating a problem that would allow us to explore how well students integrated their force/dynamics knowledge with their energy/work knowledge. The original version was written by Sabella and given to undergraduate students in first semester calculus-based physics at the University of Maryland as a supplemental homework problem and to a small group in exploratory semi-structured one-on-one interviews. The results of these observations suggested that students were having trouble integrating their knowledge of force and energy. As a result of what we learned, we revised the problem and carried out two studies.

In our first study, we gave the revised problem in a semi-structured one-on-one interview to six advanced physics students (senior physics majors and graduate students). The results demonstrated that some students

had locally coherent force/dynamics knowledge that excluded significant linkage to their work/energy knowledge. To determine the extent to which this phenomenon was present in our population of introductory physics students, we carried out a second study.

In our second study, we look at the way classes of undergraduate students in the engineering physics course performed on two variants of the revised problem that we presented to the advanced students. Both variants of the revised problem are shown in figure 2 along with a model solution (in gray text). The undergraduate students had received traditional lecture instruction combined with instruction on qualitative reasoning using a pre-publication version of the University of Washington Tutorials[29] before being given the problem as an ungraded quiz.


A hand applies a force to a small 1 kg block from "A" to "C." The block starts at rest at point "A" and then comes to rest at point "C." The block moves along a frictionless surface from "A" to "B" and then travels an **equal** distance along a surface with friction from "B" to "C" with the force of the hand **remaining constant**. The force of the hand is 2 N to the right and the distance from "A" to "C" is 2 m. (See figure above.)

a) **Draw a free body diagram for the block when it is at "P."**

b) Is the magnitude of the net force acting on the block at "M" greater than, less than, or equal to the magnitude of the net force acting at "P"? Explain your reasoning.

Since the change in kinetic energy from A to B and from B to C are equal, the magnitudes of the net works are equal therefore the magnitudes of the net forces are equal.

c) i. Draw a vector representing the acceleration of the block at "P." If the acceleration is zero state that explicitly.

Since the block is coming to rest at C and the force of the hand and the force from the friction are constant the acceleration must be toward the left.

ii. Does the magnitude of the acceleration increase, decrease, or remain the same as the block moves from "B" to "C"? Explain.

The magnitude of the acceleration vector remains the same since the two forces acting on the block are constant.

d) **Calculate the coefficient of kinetic friction μ.**

From (b) we know that T - f = - T and f = μmg therefore μ=.41

Fig. 2: Revised version of the hand-block problem with a model solution. This problem was asked as an interview with advanced students and as an ungraded quiz in the introductory mechanics class

Study 1: Advanced Students

We presented the dynamics-work energy problem shown in figure 2 to six students enrolled in graduate classes at the University of Maryland as a problem-solving interview. A model solution to this problem would involve the application of principles of force and energy and what we might call a globally coherent knowledge structure. In particular, we would like to see the students tying the work-energy theorem to their knowledge of force. The students were given the short version of the question, consisting of parts a) and d). The complete problem, shown in the figure, was given to one of the students after he had difficulty solving the shortened version.

In these interviews the researcher provided the student with a paper copy of the problem and asked the student to solve it, while explaining out loud what he or she was thinking and writing as the solution progressed.[30]

The Student Responses

The volunteer participants in this study were one upper-level undergraduate student who was enrolled in graduate level classes, three first-year graduate students, and two second-year graduate students.

The three first-year graduate students exhibited many qualitative difficulties when answering the problem, while the two second-year graduate and the upper-level undergraduate students answered the question correctly, with little or no prompting. The undergraduate student seemed to exhibit the most coherent knowledge. He continuously went back and forth between force and work-energy ideas. We present here a detailed analysis of the interviews with four representative students: one who displayed an integrated knowledge structure containing both dynamics and energy (Mark), one who showed a strongly integrated knowledge structure of dynamics (John), and two who displayed a restrictive local coherence (Tom and Dee-Dee). The transcript excerpts presented include a gender specific code name for the student and use the following short-hand notation: [ ] indicate comments added about the interview by the researcher after the fact, {—} is a short pause, [pause] is a long pause, {…} indicates that unimportant words were purposely omitted from the transcript to facilitate the reading, and (IA) indicates that the words were inaudible.

Mark

Mark is an upper-level undergraduate. He successfully integrates the concepts of force and energy into a coherent knowledge structure that he uses to solve the problem. In this section of the interview, Mark is looking for the coefficient of kinetic friction. He has already drawn a correct free-body diagram.

M: Let's see— the block travels an equal distance (IA) with the force remaining constant — Let's see — (IA) — let me think — does it say anything about the speed — it doesn't — oh okay I see — I suppose the force is being applied until the end of the trajectory and the block stops due to the friction and not that the hand stops.

I: The hand keeps applying from A to C.

M: Okay — So I'm going to calculate the kinetic energy that the block has until point B —

I: How come you're doing that?

M: To find out what the total — what the energy it loses on the friction surface is — which should tell me —yes of course — what the force acting against it was. So that is going to be 2 Newtons times 1 meter, which is [2] Joule[s] and that is equal to ½ m v squared — ... v being the velocity of the block — and that is exactly what it is going to lose which means the force — the friction force should be equal to 4 Newtons in the other direction — meaning ... in the direction C to A, or to the left — So that the net force being applied on the block is 2 Newtons in the other direction so that the loss of energy is equal to the gain of energy in the first half of the trajectory.

At this point Mark connects his knowledge of force and work-energy and uses them to solve the problem. He makes these connections throughout, indicating coherence among these knowledge elements.

I: How did you know the loss in energy was the same as the gain in energy?

M: Because it started at rest and it ends at rest — I suppose. I assume that is what it means when it travels an equal distance — ... [Rereads part of the question.] So the force is 4 Newtons, which is equal to the magnitude of the normal force times μ — the kinetic friction coefficient— ... so the normal force is equal to the weight of the block which is 1 kilogram times 9.8 so — □ is equal to 4 over 9.8 which is about .4.

I: ... Can you compare the magnitude of the net force at M to the net force at P — how would they compare?

M: The magnitude of the net forces? — well they should be equal and opposite —

I: And how did you know that?

M: By the same argument — because I assume that the force due to friction — which is constant along the whole surface since the weight of the block doesn't change — I assumed that it was equal and opposite to the force — I'm sorry — I mean the sum of the force being applied by the hand and the friction should be totally equal and opposite to just the force applied by the hand on the block so that the loss of energy is equal along the same distance traveled — so they will be equal and opposite.

The question concerning the magnitude of the net forces on the two regions was particularly difficult. Mark answers correctly without hesitation by applying an integrated knowledge of dynamics and energy. Even some of the graduate students who solved for μ correctly answered the question about the net force incorrectly (at least at first). A pictorial representation of Mark's interview is given in figure 3.

John

John is a second-year graduate student. Like Mark, he solves the problem correctly in a short amount of time. Unlike Mark, he only uses a tightly linked set of force/dynamics knowledge to solve the problem, instead of a knowledge structure containing both dynamics and work-energy. It is interesting that even though John correctly solves the problem, he first states that the magnitude of the net force at P is smaller than the magnitude of the net force at M. He corrects himself soon after. The excerpt below is taken from the interview.

J: ... so it will be minus 4 over 2 — which is the same acceleration — but opposite sign ... and now we can plug it back into this equation (points to $F - \mu_k N = - ma$) for the force and the coefficient of friction and so we have 2 Newtons — we are going to put numbers immediately — 2 minus μ_k will be equal to F plus ma over N — μ_k is 2 Newtons plus ... I'm using here the absolute values of the acceleration over the 10 ... so we are going to have 0.4.

I: How does the net force at M compare to the net force at P?

J: Umm — the net force at P is smaller than at M — by the amount of the kinetic friction. The net force is smaller by this amount because — the y components of the two forces are canceled out. So the only difference — they will be cancelled out at point A too — and the only difference comes at point P because of the introduction of the force of friction, which is directed opposite to the applied force.

I: So can you draw me a vector for the net force at point P — how would that look?

J: It would look — almost caught me there — yeah the force of friction is bigger than the force — this is net force — since the acceleration is negative — so negative y direction — net force — according to the famous Newton's second Law should ... I'll put it here to support my statement — the net force should be in the same direction as acceleration —

A pictorial representation of John's interview is given in figure 4.

Tom

Tom, a first-year graduate student, exhibits many conceptual difficulties while solving this problem. In addition, we see a lack of coherence in his knowledge, evident from his inability to go back and forth between

the two topics and the contradictory statements in the interview.

Tom initially identifies all the forces in the free-body diagram correctly. Later, he incorrectly describes the force of the hand to be greater at point P because the block is still moving toward the right. Tom appears to have applied the force-motion p-prim in such a way as to incorrectly link force and velocity. In addition, Tom states that at point C the force of the hand would be equal to the force of friction. (Although this is true after the block has stopped, it seems that Tom makes the statement because the velocity is zero at C and not because it remains at zero.) When solving for the coefficient of friction, Tom set the two forces equal and solved for μ. But this solution does not feel right to him. Tom tries to access different physics principles when his analysis using dynamics does not seem correct, but these alternate principles lead to dead ends and Tom goes back to thinking about the forces. Much of the dynamics knowledge Tom activates in this problem is stated fairly quickly, indicating a local coherence among these elements. The difficulty lies when Tom recognizes that he needs additional knowledge to solve the problem.

T: Yeah. So wait maybe that should be right. [pause] That's not right at all — ... it doesn't seem right to me — just give me a second — I just started to solve it assuming it would be easy — then realized maybe it wasn't as easy as I thought. Could also do the work — the work from here [points to A] to there [points to B] — no that's got nothing to do with it — no — friction — 2 Newtons — the force of the hand remains constant — because according to this — this wouldn't be stopped there — it would stop there if this were the case — ... if this were the case if it was 9.8 Newtons and 2 Newtons because that is the same circumstance that we have at B — so it would stop at B the way I have it set up — why would it stop at C? There is something about the 1 meter that I'm not getting — I'm not thinking very well. There has to be something to do with ... the velocity — with the hand force I'm thinking — so I think — there is something to do with friction apparently —

This excerpt shows that Tom uses linked pieces of his knowledge to try to activate some procedure that he could use to solve the problem. In this small excerpt he brings up work, the force of the hand, the force of friction, the 1 meter, and the velocity. Unfortunately, these items lead to dead ends even though these ideas are all closely related to the work-energy theorem and the definition of work. Although Tom does activate pieces of knowledge that are associated with the knowledge structures of work and energy, the fact that Tom does not pursue this line of reasoning, despite the fact that he is at somewhat of a standstill, indicates that he is unable to activate the larger set of knowledge about work and energy that an expert would. Without having this knowledge connected by links that activate in appropriate contexts, Tom cannot get to the work-energy theorem.[31] A pictorial representation of Tom's interview is given in figure 5.

Dee-Dee

Dee-dee is another student who has serious conceptual difficulties with the material. Like Tom, she draws a correct free-body diagram for the block but seems to be confusing velocity with acceleration throughout the interview. She seems to be using the idea of a maintaining agency, which states that a force is required to keep an object moving.[32] A section of the transcript follows.

I: How does this force [friction] compare to that force [hand]?

D: Well if it's still moving forward then this [points to force of hand] is bigger then this [points to friction force] — it's not enough to stop it.

This statement is particularly interesting because Dee-Dee stated Newton's second law (N2) correctly during the interview. The excerpt below shows that Dee-Dee is very unwilling to give up her qualitative ideas about force and motion, even though she has already written down the correct algebraic form of N2. Her qualitative and her quantitative dynamics knowledge appear to be associated (they occur very close in time) but they have not been reconciled into a consistent knowledge structure. Dee-Dee's use of maintaining agency is consistent with the force-motion p-prim and directly contradicts her activation of the quantitative form of N2.

I: So what happens at point C?

D: It stops — zero — velocity equals zero — ...Oh — I see what your saying — but that is because the force is not acting anymore — the hand is not pushing anymore —

I: The hand acts all the way to point C.

D: Oh — so it just stops at point C — for no reason? — ...

I: The force of the hand is remaining constant from point A to point C

D: Then if there is no impediment there then friction is greater than the hand pushing it.

I: How did you know that? How did you know friction was greater?

D: How? Because the force of the hand is the same — so friction must have dominated that.

I: How did you know it dominated?

D: Because it came to a stop —

I: Before, you said this force [points to force of hand] was greater than the force of friction.

D: Cause it was moving that way … — The coefficient must have been — No — Not the coefficient — I don't know — if the force of the hand is the same then the friction must have been different.

A pictorial representation of Dee Dee's interview is given in figure 6..

Advanced students' reasoning maps

We present *reasoning maps* to represent some of the interview data. (Note that these maps are intended to describe the students' behaviors. They should not, based on only our data, be interpreted as static models "present in the students' minds." They may be created dynamically in response to the given context. In addition, we are using the students' verbalizations to infer what the students are thinking although we do not expect that this is a direct mapping.) Each statement of physical reasoning from the interview is presented in the map, with a link to the next statement and each statement is coded. Statements are shaded lightly if they are based on ideas that come from dynamics knowledge, dark gray if they come from work and energy, speckled if it is unclear where they come from, and shaded darkly with words written in italics if the statement was made by the interviewer.

Maps for two of the students who solved the problem correctly are shown in figures 3 and 4. Even though neither student has difficulty with the problem and each student's statements are consistent and correct, the two solutions are quite different. Mark (figure 3) goes back and forth between his knowledge of dynamics and his knowledge of work and energy. John (figure 4) primarily uses his dynamics knowledge structure. When John needs some additional information he uses a formula from kinematics. (This formula could also be interpreted as coming from the work-energy theorem but John did not explicitly make this connection.) We have shaded the statement lightly because he uses the form of the equation usually introduced in kinematics and dynamics. John's reasoning map shows mostly dynamics knowledge, suggesting that he has a locally coherent knowledge structure of force and motion, but we cannot say anything about his work-energy knowledge structure or how well these topics are integrated.

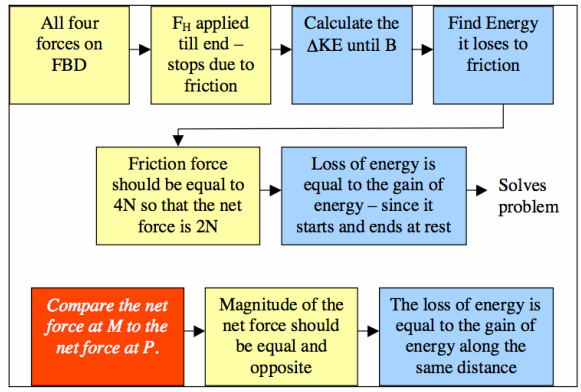


Fig. 3: Mark's reasoning map showing that he was able to go back and forth between his knowledge structures for force and work and energy.

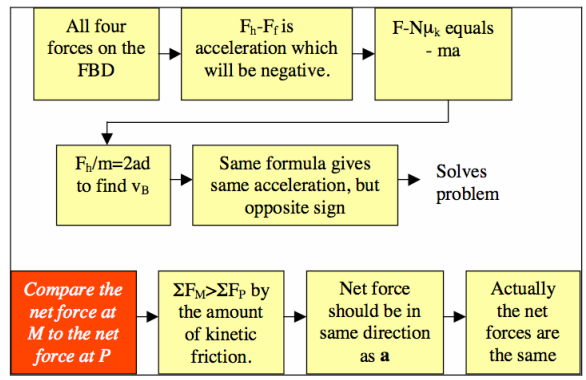


Fig. 4: John's reasoning map showing that even though he used the knowledge from the dynamics schema exclusively he was able to solve the problem in very few steps.

Figure 5 is a map of Tom's interview. We can see that he primarily uses his knowledge of force and dynamics. Although he mentions statements about work and energy, they lead to dead ends and he goes back to thinking about the forces and the motion of the block. The map also shows that Tom makes many contradictory remarks in his interview. Some of these inconsistencies could have been resolved if the appropriate links were made between different knowledge structures.

Dee-Dee's map is shown in figure 6. It shows that she also tries to solve the problems by thinking about forces through most of the interview. Here, knowledge structure of forces appears to be poorly connected to her knowledge of work and energy (and internally inconsistent, mixing formal knowledge with contradictory p-prims). The interviewer asks her three questions relating to the work-energy theorem but she has difficulty tying these ideas into her analysis.

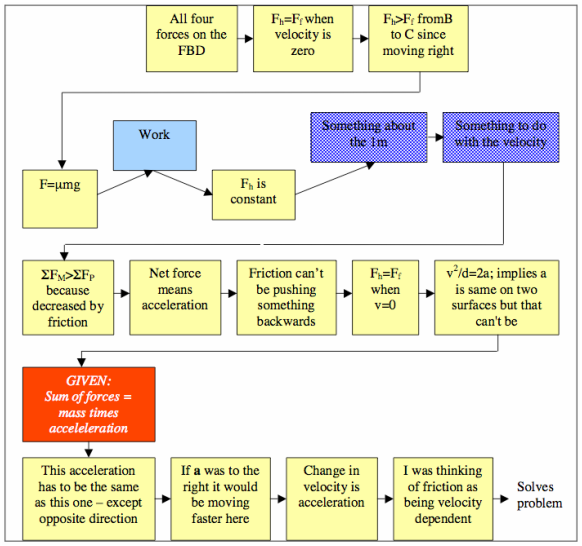


Fig. 5: Tom's reasoning map shows that although he tried to bring up the ideas of work and energy they lead to dead ends.

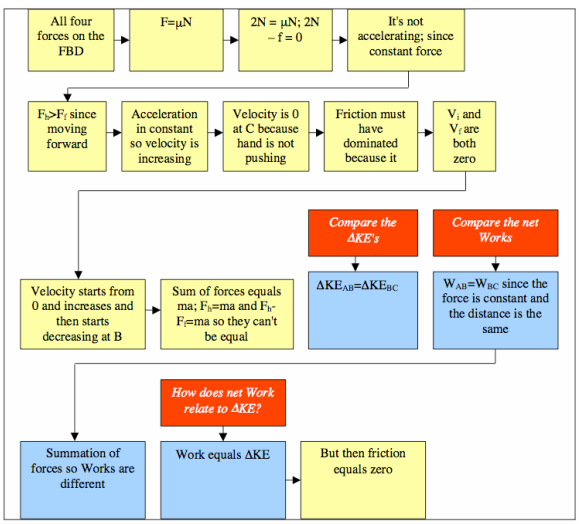


Fig. 6: Dee-Dee's reasoning map showing that even with hints from the interviewer it was difficult for her to reconcile the dynamics information.

The interviews and the reasoning maps illustrate what we mean by knowledge structures[33] — they are strong patterns of association between particular knowledge elements in response to a given context. The maps are consistent with the fact that for different individuals the same knowledge can be connected in different ways. In addition, for different individuals, connections can be strong or weak. Tom attempts to activate different types of knowledge, from different physics topics. (Recall the excerpt where he brings up work, velocity, and the distance the block travels.) The fact that these attempts lead to dead ends leads us to believe that his relevant knowledge structures (for dynamics and work-energy) are relatively isolated.

Conclusions of Study 1

From numerous personal, informal interactions, we are aware that graduate students at the University of Maryland have both dynamics and work-energy knowledge structures and are very capable of using these each of these structures separately in solving traditional problems on these topics. A model solution to this problem would involve the application of multiple physics principles and ideas or what we might call a globally coherent knowledge structure. In particular, we would like to see the students tying the work-energy theorem to their knowledge of force. However, these interviews provide evidence that even some advanced students exhibit strong local knowledge structures (some of which are internally inconsistent) and do not combine their locally coherent force knowledge with their energy knowledge. Three of the six students were unable to go back and forth between the ideas of force and work-energy and even when prodded towards energy, one student (Tom) made only brief forays into the topic, resisting strongly a change of intellectual venue. Some of the students also made contradictory remarks that they found difficult to resolve. (The evidence for local coherence lies in the fact that these bits of knowledge are brought up rapidly during these interviews, one statement leading directly and easily to the next.)

These results suggest that local coherences may set up barriers to the activation of other related physics topics. In order to probe this issue more explicitly, we set up a controlled study with a larger group of introductory physics students.

Study 2: Engineering physics students

Overview

In our second study, we compared students' performance when they were presented with one of two versions of the same mechanics problem that we used in study 1. Students were asked to solve either a short version or a long version of the problem shown in figure 2. The short version asked two questions, the long version four. The questions in boldface appeared on both versions. The purpose of the non-boldfaced questions was to see whether cueing a qualitative analysis of the forces in the problem would improve or harm students' access to the appropriate knowledge needed to solve the problem: either quantitative treatments of force or the work-energy theorem.

Students in the mechanics term of calculus-based physics were given the problem as an ungraded quiz in recitation sections. The short version was administered to 40 students, and the long version was administered to 69 students. Because the quiz was asked in the recitation sections and not all students attended recitations,

not all students in the class participated in the study. Students were given 15 minutes to complete the quiz.

Many of the undergraduate students exhibit patterns of association that are characterized by local coherence but not by the global coherence that would characterize an expert problem-solver. In particular, the inclusion of qualitative force-based questions (long version of the problem) resulted in an increase in the fraction of the students activating a pattern of association about forces and a reduction in the fraction using energy methods. This suggests that, for these students, force knowledge was isolated from work and energy knowledge. Students who were not presented with the qualitative force questions (short version of the problem) were more likely to activate their knowledge of work and energy to solve the problem.

We first discuss the student responses on the long version of the problem. We then show the results of a comparison on the responses to the last part of the problem, which was answered by students with both versions.

Student responses to the qualitative force question: long version question (b)

Part a) of the problem asked students to draw a free-body diagram for the block when it was on the surface with friction. Almost all students correctly identified all four forces on the block, although in most cases it was difficult to check the relative magnitudes of the forces.

N=69

	Correct: The net force is equal	Incorrect: F_{net} greater on non-friction surface	Incorrect: F_{net} greater on friction surface
Compare magnitude of net force at **M** to **P**	12% ± 4%	56% ± 6%	26% ± 5%

Table 1: Performance on the question asking to compare the magnitudes of the net forces in the two regions.

Question b) proved to be extremely difficult for these students given the long form of the problem. Only 12% answered correctly. The results are shown in Table 1. (Uncertainties in the reported results are estimates of sampling error calculated as $(pq/N)^{1/2}$.) The most common error, given by 56% of the students, was that the magnitude of the net force on the non-friction surface was greater than the magnitude of the net force on the friction surface. One way this error commonly arises is through activation of diSessa's Ohm's p-prim.[17] The Ohm's primitive comes from the compensating type of reasoning that is associated with Ohm's Law. A part of the Ohm's primitive states that an "increased resistance leads to less result." Because the block first travels over a non-friction surface and then over a surface with friction, the resistance on the block increases thereby decreasing the result, which in this case can be interpreted as the net force. We commonly saw this type of response in interviews with both the undergraduate engineering students and the advanced physics students.

In addition, 26% of the students stated that the net force on the frictionless surface would be less. One way this result could come about is if students were not considering the vector nature of the forces and were just thinking of the net force as the number of forces acting on the block. This interpretation is supported by evidence from some of the students' written responses. Two examples of student responses showing each type of incorrect response are shown in Table 2.

Two sample student responses	Case 1: "It is greater at M because there is no frictional force working against the F_{hand}."	Case 2: "[It is] less than [at M] because friction is being invoked at point P in addition to the 2N."

Table 2: Sample student responses comparing the net force on the friction surface to the non-friction surface

Student responses to the qualitative acceleration questions: long version question (c)

In part c) the students were asked two conceptual questions about the acceleration vector on the surface with friction. The results, shown in Table 3, indicate that 41% of the students answered correctly, that the direction of the acceleration vector was to the left. On the other hand, 32% of the students stated that the magnitude of the acceleration vector does not change as the block moves from point B to point C, where it comes to rest. These results are consistent with previous work indicating that students often treat acceleration as if it were proportional to velocity.[34] Another 20% of the students stated that the acceleration of the block at point P was zero.

The results on the qualitative questions indicate that students still have many difficulties with N2, even though instruction on Newton's laws was completed a few weeks before this study was conducted.

We are now in a position to examine the student responses on the final part of the problem, where students were asked to calculate the coefficient of kinetic friction.

N=69

	Correct: Vector directed to the left	Incorrect: Zero	Incorrect: Vector in the direction of motion
Acceleration vector at point **P**	41% ± 6%	20% ± 5%	19% ± 5%

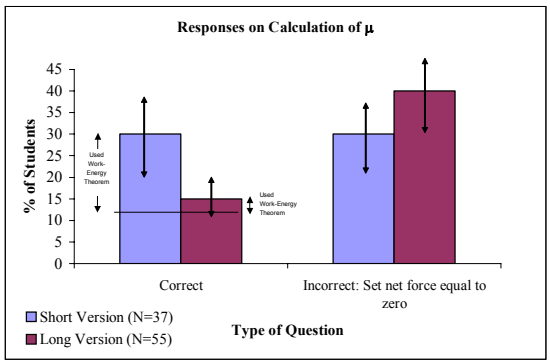
	Correct: Acceleration is constant	Incorrect: Acceleration is decreasing
How does the vector change?	32% ± 6%	55% ± 6%

Table 3: Performance on the questions concerning the acceleration vector on the surface with friction.

Student responses to the quantitative question: comparing short and long versions (d)

Figure 7 shows the results on the quantitative part of this problem for the students taking both the long and the short version. It shows the percentage of correct responses as well as the percentage of students setting the net force on the block equal to zero in the region with friction. The percentages listed include only the students who had enough time to attempt the final question.

Even though students had covered this material in lecture and had homework assignments on the material, fewer than 30% of the students answered this question correctly. The results show that students performed better on the short version of the problem.



Data does not include students that did not complete the problem.
Unfinished: 8% on short version, 20% on long version

Fig.7: Performance on the quantitative part of the problem showing that students performed better on the short version. The graph also indicates that the methods of solution on the long and short version are different.

We can see from figure 7 that more students used the ideas of work-energy to solve the question on the short version of the problem. Most of the students who did not solve the problem using work-energy used ideas from kinematics to try to solve for the acceleration of the block from point B to point C.

These observations provide evidence of the dynamic nature of student knowledge structures. The data support the idea that, for these two populations of students, particular elements in the problem sometimes activated distinct sets of knowledge.[35] If the conceptual questions lead students into a dynamics/force knowledge structure that is not linked to their work/energy structure, the students may try to solve the problem using only force. This extends the observation that novice problem solvers tend to cue on the surface features of a problem.[36]

Since all the conceptual questions focus on force, it is possible that our students responded by cueing knowledge structures with strong links only to dynamics. We had hoped that our students would use their dynamics knowledge to activate work-energy knowledge. This does not appear to be the case for these students, even though they had received a mix of conceptual and quantitative instruction.

Qualitative vs. quantitative methods

We note that, although the results in study 2 are consistent with those in study 1, suggesting that a significant fraction of the students in calculus-based physics have trouble integrating knowledge of force and knowledge of energy, there is a complicating factor for this population. In addition to knowledge structures for organizing specific physics content, students at the introductory level may also have created isolated knowledge modes (epistemological control structures that affect how they use their conceptual knowledge) for their qualitative knowledge and their quantitative knowledge within these physics topics.

Although many education reformers (and some instructors) believe that conceptual questions help students cue the correct knowledge needed to solve a complex problem, our data suggest that if the students have not adequately integrated their conceptual and formal knowledge, conceptual questions can actually hurt their performance on quantitative problems. Cues that help a physics instructor activate a set of different interrelated knowledge structures do not necessarily help students activate those structures. In addition, some of these cues can cause students to activate a particular knowledge structure that may be isolated from the relevant knowledge structure for a given task.

These students made two main types of errors. Some solved for the coefficient of kinetic friction correctly yet demonstrated a serious qualitative misunderstanding. These observations are consistent with those of Mazur who shows that students can succeed on quantitative problems but have difficulty with qualitative discussions of the same or similar situation.[37] Other students gave appropriate qualitative responses that they did not apply when answering the final quantitative question. Our results extend Mazur's point a step further, showing that cueing qualitative knowledge, even when the students have that knowledge, may actually inhibit appropriate quantitative responses.

We cannot distinguish in this experiment between the activation of the conceptual knowledge of forces inhibiting student use of the knowledge of energy and the activation of a qualitative mode inhibiting the use of quantitative reasoning. Either one makes our point that cueing one kind of knowledge structure can inhibit the activation of others. Note that these issues (force vs. energy and qualitative vs. quantitative) are not "either-or." Both are relevant and it may be difficult to disentangle them in any particular example. Some de-

tail from the student responses can help us get some insight as to how these issues are related here.

Table 4 shows examples of student inconsistencies on our mechanics problem. Even though some of the students could solve the quantitative question correctly, many of them still had serious qualitative difficulties. Answering the quantitative question correctly requires that the student use the fact that the magnitudes of the accelerations are equal in the two regions. Note that only eight students answered part d) correctly on the long version. We observe that five of the eight students who answered the final part correctly stated incorrectly that the magnitude of the net force was different in the two regions. Only three of the students who solved for the coefficient of friction correctly stated that the acceleration vector was decreasing from point B to point C. Perhaps an even more surprising result is that seven of the 41 students who drew a non-zero acceleration vector in part c) set the net force equal to zero when solving the quantitative question – showing a clear disconnect in their qualitative and quantitative knowledge.

Of the students who could calculate μ correctly many made serious conceptual errors.	Incorrect: Stated that the magnitude of the net force in the two regions were different on part B	Incorrect: Stated that the acceleration vector decreases as the block moves from B to C	
	5 students	3 students	**N=8**

Of the students who had a non-zero acceleration vector some answered inconsistently on the quantitative part.	Incorrect: Stated that the net force was equal to zero when solving for μ.	
	7 students	**N=41**

Table 4: Inconsistencies in the student responses on the hand-block problem.

Conclusions of study 2

The data from the ungraded quiz suggest that, for these students, physics knowledge may be organized in structures that are only weakly linked. If these students were developing global coherence in their knowledge, we would expect qualitative questions to help in solving these problems, instead of hurt. For the problem shown in figure 2, qualitative questions that lead students to a force/dynamics knowledge structure seem to isolate them from other pieces of knowledge that could have been helpful in solving the problem. Students who were given the problem without the qualitative component were more likely to activate their work-energy knowledge.

Implications for Instruction and Research

The resource model of knowledge elements connected in associational patterns that has helped us make sense of our data has a number of implications for our interpretation of what we see in the classroom, what our goals might be for instruction, and how we might further explore student knowledge in our research. We classify these implications briefly as recognition, organization, and coherence.

Recognition

Having knowledge is not sufficient: it must be activated in appropriate contexts. Students who are learning new knowledge often have trouble recognizing that some knowledge they know is appropriate when needed in a different context from the one in which the knowledge was learned. This leads to an apparent *context dependence* of student knowledge. This feature of the model reminds us that if a student does not use a particular fact or process in a given situation, that does not necessarily mean the student doesn't possess that knowledge. It may mean that the student has not correctly associated the knowledge to the conditions and circumstances relevant to its use. All knowledge is context dependent; the critical factor is whether knowledge is activated in appropriate contexts. Given simple physics problems, experts may activate relevant fundamental principles while novices may activate inappropriate knowledge of what equation was used in a problem with similar surface structure.

We not only have to teach our students knowledge, we have to make it functional by helping them learn to recognize the situations in which it is appropriate to use that knowledge. We not only have to do research to understand what "difficulties" or inappropriate responses our students have, we need to do research to understand how to cue appropriate links so that our students can learn to build appropriate activations.

Organization

These studies, particularly the detailed interviews with the advanced students and the failure of the introductory students to see inconsistencies in qualitative and quantitative responses, show that having lots of accumulated knowledge does not suffice. Knowledge has to be organized in appropriate ways to allow relevant knowledge to be activated appropriately. Our instruction often focuses on "getting the physics" or "learning the concepts" and fails to help students integrate the ideas they are learning into a usable whole. Straightforward exercises that only activate a single physics principle or idea send our students the wrong message: that only the "facts" they are learning are important. More complex problems, especially carefully chosen ones, may do a better job in helping our students learn to integrate their knowledge. We need more research to explore how students integrate their knowledge and how the activation of that integration depends on context.

Coherence

Building a coherent knowledge structure has to be one of the goals of a scientific education. Our studies indicate that traditional training may leave our students (even after a full undergraduate physics major's program) with inadequate cross-connections and consistencies between the different parts of their knowledge. Studies using survey instruments (the MPEX[38]) indicate that students' sense of the importance of coherence does not generally improve as the result of a traditional introductory physics course or even as the result of an introductory course reformed to improve conceptual knowledge. These self-reports do not necessarily reflect the actual coherence of the students' knowledge structures, but if students don't consider coherence important, it is unlikely they will pay much attention to the integration of their knowledge. More explicit instructional effort towards building coherence appears to be required and research on what helps students learn to seek coherence could be of considerable value.

Conclusion

Instructors and curriculum developers can use the idea that students often form isolated sets of knowledge during instruction. At the simplest level, the associational character of the resource model highlights the importance of helping students develop explicit links to related topics in the physics courses. In addition, instructional materials, as well as exams, should be designed to help students develop the necessary connections between various physics concepts as well as connections between qualitative and quantitative knowledge. Although this may be obvious to many, anecdotal evidence suggests that the vast majority of physics exams are dominated by questions that each deal with a single specific topic or that do not require students to link their qualitative understanding to quantitative problem solving.

In this paper we have presented evidence for the existence of locally coherent student knowledge structures: strongly related sets of knowledge that are brought to a problem-solving task. The knowledge structures that our students form, unlike those of an expert, are often characterized, at best, by local rather than global coherence – isolated from other appropriate and related knowledge structures. These characteristics can hinder students when they attempt to solve complex and challenging problems. In addition, even in reformed instruction where there is an emphasis on qualitative reasoning about a particular topic, students may tend to group this qualitative knowledge separately from the quantitative knowledge, forming isolated, weakly linked knowledge structures.

Our results suggest that these weak links can actually cause our students to perform more poorly when they are presented with qualitative questions before a final quantitative part. This may be due to students activating a particular knowledge structure as a response to the qualitative cue and then getting trapped in knowledge elements strongly linked to that particular structure. If the knowledge structure that is activated does not contain all the information needed for the problem, the student may not be able to access the needed knowledge, even if he or she possesses it.

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