Chapter 7: Investigating the Dynamics of Student Reasoning

Introduction

A goal of our research has been to go beyond simply categorizing student difficulties. In addition, we would like to investigate the dynamics of student responses during instruction. In chapter 6, data were presented to show that individual topics of student understanding were affected by a modified curriculum, but the overall picture of student understanding of wave physics was not discussed. The data were discussed in terms of two reasoning methods that students use when answering a specific set of questions. We can describe students as using either:

- the correct model of waves (Community Consensus Model, or CM), elements of which students learn during the semester, or
- the more problematic pattern of associations to Newtonian (mechanical) particle physics (loosely referred to as the Particle Model, or PM, of waves), elements of which students bring to the classroom.

To investigate the dynamics of student reasoning in more detail, we have developed a diagnostic test that can be administered before and after student instruction. Though the developed diagnostic test included many questions that did not specifically address the distinction between student use of the CM and PM, the discussion in this chapter focuses on questions that elicited student difficulties related to their use of the two models. The questions not discussed in this chapter will be part of future work in investigating student understanding of wave physics.

In this chapter, I discuss the development of the diagnostic test and its usefulness in coming to an understanding of how individual students and an entire class develop an understanding of wave physics. The distinction between the CM and PM gives one example of many different aspects of reasoning that students use when thinking about waves. The questions chosen in the diagnostic represent questions that possibly elicit both CM and PM responses (and possibly both). The preliminary and pre- and post-instruction final diagnostic tests are given Appendix D of the dissertation.

The analysis of the data is similar to the analysis described in chapters 5 and 6. Student responses are categorized as representative of either the CM, the PM, both, or neither. We find that students begin the semester using primarily the PM but move to a mixed state where both the CM and the PM play a role in their thinking. This mixture of reasoning patterns is not due to understanding one topic of wave physics with the CM and another with the PM but seems to exist within a single wave physics topic.

Preliminary Diagnostic Test

A preliminary version of a wave diagnostic test was designed for use in interviews in the S97 semester. The interview setting was chosen to give the opportunity to probe student responses in more detail. By following up on student responses, we were able to compare the reasoning in their responses to the reasoning we believed was occurring. Our beginning assumptions were based on previous research into student difficulties. The diagnostic test was developed using questions we had found were effective in uncovering student use of the CM and PM in reasoning about waves.

The diagnostic was prepared with multiple-choice versions of questions previously used in free response formats in interviews or written tests. The multiple-choice questions were designed with research-based distractors so that students would have the opportunity to make common errors. The distractors were based on responses that students had given during our previous research projects. An example of a question with research-based distractors has been discussed extensively in this dissertation in the context of the free response (FR) and multiple-choice multiple-response (MCMR) question on wave propagation speed (see chapter 3, 5, and 6).

Many of the multiple-choice questions were in an MCMR format. In many of these questions, students were offered a series of questions and a series of possible responses for that set of questions (for example, questions 1 to 4 might have possible responses *a* to *f*). Students were told that they could use one response more than once to answer more than one question.

During the implementation of the interview diagnostic test (see below for more details), two free response versions of MCMR questions were added. This gave us the opportunity to ask nearly identical questions using different formats, which we had found effective in previous research settings.[1] The FR questions were asked at the beginning of the diagnostic, so that students would not use offered responses from the MCMR questions in their FR responses.

The preliminary diagnostic test is presented in Appendix D-1. The entire diagnostic test is presented, but only those parts which directly deal with student use of the CM and PM are discussed in this chapter (as discussed above). The wave physics issues included in the diagnostic are:

- wave propagation (both mechanical waves and sound waves),
- superposition of mechanical waves,
- the mathematics used to describe waves (both mechanical and sound),
- the motion of elements of the system through which the wave propagates,
- reflection of mechanical waves from a boundary.

Most of the questions used in the diagnostic had been used in previous research and common student responses to these questions were well known. For most of these questions, we did both interviews and written tests. In many cases, we explicitly used interviews to check to see if students answered the written questions consistently with the way that they actually thought about the situation. We had found that students answered the questions in the preliminary diagnostic test consistently. (These questions are said to be validated in such a situation.)

As stated above, the design of the preliminary diagnostic test changed during the course of its implementation. Some questions were rephrased due to student comments, some questions were added, and others were dropped. The questions on the last version of the preliminary diagnostic test that were most effective in uncovering student use of the PM are shown in Table 7-1. For each question from the diagnostic test in Appendix D-1, the correct (CM) response is given along with the possible PM responses offered as distractors to the students.

Table 7-1

S97 Wave Diagnostic Question	Possible PM Response(s)	Possible CM Response(s)
Wave Propagation		
FR1 and 5	a, b, c, d, i, j	e, f, g, h
Wave-Math		
FR2a) and 9	a, f	g
FR2b) and 10	a, b, c, d, e	f
Sound Waves		
1	b (if without c)	c
3 and 4	depends on response to question 1…	depends on response to question 1…
Superposition		
12	a or b (if a bouncing explanation given), g	c
17	f, g	e
Reflection		
20	h	e
22	a (bouncing), i (pulse absorbed into wall)	d
23	a, i (see above)	b

Table of S97 wave diagnostic questions that were used to determine if students were answering using the PM or CM. The diagnostic test can be found in Appendix D-1. Not all questions had clear PM and CM responses and are therefore not included in this analysis.

The interview diagnostic test was administered to 20 students. Five of these students had completed traditional instruction in which their recitation sections were led by a department professor. Fifteen students had completed tutorial instruction. Due to time limitations, not all students answered all questions. Also, for reasons stated above, the test itself was changed during the course of the interviews. Although all the interviewed students had completed instruction on waves, many still used problematic reasoning and showed difficulties with the material.

Each student's response was categorized according to the type of reasoning used. The criteria involved have been discussed in chapters 3, 5, and 6. Student responses were first categorized according to the difficulties that they had with the problem. These difficulties were tabulated using a spreadsheet program. Then, the various classes of difficulties were organized. Finally, each student and each question were analyzed according to:

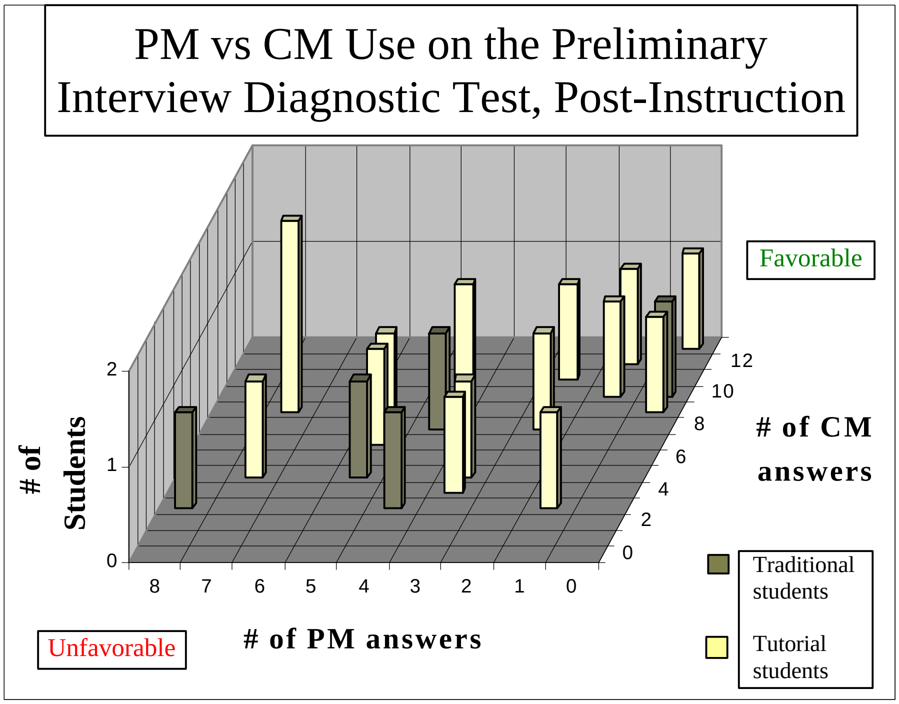
- the number of correct responses (those categorized best by the CM)
- the number of responses best categorized by the PM,
- the number of responses not categorized by either CM or PM, and
- the number of unanswered questions.

The organization of student responses into these 4 categories was used for all the wave diagnostic tests that will be discussed in this chapter.

The summary of how students responded to the 15 questions most likely to show evidence of the PM is shown in a two-dimensional histogram plot in Figure 7-1. The data in the table represent student performance according to how many questions they answered using a specific number of responses that are best classified as either PM or CM responses. For example, we classified the 13 responses of one tutorial student as indicative of the CM and none as indicative of the PM. We consider this a generally favorable result (i.e. we would like all our students to show such performance). We consider a student using primarily PM-like responses as showing unfavorable performance. Note that the sum of student PM and CM responses does not add to 15 in many instances, for reasons stated above and the additional reason that not all responses were classifiable with the CM or PM.

Consistent with our previous findings, we observe that many students consistently misapply otherwise reasonable primitives in their reasoning about wave physics. They are not consistent in their use of the PM, though. If we consider the responses given by Kyle (previously discussed in chapter 5), we see an example of this mixture in student reasoning. Kyle answered six of fifteen questions in a manner best classified by the PM and four in a way best described by the CM. In one question, he used reasoning that was indicative of both the CM and the PM. Thus, he answered a

Figure 7-1



Comparison of post-instruction PM and CM use by 15 tutorial and 5 traditional students on 15 questions from the preliminary wave diagnostic test. PM = Particle Pulses Pattern of Association. CM = Community Consensus (correct) Model. Mixed responses were counted as both PM and CM responses. “Favorable” describes a student who gives only answers best classified by the CM.

total of seven questions using the PM and five using the CM. In addition, four of his responses were not indicative of either the PM or CM. Thus, a student who clearly showed that inappropriately applied primitives guide part of his reasoning in wave physics also showed that he used multiple reasoning methods when thinking about the physics.

If we compare tutorial and traditional instruction students' performances, we note that most tutorial students performed better than the non-tutorial students on the material. This result is consistent with the results presented in chapter 6, which show that the tutorials effectively address student use of the PM. But, with the small number of students participating in the interviews, these data are merely suggestive and not conclusive.[2]

The preliminary diagnostic test was designed as a precursor to a written diagnostic test that would be applied in future semesters. To determine the effectiveness of the questions in uncovering student difficulties with the use of the PM in their reasoning, we counted how many students used the CM, PM, or other explanations when answering each question. The results are shown in Table 7-2. Note that the results show student performance after (both traditional and tutorial) instruction.

Students seemed to have the greatest difficulty with the wave propagation questions. The common "mixed" responses on the MCMR questions stand in contrast to the very polarized responses on the FR question, consistent with results from other investigations. Both wave propagation questions were effective in uncovering student

Table 7-2

# of responses Question	CM	Mixed CM/PM	PM	Other	Total
Propagation					
FR1	8	2	8	0	18
5	9	9	1	1	20
Sound Waves					
1	13	0	7	0	20
3	12	0	6	2	20
4	11	0	2	7	20
Wave-Math					
FR2 a	8	0	2	0	10
FR2 b	5	0	2	3	10
9	7	1	2	0	10
10	6	0	2	2	10
Superposition					
12	17	0	1	0	18
17	7	0	11	0	18
Reflection					
20	9	0	2	6	17
21	7	0	4	6	17
22	14	0	1	2	17
23	10	0	7	0	17

S97 wave diagnostic test responses split by topic. For each question, the number of CM, PM, mixed, and other responses is given, followed by the total number of students who answered that specific question.

use of the PM in their reasoning after instruction.

The sound wave question in which students describe the motion of a dust particle in front of a loudspeaker also elicited many PM responses. Of the five traditional instruction students, four answered this question using the PM (the fifth gave a CM response). Many students had difficulty with the question about the effects of a change in frequency on the dust particle, but did fewer used the PM when describing the effects of a change in volume.

The superposition questions were partially successful in uncovering student use of the PM. Consistent with previous results, very few students state after instruction that waves permanently cancel each other. Thus, they answered one question very well. But, many students showed a lack of functional understanding of the point-by-point superposition of displacement from equilibrium when the wavepulses coincided but their peaks did not overlap.

Finally, many students had difficulty with the wave reflection question in which they described the shape of the wavepulse reflected from a free end. Many stated that the wavepulse was absorbed into the. Few had specific problems with the other wave reflection questions, though many "other" responses were given. These questions were only partially successful in uncovering student use of the PM.

The difficulties students had with the questions on the preliminary diagnostic test were consistent with previous results. Therefore, the diagnostic test was not modified very much when it was next used.

Final Diagnostic Test

Based on the results from the S97 preliminary diagnostic test interviews, we developed a final version diagnostic test for F97. We asked a pre-instruction and post-instruction written diagnostic test of two Physics 262 classes at the University of Maryland. In the following sections, I discuss the design and implementation of the pre-instruction and post-instruction diagnostics. The design changes that led to the post-instruction test were partially based on a data analysis of the pre-instruction test, which will be described below. I end the chapter with a comparison of student performance on the two diagnostic tests and draw conclusions about student reasoning and classroom performance based on the data.

Pre-Instruction Diagnostic Test, Final Version

The pre-instruction written diagnostic test is presented in Appendix D-2. The pre-instruction diagnostic consisted of the 15 questions shown in the preliminary interview diagnostic to often elicit responses that could be classified as PM responses.

Two new questions were written for this test. The purpose of these questions was to raise issues from the dust particle sound wave question (discussed in chapters 3 and 6) in a different setting. One question asked students to compare the speed of sound of two people's voices, given that one person's voice was deeper and louder than the other's. A correct response would state that the speed of sound for the two is the same, regardless of volume or frequency of the sound. Students also answered variations on this question (e.g. how does your answer change if the volumes are equal). A second question asked students to consider the motion of a dust particle close to a wall when a sound wave reached the wall and reflected from it. A correct answer would state that the air near the wall is incapable of moving from its equilibrium position due to the wall, and therefore the dust particle will not move. This question also asked students the effect of a louder volume on the speed of sound. For both questions, we expected students to make similar errors to the ones they made when describing the motion of a dust particle in front of a loudspeaker. We expected students to show an incomplete understanding of the relationship between frequency, volume, and speed of sound. We also expected students to have difficulty describing the motion of the medium through which the sound traveled, especially in a setting involving reflection from what is effectively a "fixed end."

One important difference between the preliminary (interview) diagnostic test and the final (written) diagnostic test was that the final version consisted of primarily free response (FR) questions. The predominantly multiple-choice multiple-response (MCMR) format of the preliminary test had been feasible in an interview setting, but students had many difficulties with certain questions. As a result, it was decided to make the F97 test a free response test. One exception was the wave propagation question already discussed in chapters 3 and 6. This question was asked in both FR and MCMR formats

The pre-instruction diagnostic was administered during the first week of the semester during the tutorial period. Tutorials had not originally been scheduled for that week because the first day of the semester was a Tuesday. Those students who had tutorial period on a Monday were asked to come to another section during the course of the week. During the tutorial period, students were asked to answer all the FR questions, turn them in, and were then handed the wave propagation MCMR question separately. In this way, we were assured that they could not change their responses on the FR question due to the offered responses on the MCMR version.

Most of the analysis of the final diagnostic test involves a comparison of pre- and post-instruction data that will be discussed below. The analysis that led to the post-instruction diagnostic must be discussed before introducing the post-instruction diagnostic.

Many of the questions had similar content, though their surface features were different. For example, students answered questions about the speed of sound in the context of two people yelling (at different volumes and frequencies), sound waves created by a loudspeaker, and sound waves created by a clap. Students took much longer to answer the pre-instruction written diagnostic test than had been expected. As a result, we were aware that we had to shorten the test for its use after instruction. Thus, we planned to drop questions in which students gave consistent responses. For example, if students consistently used the same reasoning to describe the effect of a change in frequency on the speed of sound, then we would use only one question addressing that issue.

To see the correlation between student responses on the written diagnostic test, we first classified all student responses according to their use of the PM, CM or other explanation. We then compared how the students answered sets of questions. (In the discussion below, I will refer to the parts of questions as *a*, *b*, and *c*, even when that distinction did not exist in the actual numbering of the questions.) For the three parts of question 1 (on the speed of sound waves), we found that students used the same explanation to answer parts *b* and *c* as they used to answer part *a* (80% and 90%, respectively). Similarly, on question 3 (on the motion of the medium due to sound waves), we found that students answered parts *b* and *c* consistent with part *a* 80% and 90% of the time, respectively. But, they did not necessarily respond to parts *a* of questions 1 and 3 consistently (only 50% of the time). Thus, their responses showed that they were consistent when answering a single problem, but not when thinking about a single physics topic.

Students were also not consistent across physics topics. Students might answer use the PM consistently when describing how to change the speed of a sound wave but use the CM to describe how to change the speed of a mechanical wavepulse. The correlation in student responses between the part *a* of question 1 (on the speed of sound) and the part *a* of question 4 (on the speed of mechanical waves) was only 45%. This could be interpreted as saying that the questions are inconsistent and do not give us insight into student understanding of the material. Such an interpretation would assume that students use only one form of reasoning when thinking about wave physics. Results discussed in previous chapters indicate that students are inconsistent in their reasoning. Therefore, we believe that the questions are accurately uncovering areas in which students think inconsistently about the physics.

Post-Instruction Diagnostic, Final Version

The post-instruction diagnostic was shortened from the pre-instruction diagnostic due to time limitations. Based on the analysis of questions described above, we only used questions that gave unique information about student reasoning. For example, we asked students only part *a* of question 1 from the pre-instruction diagnostic. The wave reflection questions were completely dropped, though their inclusion would have been interesting because it would have given us insight into how student performance changed when there was no tutorial instruction on a given topic. The diagnostic is given in Appendix D-3.

The post-instruction wave diagnostic test was given in two parts. In the week before Thanksgiving (roughly 6 weeks after students had taken a mid-term examination on waves), students answered the FR wave propagation and dust particle (sound wave) questions as part of that week's tutorial pretest. (The material usually covered in that week's pretest was shortened and the extra space used for the wave diagnostic test.) During Thanksgiving week, students took the remainder of the wave diagnostic test during the commonly scheduled pretest time. There was no pretest because tutorials are not held during a holiday week. On this part of the wave diagnostic test, students answered the MCMR wave propagation question, the two "real world" sound wave questions, the superposition question with asymmetric waves, and an MCMR version of the dust particle question. This MCMR question had not been asked on the pre-instruction diagnostic.

Comparison of Student Pre- and Post-Instruction Performance

Two different types of analysis were done on the data. First, one specific topic of the wave diagnostic test was investigated according to the modes of reasoning (CM or PM) students used to answer four very similar questions. The evidence suggests that students use multiple reasoning methods within individual topics of wave physics rather than separate consistent reasoning methods for different topics. Our result is consistent with the analysis of FR and MCMR questions described in chapter 3 and it is more robust than the results from preliminary diagnostic test, described above. Second, a statistical analysis of the data was developed. Using a mathematical description allowed us to parameterize the results and compare parameter variables from before and after tutorial instruction.

Because a subset of pre-instruction questions was used on the post-instruction diagnostic, only those questions used in both pre- and post-instruction diagnostics are compared. In addition, only those students who answered a majority of these eight questions before and after instruction are compared.[3] This lets us restrict the discussion of the data to only those students who answered identical questions before and after instruction.

Inconsistent reasoning to describe a single wave physics topic

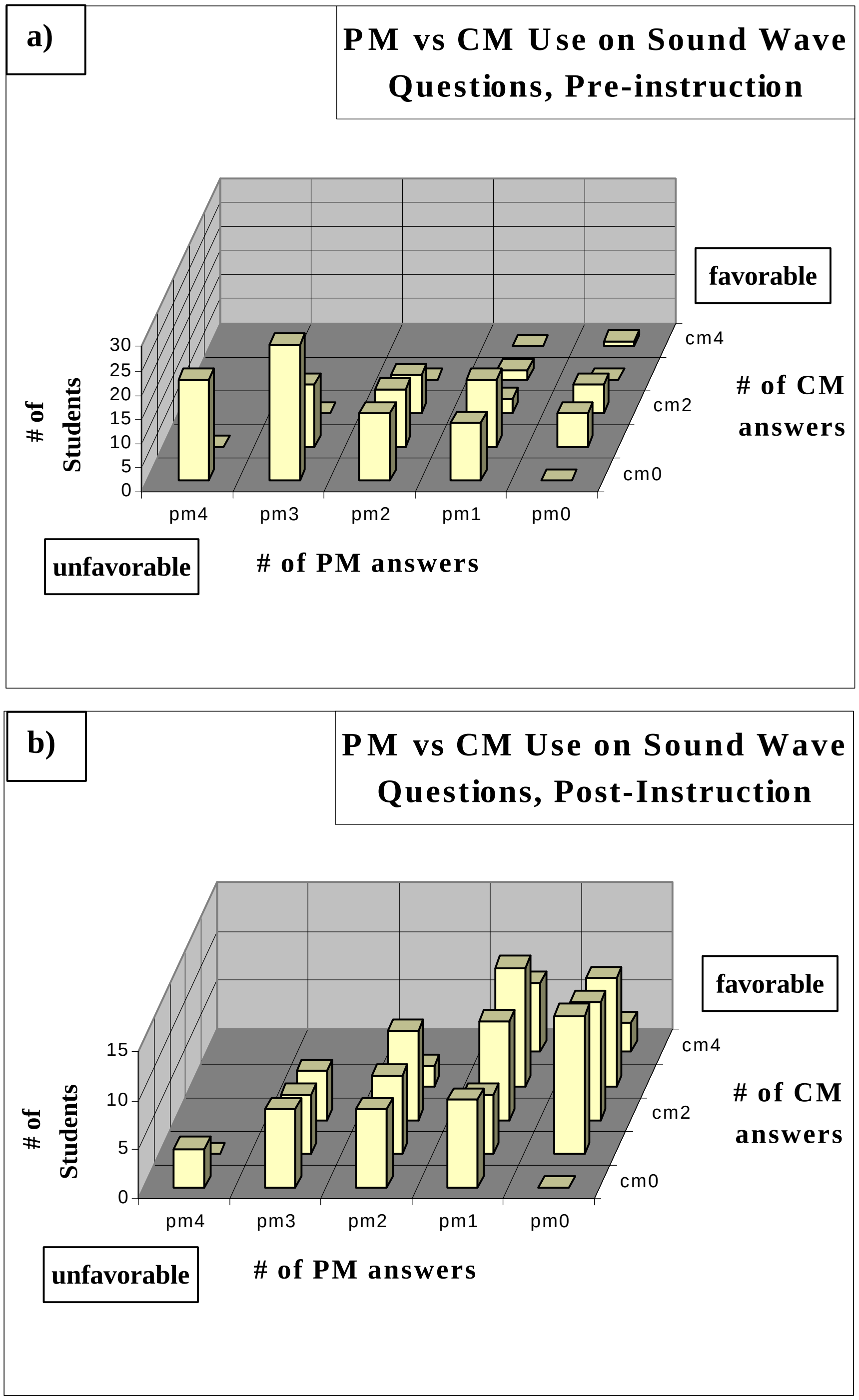
By focusing on student responses on a single topic (such as sound waves), we can see how students use multiple reasoning methods to describe a single physics topic. We find that most students begin the semester using the PM in their reasoning, but students at the end of the semester are more mixed in their responses.

On the pre-instruction diagnostic, questions 1a, 3a, and 6a have the same physics (how a change in the creation of a wave affects fast the wave propagates). Of the 182 students who answered each question, 99, 94, and 108 students (respectively) gave a PM response. Of the other students, 68, 16, and 14 (respectively) gave CM responses. The second most common category (after PM reasoning) for the last two questions was “other.” This implies that students are reasoning in different ways about the same physics situation. Students seem to be reasoning about a single physical topic in many different ways (though most consistently use the PM).

Further evidence comes from looking at a plot which shows how many students gave a specific number of PM and CM responses for all four sound wave questions asked before and after instruction. We find that students are neither consistent nor coherent in their understanding of individual topics of wave physics. Figure 7-2 shows data from pre- and post-instruction wave diagnostic test questions that deal with sound waves. Only matched data are included (i.e. 136 students answered a majority of the questions both before and after instruction). The histogram is like the one discussed in relation to the preliminary diagnostic test. Each histogram bar shows how many students gave a specific number of PM and a specific number of CM responses.

Note that most students begin the semester answering predominately with PM reasoning, but many use mixed reasoning. After instruction, we find that students still answer the four sound wave questions using both reasoning methods. They have moved toward CM reasoning in their responses but have not stopped using PM reasoning.

Figure 7-2



Histograms of student PM and CM responses on sound wave questions in the final version wave diagnostic test. Figures show data from a) before instruction, b) after tutorial instruction. Data are matched, N=141 students. "Favorable" describes a student who gives only answers best classified by the CM.

Students use inconsistent reasoning when thinking about a single wave physics topic such as sound waves both before and after tutorial instruction. It seems that the effect of tutorial instruction was to move students to a hybrid form of reasoning that includes both the CM reasoning that we would like them to have and the PM reasoning with which many enter our courses.

Multiple reasoning methods to describe wave physics

Students also use inconsistent reasoning when describing the investigated wave physics topics both before and after tutorial instruction. Figure 7-3 shows separate histograms of student pre- and post-instruction responses. Again, each column represents the number of students who gave a certain number of CM and PM responses. For example, before instruction, two students answered the eight analyzed questions using one PM response and seven CM responses. We consider this favorable student performance.

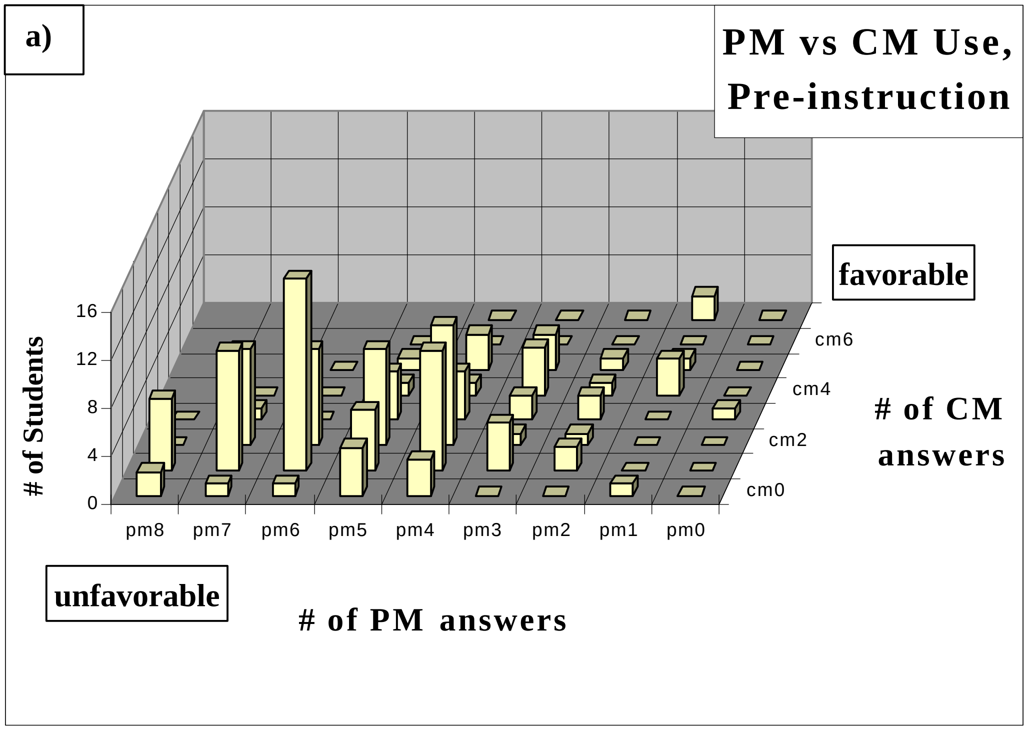
At the beginning of the semester, most students use primarily PM reasoning. They use CM reasoning for only one or two questions. Based on these results, we conclude that students are inexperienced with wave physics and are using the previously learned mechanics to help guide their reasoning for most topics. As stated in a previous chapter, student attempts to use their previous knowledge to guide their reasoning on unfamiliar topics is a quality that we would like them to develop in the classroom. The difficulty in this setting occurs from the incorrect application of otherwise useful primitives to waves.

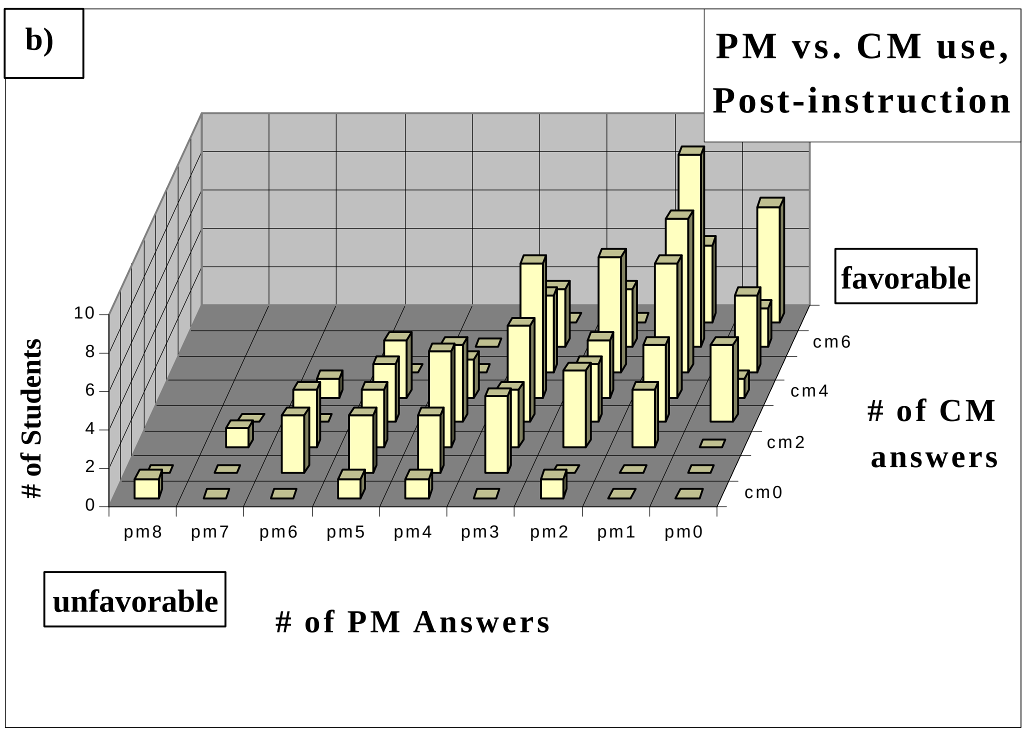
At the beginning of the semester, some students are located in the middle region of the graph, answering between 3 and 5 questions using both the PM and the CM. This indicates that the students are in a mixed state of knowledge about the physics when they enter our course. As was suggested in chapter 3 in the context of the FR and MCMR wave propagation questions, students have difficulty being consistent in their descriptions of physics topics. In the discussion of FR and MCMR responses, we found that students often recognized the correct responses but were unable to call them up on their own. It may be that pre-instruction student performance shows evidence that students are aware of a few correct ideas in wave physics, but predominantly use the PM to guide most of their thinking.

At the end of the semester, students have begun to use more CM reasoning, but still use PM reasoning heavily. Where they began the semester predominantly in the high-PM, low-CM region of the plots, they end the semester spread out in the middle- to high-CM region of the plots. The data as presented do not show that the number of responses categorized as “other” has stayed roughly the same as at the beginning of the semester. Most of the movement in student responses during the semester seems to occur between PM and CM reasoning.

Our results suggest that students have difficulties when learning to describe new phenomena in physics. Students bring to the discussion an ability to make analogies to the knowledge they do have. These analogies are guided by their experience (limited, in the case of waves, since most wave phenomena that we deal with on a daily basis are not visible), and often the analogies are incorrectly applied

Figure 7-3





Histograms of student PM and CM responses on the final version wave diagnostic test. Data are from a) before instruction, b) after tutorial instruction. Data are matched, N=136 students. “Favorable” describes predominantly CM responses.

without a functional understanding of the physics. As students go through our courses, they learn aspects of the correct model of physics, but do not let go of their previous knowledge in all cases.

Describing class use of different reasoning methods

The previous two analyses have focused on an overview of changes in student performance on the wave diagnostic test, but an analysis and description of an entire class's performance is also possible. We have carried out this analysis by considering the use of the PM and CM separately, rather than in a two-dimensional histogram. We can look at the average use of the PM or CM within a class and use these criteria to categorize classroom performance in more detail. By providing a statistical language, this method summarizes the data and allows a discussion of classroom use of multiple reasoning methods that goes beyond a description of student movement from favorable to unfavorable responses.

Each data set consists of a count of how many students answered a specific number of questions using a specific reasoning method (either CM or PM). This is essentially the sum of each row of data in Figure 7-3. These data were plotted on a graph where the number of student responses was compared to the number of responses using the given reasoning method. By fitting equations to the data sets, we are able to parameterize the results in a way that lets us quantify any changes in student reasoning that occur due to instruction. This analysis is data-driven, in the sense that the data fits are chosen based on reasonable descriptions of the population and of the situation. Figure 7-4 shows the data fits for the number of questions to which students responded with the PM before and after instruction. In Figure 7-5, the data for the number of CM responses on the wave diagnostic tests from both before and after instruction are presented. A variety of methods was used to determine the best fits of the data. For example, the pre-instruction PM data were plotted on a lin-log plot to help determine the best data fit. The plot is shown in Figure 7-6. The best fit for the data was parabolic with negative curvature, indicating that the best fit for the actual data would be a Gaussian or normal distribution. For the lin-log plot, a parabolic fit gives an equation with the form

$$\ln(y) = -N(x - x_0)^2 + y_0 \tag{7-1}$$

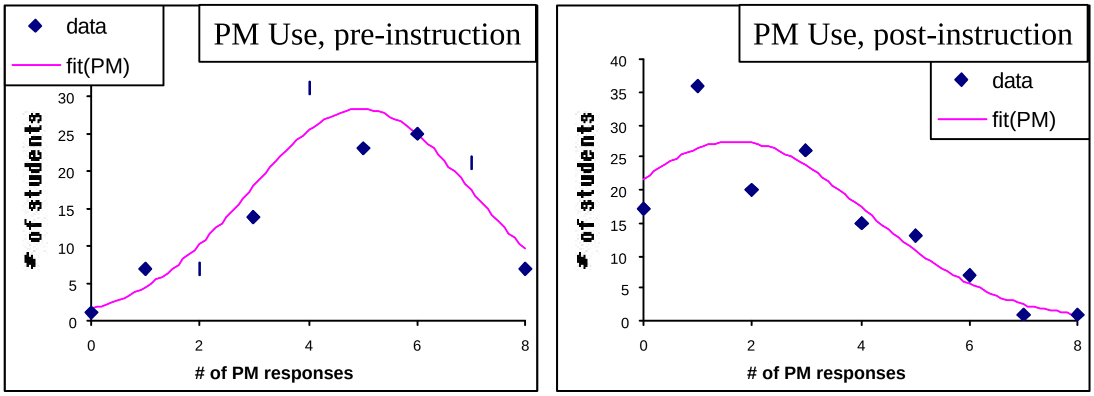
where the parabola has a width determined by N and has its maximum (or minimum) is located at x_0,y_0. The negative sign determines the downward shape of the parabola.

Solving for y gives the normal distribution,

$$y = Ae^{-\frac{1}{2}((x-x_0)/s)^2} \tag{7-2}$$

where $A = e^{y_0}$ and $s = (2N)^{-1/2}$. In this situation, s^2 gives the standard deviation of the data around the mean, x_0. Three of the four data plots (pre-instruction and post-instruction PM use, post-instruction CM use) were best described with normal distributions, as determined by the method described above.

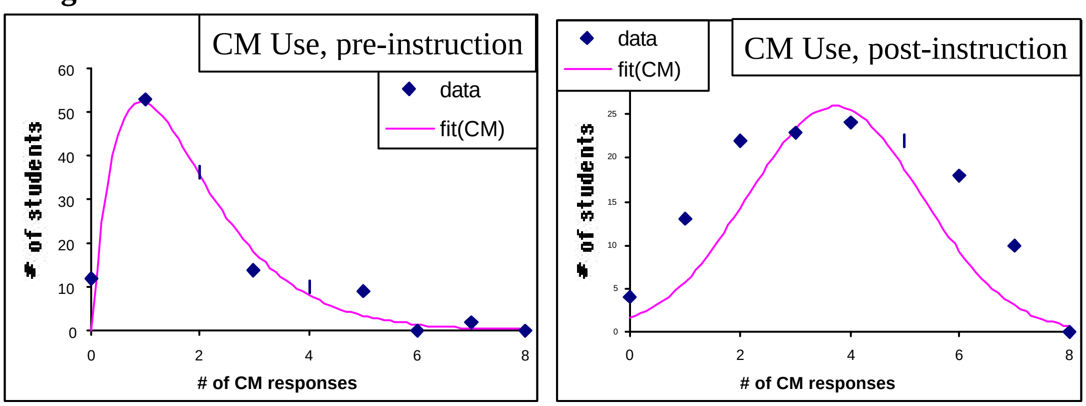
Figure 7-4



Pre- and post-instruction PM use on the wave diagnostic test by N=136 students, F97.

Fits for both sets of data are given by a Gaussian distribution, $y = Ae^{-\frac{1}{2}((x-x_0)/s)^2}$. For the pre-instruction data, A = 28.0, x_0 = 5.03, σ = 2.02. For the post-instruction data, A = 27.4, x_0 = 1.68, σ = 2.43.

Figure 7-5



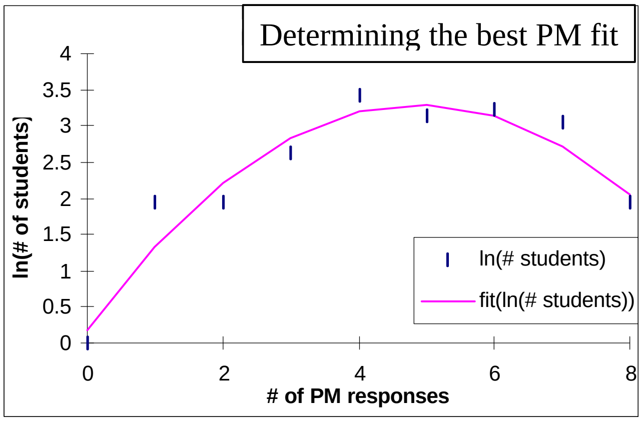
Pre- and post-instruction CM use on the wave diagnostic test by N=136 students, F97. The fit of the pre-instruction data is the integrand of a Gamma function distribution, $y=Ax^{b-1}e^{-x/c}$, where A = 154, b = 2, c = .926 (mean: bc = 1.84, variance: bc^2 = 1.71). The fit of the post-instruction data is given by a Gaussian distribution,

$y = Ae^{-\frac{1}{2}((x-x_0)/s)^2}$, where A = 25.9, x_0 = 3.73, σ = 2.23.

For the fourth data plot, a different function had to be found. The pre-instruction data indicating CM use is heavily skewed to the left. Most of the students did not use any CM responses at the beginning of the semester (which is not necessarily surprising, since the investigation preceded any study of waves in the classroom). But, very few students used *no* CM responses (recall that on the MCMR wave propagation questions, 85% of the students entering the course included the CM in their responses). Thus, a function had to be found that went to zero at the origin but also decayed very quickly to zero as the number of questions answered using the CM increased. A function of the type

$$y = Axe^{-nx} \qquad (7\text{-}3)$$

Figure 7-6



F97 pre-instruction PM data plotted on a lin-log plot. A parabolic fit was the best fit for the data, indicating that the appropriate fit for the lin-lin plot of pre-instruction PM use vs. # of PM responses would be Gaussian (normal distribution).

would provide this structure. Equation 7-3 is closely related to the integrand of the Gamma function distribution,

$$y = Ax^{b-1}e^{-x/c} \tag{7-4}$$

with values of $b = 2$ and $c = 1/n$. For such a function, the mean value is given by $x_0 = bc$ and the variance (the equivalent of σ^2 for a Gaussian distribution) by bc^2. As opposed to the normal distribution, the mean of the Gamma function distribution is not located at its maximum value (which is $x_{max} = bc - c$, as determined by setting the first derivative of equation 4 equal to zero).

Once the functions had been chosen for the fits, the various parameters had to be fit correctly. Using a spreadsheet program, the sum of the squares of the differences between the actual data value and the fitted value was computed. Using macros in the program, the lowest value of the sum of the squares was computed by varying the mean, standard deviation, and normalization values. Since the normalization values were primarily determined by the size of the class (N=136 students), the normalization is not a measure of student performance on the wave diagnostic tests while the mean and standard deviations are.

Table 7-3 shows a comparison of mean and standard deviation values for PM and CM use before and after instruction. These data describe the class performance rather than an overview of individual students' performances. Students start the semester using the PM to answer most of the questions (5.03 ± 2.02 questions), and only answer a few questions using the CM (1.84 ± 1.71 questions). The sum of PM

Table 7-3

Data fit Value:	Pre PM use	Post PM use	Pre CM use	Post CM use
Mean	5.03	1.68	1.84	3.73
Standard Deviation	2.02	2.43	1.71	2.23

Summary of data presented in Figure 7-4 and Figure 7-5. Class averages are from matched data of 136 students who answered identical wave diagnostic test questions before and after instruction in F97.

and CM means before instruction is nearly 7 out of 8 questions (though many students answered one with mixed MM use, which was then counted for both PM and CM use), showing that for the class average, most students are using these two models. Also, the standard deviations of the pre-instruction data overlap only slightly. This shows that the data are separated far enough to show that there is little overlap in student model use.

At the end of the semester, the mean value of PM use is nearly as low as the pre-instruction mean of CM use, but the standard deviation is larger (1.68 ± 2.43 questions). Thus, the spread in PM use for the class is larger, showing that students have moved away from their pre-instruction reasoning but not in a consistent fashion. The class is still using the PM more after instruction than they used the CM before instruction.

The post-instruction CM data has also not moved as far from the pre-instruction as we would like as instructors (3.73 ± 2.23 questions). The mean value of the data shows that the class as a whole uses the CM for slightly less than half of the questions. Again, the spread of the data is large, implying that students are spread out over a many different levels of CM use in their responses. Class use of the CM after instruction is less than class use of the PM before instruction, implying that the model of waves which they learn in our classes is used less than the inappropriately applied reasoning with which students enter our classes.

Finally, when looking at the sum of the means of post-instruction mental model use, we see that 5.4 questions are answered using one of the two mental models. Recall that on the post-instruction wave diagnostic test there were 9 questions and that mixed reasoning was common to the MCMR wave propagation question. Thus, many students are using other explanations after instruction, and many are leaving questions blank. The latter occurred often on the FR sound wave question, for example, since it was at the end of a lengthy pretest and many students did not complete it. Still, 5.4 out of 8 questions is still very low. In general, we see that students have a strongly mixed understanding of wave physics, with the PM and CM being their predominant models but other explanations also playing a role. In F97, they did not leave our classes with a coherent understanding of wave physics.

Summary

In this chapter, I have described how the development of a diagnostic test to investigate the dynamics of student reasoning about wave physics. The diagnostic was developed to elicit the most common difficulties we saw students having in our previous investigations. These difficulties have been organized in terms of the Particle Pulses Pattern of Association (PM). We have also characterized the model we would like students to learn in our introductory physics classes as the Community Consensus Model (CM).

A preliminary version of a diagnostic test showed that students used both the PM and the CM to describe wave physics after instruction. Based on these results, we developed a written wave diagnostic test that could be administered both before and after instruction. This would let us evaluate the effectiveness of instruction with respect to the broad picture of student understanding of waves.

Results on the F97 wave diagnostic test were interpreted in terms of both student and class performance. Student performance was described by considering two-dimensional histograms which showed both PM and CM performance. The histograms indicated that most students were moving from predominantly PM to mixed PM and CM reasoning. Students seem to leave the classroom with a less coherent (though more correct) model of waves than the model with which they entered our course.

Class performance was described by analyzing use of a single reasoning method (PM or CM) and fitting functions to the data. Again, we saw that the class as a whole began the semester using predominantly the PM with only weak use of the CM. Out of eight questions, an average of 5.03 (±2.02) were answered using the PM and less than 1.68 (±2.43) were answered using the CM before instruction (numbers in parentheses are the standard deviations of the distribution functions used to fit the data). After instruction, the PM is still used to answer some questions (1.84 ± 1.71), and the CM is used more often (3.73 ± 2.23). Still, the CM is not used as often as one would hope, and class performance indicates that most students are finishing instruction on waves with a mixed understanding of wave physics.

Both analyses indicate that students go through a transition in their understanding of wave physics. Students bring previous understanding to the classroom. We can say that they apply their previous understanding to new settings in an attempt to make sense of the material. We have found that students enter our classes using mechanics-based reasoning inappropriately applied to wave physics. They leave our classes using the correct model of physics but still holding on to their original analogies and reasoning patterns.

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[1] See chapters 3 and 6 for a discussion of the use of FR and MCMR questions to gain insight into student understanding wave propagation.

[2] We have not yet had the opportunity to investigate the difference in student performance in the two instructional settings in more detail.

[3] Some students were not present during one of the weeks the post-instruction diagnostic was asked. Others did not complete large parts of the diagnostic.