

# CHANGING STUDENT WAYS OF KNOWING: WHAT SHOULD OUR STUDENTS LEARN IN A PHYSICS CLASS?

EDWARD F. REDISH

*Department of Physics, University of Maryland  
College Park, MD, 20742-4111 USA*

Over the past twenty-five years, educational research on introductory university physics classes has demonstrated that student learning is often significantly less than we hope and expect. Specific conceptual difficulties have been identified in a wide variety of topics. Research-based curricula designed to improve student conceptual learning can yield substantial gains over traditional instruction. I review some of the results of this research and development and describe a project at the University of Maryland that explores the next step: understanding the deeper and less explicit elements of science learning that I refer to as “the hidden curriculum.” The results of this project indicate that it is possible to provide explicit instruction to change students’ ways of knowing even in a large lecture environment.

## **1. Physics Education Research: Learning what students do and don’t learn**

Beginning in about 1980, extensive research has demonstrated that in most areas of physics, introductory students often have many more difficulties learning physics than their instructors realize. This could be intractable if every student had different difficulties that required special treatment. Few institutions have the resources to instruct students individually. We would have to be satisfied with reaching a small fraction of our students.

Fortunately, research shows that most student difficulties can be described in terms of a small number of common conceptual misunderstandings. These can be very robust and students often retain them even after traditional instruction. Research-based curriculum development has been able to produce improvement on conceptual surveys and exams without increasing the time-on-task, but this is only one step in a long process of improving our physics instruction.

We want our students to know more physics than just algorithmic problem solving and more than just conceptual knowledge. We want them to be able to understand and reason about complex physical situations. Many of the goals of our curriculum concern the development of skills we rarely explicate or discuss – a “hidden” curriculum.

In a recent project, the University of Maryland Physics Education Research Group transformed a large-lecture algebra-based physics class so that much of the student learning took place in environments where they could be videotaped and studied. Observations throughout the year demonstrate that a coherently transformed instructional environment can help students learn physics concepts and make gains on hidden curriculum items such as building physical intuition, and developing their ability to reason scientifically.

### 1.1 PER: Goals and tools

Physics Education Research (PER) is an inter-disciplinary area of study. Many of us who do PER are physicists who have turned our attention to trying to improve the effectiveness of physics teaching. Our goal is to use as scientific an approach as we possibly can to help us do this. As in any scientific endeavor, the first step is to better understand the phenomenon. For this purpose, we have to figure out:

- What is really going on in our classes?
- What are our real goals for our students?
- How we can accomplish those goals more effectively?

To answer these questions, we have to observe the process of teaching and learning in detail. Some of the observational tools of PER include:

- detailed interviews with individual students
- videotaping student work in group environments (labs, tutorials, solving problems)
- collecting students' written answers to homework, exams, and ungraded quizzes
- having students complete pre- and post-testing with standardized tests.

One of the first and most important results revealed by this research is that even students who can solve problems algorithmically may not have good conceptual knowledge.

### 1.2 Getting students to solve algorithmic problems is not enough

Physics education researchers have demonstrated that many students learn to do algorithmic problem solving quite successfully without having a good understanding of the phenomenon the algorithm describes. They can “turn the crank” but not “make sense.”

A nice example of this is provided in the research of Eric Mazur at Harvard [2]. Teaching some of the very best college students in the USA, Mazur assumed that his students made the same sense of formal problem solving that he did. But when he presented the following two problems on one of his exams (see Figs. 1 and 2), he was startled by the results.

The first problem is a complex quantitative problem and requires knowing Kirchoff's laws, setting up and solving two equations in two unknowns (currents), and then figuring out the potential drop. Mazur's students were quite comfortable with it and did reasonably well. The average score on problem 1 was 75%.

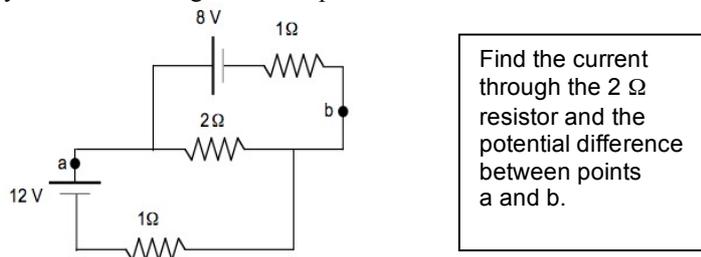


Fig. 1: A complex problem that requires the use of Kirchoff's laws and the setting up and solution of two simultaneous equations. Students in Mazur's class at Harvard had little difficulty with this, scoring an average of ~75% [2]

The second problem (Fig. 2) was a different story. Although this is a qualitative problem with a simple short circuit that most physicists can solve more easily than they can the first problem, Mazur's students found it exceedingly difficult and did poorly. The average score on problem 2 was 40%.

This is just one example of what is now a very well documented result: students can learn to solve complex quantitative problems algorithmically, but are largely unable to interpret their results or understand what the algorithms they have learned mean or why they hold. On one level, many teachers are familiar with this phenomenon. They see that many of their students can produce answers that are quite bizarre but fail to see that there is a problem or consider the possibility that they might have made a mistake.

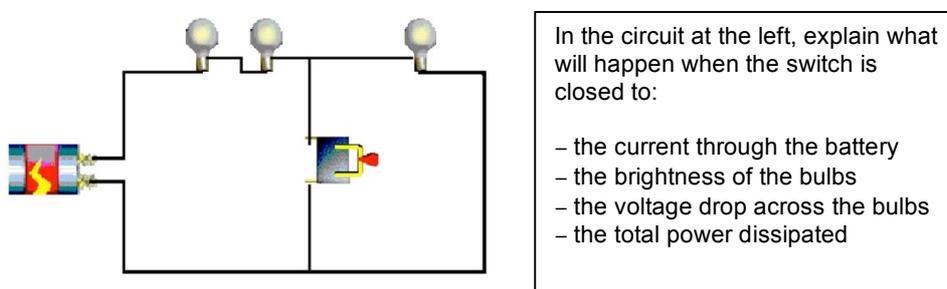


Fig. 2: A simple qualitative problem with a “short circuit” that requires a qualitative understanding of what happens in an electrical circuit. Students in Mazur's class at Harvard had great difficulty with this, scoring an average of ~40% [2].

### 1.3 Why concepts are important

Without a good understanding of the basic concepts, students may generate results in physics without understanding what they are about. Memorizing equations and definitions without having a conceptual understanding is like learning a language by memorizing text without knowing its meaning.

If we want to provide a more effective physics instruction for more of our students, we have to get some idea of the nature of their difficulties and whether a lack of conceptual understanding is limited to a few students or is widespread.

### 1.4 Concepts don't come from free

Teachers who know their physics well often assume that students will learn sense-making in physics as a natural consequence of learning quantitative problem solving – eventually; that the learning of conceptual physics is “automatic” – that it comes along “for free” as our students do the quantitative problems we assign. The problem is that the qualitative understanding we call sense-making sometimes does not develop until graduate school or until the student teaches the subject themselves. But many of our students don't reach this level and never intend to. Without that “eventual” sense-making, the algorithmic problem solving skills learned by many of our students can turn out to be of little value.

### 1.5 How we study student learning

In studying student understanding of physics, researchers listen carefully to students in demonstration or problem-solving interviews and analyze their thinking in written questions containing the phrase “explain your reasoning.” Through activities such as these, they have learned a lot about student conceptual difficulties. Standardized concept surveys based on this research began to appear about 1985 [3]. More than 20 such surveys now exist and cover topics ranging from kinematics to electromagnetism.<sup>†</sup> These surveys present apparently simple questions on fundamental conceptual issues with distractors chosen to match common student misconceptions discovered by researchers. The result is a set of problems that look simple to the instructor (because they know the answer well and are not tempted by the distractors) but that can be quite difficult for the students (who may be unsure when to use their physics knowledge and when to call on their everyday knowledge). These surveys indicate that there is widespread student confusion about some of the most fundamental concepts even after traditional instruction.

#### *Example: The Force Concept Inventory*

One of the most widely used conceptual surveys is the Force Concept Inventory (FCI) [4]. This is a 30-item multiple-choice test to probe student's understanding of basic concepts in mechanics. The choice of topics is based on careful thought about what are the fundamental issues and concepts in Newtonian dynamics. It mostly uses common speech rather than cueing specific physics principles, and the distractors (wrong answers) are based on students' common misconceptions.

Imagine a head-on collision between a large truck and a small car. During the collision:

- (A) the truck exerts a greater amount of force on the car than the car exerts on the truck.
- (B) the car exerts a greater amount of force on the truck than the truck exerts on the car.
- (C) neither exerts a force on the other, the car gets smashed simply because it gets in the way of the truck.
- (D) the truck exerts a force on the car but the car does not exert a force on the truck.
- (E) the truck exerts the same amount of force on the car as the car exerts on the truck

Fig. 3: A problem from the FCI [4]. Note the presence of responses (such as (C)) that are commonly held among high school students entering a physics course but that would be unlikely to be offered by most teachers writing a test.

An example of an item is given in Fig. 3. My students and I gave the FCI to classes of students taking first semester calculus-based physics at the University of Maryland. Some of the classes received traditional instruction and some classes had one hour of instruction per week modified to use the research-based Tutorials described below instead of

---

<sup>†</sup> Seventeen of these surveys are included on the resource CD that comes with my book *Teaching Physics with the Physics Suite* [1]. A list of these and contacts to their authors are available online at <http://www.physics.umd.edu/perg/restools.htm>.

traditional problem solving recitations. The students were mostly engineers and all had successfully completed a high school physics course. As shown in Table 1, on the pre-test, only 26% of the students chose the correct answer. A startling 70% chose a common misconception: that the larger, more active truck exerts a greater force on the car than the car does on the truck.

Traditional instruction improved this result, increasing the number of students who chose the correct result to 51%. On so fundamental a topic, 51% is still not very satisfying. By modifying one hour of instruction per week using research-based materials we were able to improve the percentage of students selecting the correct choice to 83%.

Table 1: Engineering students' choices on the FCI item shown in Fig. 3 before and after traditional instruction and instruction modified to include one hour of UW-Style Tutorials. The correct answer is shown in bold.

	With traditional instruction (N=178)		With modified instruction (N=280)	
	Pre	Post	Pre	Post
(A)	70%	46%	62%	16%
(B)	3%	1%	2%	1%
(C)	1%	0%	1%	0%
(D)	1%	2%	0%	0%
<b>(E)</b>	<b>26%</b>	<b>51%</b>	<b>35%</b>	<b>83%</b>

### ***1.5 Conceptual difficulties are widespread***

This sort of problem has been demonstrated in introductory physics classes for almost all populations of students in countries around the world [5]. In addition, similar kinds of difficulties have been observed in essentially every topic that is commonly taught in introductory physics ranging from simple kinematics to modern physics. Many researchers have made substantial contributions to this literature. There are now a number of useful bibliographies that provide lists of or discussions of research papers on these topics [6][7][8][9].\*

## **2 Curricular options: Research-based materials**

The research that helps us understand students' difficulties also provides guidance to curriculum developers. A wide variety of research-based reform materials have been created that help students learn physics concepts more effectively. These take into account common confusions and difficulties students have with the subject and help students build their understanding through well-designed activities. A discussion of about a dozen such methods is given in my little book, *Teaching Physics* [1].

---

\* Most of these are available on the internet. For a list of links, go to <http://www.physics.umd.edu/perg/tools/rl.htm>.

### ***2.1 It's what the students do that counts!***

By now it has been very well documented that for most students, most of the traditional techniques we employ in university level physics teaching are only effective for helping them memorize some vocabulary and learn to solve a limited class of algorithmic problems. Our traditional techniques do not work well for the development of conceptual understanding and sense making [10]. For this purpose, for most students, in most environments, teaching by telling does not work well, repeated simple exercises do not work well, watching teacher-performed demonstrations do not work well, traditional labs do not work well.

Throughout the 1990's, attempts to design more effective learning environments all converged on a single consensus: to learn physics effectively students have to be "mentally engaged". For many of us who now teach physics, it was never a problem to be mentally engaged with physics. We loved the subject, were interested, and could get our brains turned on and active even in response to a weak or incoherent lecture. Many students today, however, study physics not because they love it but because it is a requirement for something else they want to do or because they see it as a step to a secure future. For these students, turning their brains on to physics is neither automatic nor easy.

What physics education researchers and curriculum developers have learned is that many more students can be brought to mentally engage the physics if they can be involved in appropriate tasks. Creating such tasks is more difficult than it sounds. The appropriateness of the task for engaging the student lies in the mind of the student, not of the instructor. Therefore, figuring out what engages the student can take considerable effort – extensive observation and deep probing of student thinking.

A number of research-based reform curricula have been created that provide environments in which students are helped to "turn on their minds to physics" by engaging in effective learning activities. The keys seem to be to get the students to

- make sense of the physics for themselves,
- learn to seek coherence, and
- reflect on what they are learning.

Activities that have the effect of getting students to do these things are often referred to by the shorthand phrase *active engagement*.\*

### ***2.2 We can do better!***

More than a dozen active engagement environments for introductory physics teaching were developed and disseminated in the USA during the 1990's. A dozen that have been tested and proven to be effective in helping to promote concept learning and sense-making are described in my little book [1]. I'll briefly describe three examples here in

---

\* One needs to be careful in using this term. It is easy to mistake an ineffective "hands on" activity from a more useful "minds on" one. Activity is not the key here, mental engagement is.

order to show how the active engagement focus changes a learning environment: Tutorials, Cooperative Group Problem Solving, and Workshop Physics.

Both Tutorials and Cooperative Group Problem Solving are transformations of a small-group recitation environment that attempt to get the students actively engaged. In a traditional recitation in the USA, an instructor (often a graduate student) answers students' questions about the week's homework and typically just solves problems for them. This may model good problem solving technique and students who are already mentally engaged may be actively processing what they hear. But in the USA, indications are that this is often a small fraction of the students in the course.

The *Tutorial* method, developed by Lillian McDermott and her collaborators at the University of Washington,<sup>†</sup> replaces the traditional recitation by a small-group discussion [11]. Students work in groups of three or four and discuss the contents of a 3-6 page worksheet. These worksheets are based on careful research that identifies common student confusions and misconceptions. The goal of the worksheets is to elicit students' misconceptions and, through discussion, help them reflect on how to refine their thinking. Many students in introductory physics classes in the USA are unfamiliar with qualitative physical reasoning and the UW Tutorials are designed to help them learn to do it.

The *Cooperative Group Problem Solving* (CGPS) approach, developed by Pat and Ken Heller and their collaborators at the University of Minnesota, retains the problem-solving character of the recitation, but instead of the students watching an instructor solve problems, they solve them themselves, working in groups of three or four on problems. The students are given general instruction in problem solving methods and the problems are not exercises. They are *context rich* problems – problems that require complex reasoning and evaluation [12].

*Workshop Physics* (WP) [13] is a more aggressive transformation of traditional instruction. The class is entirely lab-based with a detailed activity guide. Students are led to discover the laws of physics for themselves. An environment rich in computer-based data acquisition devices facilitates their exploration. The course becomes what might be described as *guided discovery learning*.

### **2.3 Research-based instructional models produce better conceptual gains**

In order to test whether these curricular reforms are effective in promoting conceptual learning, we gave the FCI before and after instruction in first semester university physics in 15 universities. These classes used four different instructional models: traditional lecture with recitation, traditional lecture with tutorial, traditional lecture with group problem solving, and workshop physics. We observed both primary and secondary im-

---

<sup>†</sup> Note that this “tutorial” method is very different from the method of the same name common in British universities. The University of Washington tutorials are heavily guided, research-based, and can be delivered to large classes.

plementations\* of the research-based CGSP and WP curricula and secondary implementations of Tutorials [14].

Our figure of merit is the fraction of the possible gain in concept learning as measured by the FCI,  $h$ , defined in Eq. (1). Our results are shown in Table 2.

$$h = \frac{(\text{posttest average} - \text{pretest average})}{(100 - \text{pretest average})} \quad (1)$$

Table 2: Fraction of possible gains,  $h$ , obtained by pre-post testing with the FCI. Uncertainty shown as SEM. Each group contained about 500 students except WP primary, which contained about 30 [14].

Traditional	0.20±0.03
Recitation modifications (Tutorial & CGPS, primary and secondary)	0.34±0.01
WP (early secondary implementations)	0.41±0.02
WP (single mature primary implementation)	0.73

The research-based curricula showed significantly better gains. By paying attention to what students know and how they learn, we can create educational environments that produce more effective concept learning than produced by traditional methods and these methods do not necessarily take additional time. (We also observed that algorithmic problem solving was not deteriorated — but it was also not improved [14].)

#### 2.4 Concepts are not enough

Concepts are important, but they are only a part of the story. We want our students to learn to think like scientists. Just as we thought that concept learning and sense making happened automatically when students learn facts and algorithmic problem solving, there is a whole set of skills that students need to learn that we usually don't talk about explicitly and which we assume "come for free." I refer these as *the hidden curriculum*.

### 3 Rethinking our goals: The hidden curriculum

When we design our courses, we tend to focus on content rather than on process or skills and assume that skill development "comes for free." This is often not the case. Some of these skills are:

- developing a strong physical intuition
- learning to reason from principle
- seeking consistency
- reasoning with mathematics
- learning to solve complex problems.

---

\* "Primary" indicates the site where the approach was developed. "Secondary" implies an implementation of the technique at some other university.

### ***3.1 Teaching students to think like a scientist***

Can we explicitly teach the hidden curriculum? Answering this question was a goal of a recently completed project, *Learning How to Learn Science* (LHLS), to study student learning in algebra-based college physics at the University of Maryland [15]. This project had two primary goals:

- To see whether students could be taught hidden curriculum goals in a large lecture class with small (but coherent) changes to standard environments.
- To create environments in which much of a student's learning took place in classroom environments where it could be observed and studied.

The class had a traditional format for large lecture introductory physics in the USA. The class ran for two 14-week semesters. Each week had three 50 minute lectures (with 100-160 students, with the lecture given by a Professor), one 50 minute recitation (with 20-24 students, run by facilitators – usually graduate students), one 110 minute laboratory (with 20-24 students run by a graduate student). Homework was assigned and collected weekly (with ~ 5 significant problems).

The population consisted largely of majors in the bio and health sciences (~85%) and was mostly upper division college students (~80% juniors and seniors). In contrast to our physics majors and engineering classes, the population was predominantly female (~65%). There were an important fraction of pre-medical students (~35%) and a large fraction of the class had research experience (~75%).

Every part of the course was modified. Lectures were made interactive with Interactive Lecture Demonstrations [16] and student response systems (individual remote answering devices) [17]. The traditional discussion was replaced by concept building and process-learning U. of Washington-style tutorials (group learning with worksheets). The traditional protocol labs were replaced by ones in which the students had to design their own experiments. Homework was a mix of qualitative, quantitative, and estimation: the focus was on reasoning, both qualitative and quantitative.

I present two examples to illustrate how our concern for helping students focus on elements of the hidden curriculum influences what it is we give students to do.

### ***3.2 Example: Teaching coherence with representation translation problems***

We want our students to learn to “make sense” of the physics. This means, in part, to see the various descriptions we use in physics – equations, graphs, figures – as different ways of looking at the same physical system. But in order to develop a coherent understanding of a physical system, you have to have the idea that you're looking both for coherence and for understanding, and you have to have the idea that everything must be in the service of describing a physical system. Many students do not understand these points. They are used to treating scientific information as a collection of independent and unrelated facts. If they are not encouraged (required) to seek consistency and to tie their knowledge to particular physical systems, they tend to simply reach for the most obvious fact and not notice that it may contradict something else they know. When we only give

problems that can be solved with one equation or principle, students don't learn to look for coherence. We need to demand more.

In order to get students to pay attention to coherence focused in a physical situation, we created problems with many parts, each of which can be done in a number of ways. One way is with representation translation problems, an example of which is shown in Fig. 4. This problem ties together a picture of a physical system, multiple representations (graphs of different physical quantities), and the use of fundamental principles (Newton's laws) to give students practice in focusing on building coherence. We give credit for consistency and give them credit if they can show that one of their answers follows by correct reasoning from another (wrong) one.

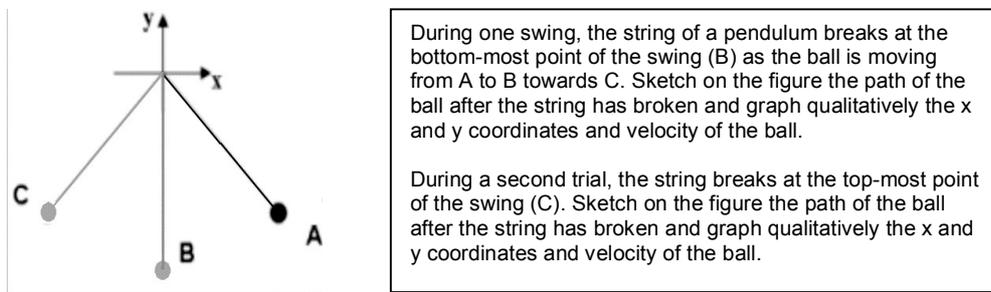


Fig. 4: A sample representation translation problem that gives students practice in building coherence.

### 3.3 Example: Refining intuitions with Elby pairs

Another technique we use to help students learn to think about coherence and to learn to evaluate, refine, and strengthen their intuitions is the method of *Elby pairs* [19]. We create paired questions, one which most students are likely to answer correctly, one which they are likely to answer with a common misconception. We then help them see there is a contradiction in their thinking and resolve it. An example is given in Fig. 5.

### 3.4 Does it work?

To evaluate our reforms, we collected a wide variety of data, both qualitative and quantitative: interviews and observations of behavior, a pre-post conceptual knowledge survey, videotapes of in-class behavior, and a pre-post attitude survey. Most of our observations supported our claim that it is possible to teach elements of the hidden curriculum in a large lecture class without substantial extra resources beyond the traditional structures (for research universities in the USA). There is not space to review all this data here, but one dramatic result is worth mentioning.

#### *Example: Fractional Gains on the FCI / Splitting*

We are certainly interested in improving our students' conceptual understanding of the physics they are learning, but we hoped to go beyond that. For example, we wanted to

have our students not only “know” the right answers, but also feel that the physics they were learning made sense – that it was intuitively reasonable.

1. A truck rams into a parked car.

(a) Intuitively, which is larger: the force exerted by the truck on the car or by the car on the truck?

(b) Suppose the truck has mass 1000 kg and the car has mass 500 kg. During the collision, the truck slows by 5 m/s. How much speed does the car gain during the collision?

2. Simulate this scenario by making a “truck” (a cart with extra weight) crash into the “car” (a regular cart). The truck and car both have force sensors attached. Do whatever experiments you want, to see when N3 applies.

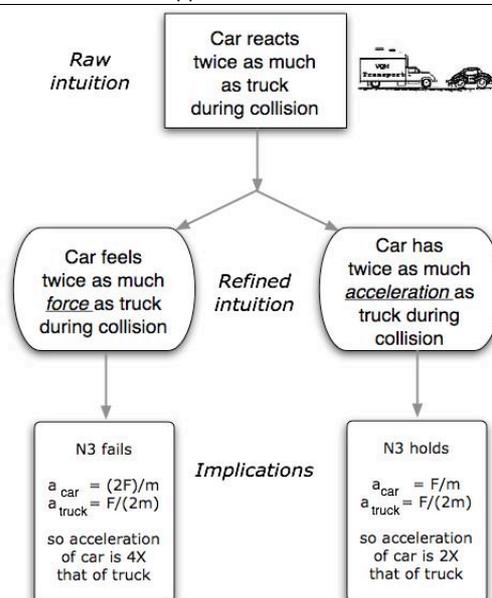


Fig. 5. Analysis of an Elby pair [19].

We did not have to give up conceptual gains in order to emphasize hidden curriculum learning. Our pre-post fractional gains on the FCI for this class was  $h = 0.47$ , higher than we had ever measured in a large lecture class at the University of Maryland. But this doesn't probe whether students were able to make their new knowledge a part of their intuition.

McCaskey and Elby [20] came up with a way of probing students' sense of whether they saw a result that they knew as intuitive. They had students make two passes through the FCI. In the first, they were told to circle the answer that they felt made the most intuitive sense. In the second, they were to put a square around the answer that they felt “a physicist would give.” Not giving the same answer for both – *splitting* – suggests that the student might know the correct answer but not feel that it made sense, intuitively.

We gave the split-task FCI to my algebra-based Physics II class at start of second term. The students (N~160) included ~1/3 who had received traditional instruction in the first term and 2/3 who had received our reformed instruction, including Tutorial lessons with Elby pairs. We isolated clusters that corresponded to the learning of particular concepts. One such is the Newton's third law cluster – four items probing students' understanding of when N3 holds.

The results were dramatic. Not only did students receiving reformed instruction do much better (~85% correct compared to ~45% correct), they were much less likely to split their answers (~10% of correct answers split as compared to ~50% of correct answers split). This totals to ~75% correct unsplit answers (“right and reconciled”) in the reformed class, compared with ~25% correct unsplit answers in the traditional class. These results are shown in Fig. 6.

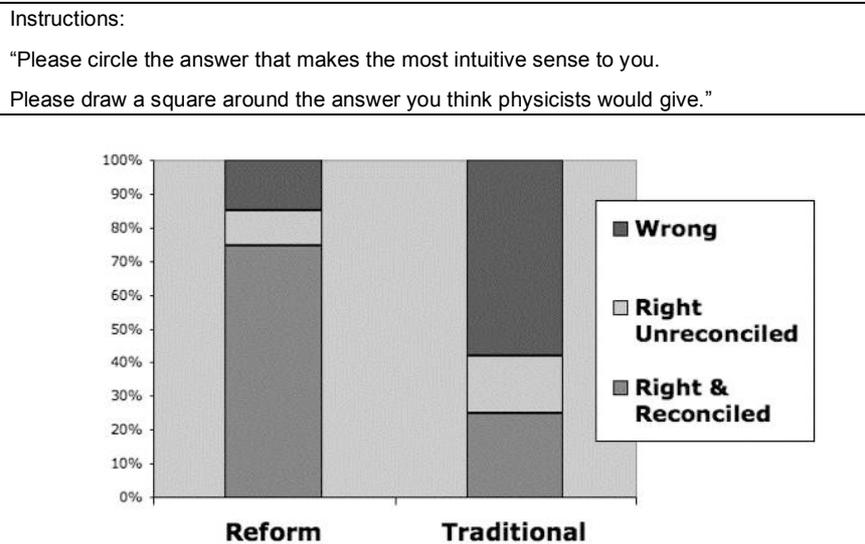


Fig. 6: Results on the 4 FCI N3 items in the LHLS reformed class and in a traditional class [20]. “Reconciled” means that the students did not split their answers.

#### 4 Some Things I Have Learned

I have been teaching now for almost 35 years and I have been doing physics education research for nearly 15. In that time, I have learned a few things.

- It's important to think about what it is you really want to accomplish in your teaching.
- It's essential to understand where your students are and what they can do.
- What we are asking our students to do is considerable harder than we sometimes appreciate.
- It's what the students do that matters most for their learning, not what the instructors do.

.... and most importantly

#### 4.1 You don't have to do it all by yourself!

Researchers and curriculum developers all over the world are creating a community of scholars and specialists. As we improve our knowledge of what our students know and don't know, of how they learn effectively, and of what they need to learn that we haven't noticed, we can better understand both what we can do and what we need to do. We can learn from each other's careful research, and we can share materials, adapting them to the needs of our local environment. Teaching physics to all our students can be difficult, but we can ease the burden and improve our effectiveness if we act and interact as a community to learn about physics learning.

#### Acknowledgments

I am very grateful to my colleagues at the University of Maryland who contributed to the work described here. This material is based on work supported by the National Science Foundation under Grant No. REC-008 7519.

#### References

1. E. F. Redish, *Teaching Physics with the Physics Suite* (John Wiley and Sons, Inc., 2003). Available online at <http://www2.physics.umd.edu/~redish/Book/>.
2. E. Mazur, *Peer Instruction: A User's Manual* (Prentice Hall, 1997)
3. I. Halloun and D. Hestenes, *Am. J. Phys.*, **53**, 1043-1056 (1985).
4. D. Hestenes, M. Wells, and G. Swackhamer, *The Physics Teacher*, **30**, 141-158 (1992).
5. R. Hake, *Am. J. Phys.*, **66**, 64-74 (1998).
6. L. C. McDermott and E. F. Redish, *Am. J. Phys.*, **67** (1999) 755-767.
7. L. Hsu, E. Brewster, T. Foster, and K. Harper, *Am. J. Phys.*, **72**, 1147-1156 (2004).
8. B. Thacker, *Repts. on Prog. in Phys.*, **66** 1833-1864 (2003).
9. R. Duit, *Bibliography: Students' and Teachers' Conceptions in Science Education*, <http://www.ipn.uni-kiel.de/aktuell/stcse/stcse.html>.
10. L. C. McDermott, *Am. J. Phys.* **61**, 295-298 (1993).
11. L. C. McDermott, et al., *Tutorials in Introductory Physics* (Prentice Hall, 1998).
12. *Cooperative Group Problem Solving*, University of Minnesota website, <http://groups.physics.umn.edu/phyped/Research/CGPS/CGPSintro.htm>.
13. P. Laws, *Phys. Today* **44**:12, 24-31(December, 1991); *Workshop Physics Activity Guide* (John Wiley and Sons, 1999).
14. J. M. Saul and E. F. Redish, "Evaluation of the Workshop Physics Dissemination Project", U. of Maryland preprint (April 1998).
15. <http://www.physics.umd.edu/perg/role/>.
16. D. Sokoloff and R. Thornton, *The Physics Teacher*, **35**, 340-347 (1997); *Interactive Lecture Demonstrations* (John Wiley & Sons, 2001).
17. W. Reay, et al., *Am. J. Phys.*, **73**, 554-558 (2005).
18. R. L. Kung, *Am. J. Phys.*, **73**, 771-777 (2005).
19. A. Elby, *Am. J. Phys.*, **69**, S54-S64 (2001).
20. T. McCaskey, PhD Dissertation, U. of Maryland, Physics Department, (2006).