Chapter 5: The Particle Pulses Mental Model

Introduction

In chapter 3, I describe specific student difficulties with physics in the context of waves. These topics included:

- a failure to distinguish between a disturbance to a medium and the manner of the propagation of the disturbance in the medium through which it travels,
- the inability to consistently describe the condition of an equilibrium state of the medium,
- the interpretation of the mathematics of waves in overly simplified terms that often show no functional dependence on variables that describe changes in both space and time, and
- the failure to adequately describe the interaction between two waves both as they meet and after they have met.

In each topic of investigation, the specific difficulties are indicative of more fundamental questions, such as how students understand and make sense of physics. In this chapter, I will use the context of student difficulties with wave physics to propose a model with which we can organize the observed student difficulties.

Although I have described student difficulties with wave physics on a topic by topic basis, there are certain similarities in student reasoning we can use in each case. In chapter 4, I described a model of learning that helps describe and organize the difficulties we see students having. This model is built from the idea that students use basic reasoning elements called primitives that are reasonable in one context but may be applied inappropriately or incompletely in another. We can describe a set of primitives and the rules that tell students when to use them as a pattern of associations that guides student reasoning in unfamiliar situations. A pattern of associations is possibly incomplete, incoherent, and self-contradictory, and serves as an example of the type of guiding structure that students might have when dealing with unfamiliar material. When a pattern of association has a reasonable level of completeness and coherence, we can refer to it as a mental model. Our analysis of the manner in which students organize primitives into patterns of association can help us understand the manner in which student beliefs about wave physics change over the course of instruction. This can serve as an example of how students come to make sense of physics in general, not just wave physics.

In the first part of this chapter, I discuss the common primitives that students use when describing wave physics. I introduce a new primitive not previously described in the literature, the *object as point* primitive. As with other primitives, it is often useful and helpful in simplifying reasoning in some areas, but problematic when misapplied in wave physics. Then, I summarize extensive interviews with four students who answered questions on a large number of wave physics topics. The interviews illustrate how certain primitives are regularly but incorrectly applied to wave physics.

Research results have been gathered using techniques and investigations previously described in chapters 2 and 3.

In the second part of the chapter, I use student responses to describe the idea of a pattern of association that we refer to as the Particle Pulses Pattern of Association (which will be loosely referred to as the Particle Model or PM of waves). This pattern of associations describes the analogies that students use to guide their use of the specific primitives. In the second part of this chapter, I will discuss how the PM is used by students to guide their reasoning. In this case, the PM has not so much predictive as productive powers, helping the student choose which primitive (or facet) to apply to a given situation. I will also compare how students use the PM in comparison to reasoning based on the correct model of wave physics, as described in chapter 2.

Some of the interview or examination quotes have been given in the previous chapter but will be repeated here for further discussion. In some interviews, we see that students use more than one guiding analogy in their reasoning. This is consistent with the results described in chapter 3, where we saw students using more than one form of reasoning to describe a single physical situation.

Student Use of Primitives in Wave Physics

In chapter 4, I discuss a variety of primitives that have been studied mostly in connection to student reasoning in mechanics. In this section, I describe the common primitives used by students who show difficulties with wave physics. In addition to those primitives describe in chapter 4, we find that in their reasoning about wave physics, students seems to use at least one additional primitive not previously included in the literature. First, I use results from chapter 3 to illustrate student use of the "object as point" primitive. Then, I give a more detailed discussion of other commonly occurring primitives in the context of interviews with four students who had difficulties with many of the topics described in chapter 3.

The object as point primitive

The object as point primitive (henceforth called the point primitive) is based on observations of student descriptions of waves, but has a more general applicability. The point primitive plays a central role in this dissertation, being the focus of the mental model which I will describe later in the chapter. Before more rigorously defining the point primitive, I will motivate why we believe it exists by quoting from interviews used in the previous chapter.

In interviews in which students described their understanding of the mathematics which describe waves (what I have called the wave-math problem), students were presented with equation 5-1 in a situation in which they were asked to describe the shape of a propagating wave.

$$y(x) = Ae^{-\left(x_0/b\right)^2} \qquad (5\text{-}1)$$

We observed students' inabilities to properly describe the variables in the equation. Most notably, students could not adequately describe and use the variables *x* and *y*. Most students who sketched a pulse whose amplitude had decreased gave the explanation that the exponent value would decrease as the value of *x* increased. They were using the variable *x* to describe the location of the peak (originally at x = 0), and then were interpreting the variable *y* to describe the peak amplitude of the wave, not the displacement of the string at all points. Many students were effectively interpreting the entire wavepulse, an extended region of displacement from equilibrium, as a single point.

Student use of the point primitive when answering the wave-math problem shows how a primitive that may seem appropriate is actually inappropriate when applied to a particular situation. First, student descriptions of decreasing amplitude are consistent with their observations of wavepulses whose amplitude decreases due to friction with the floor while propagating on springs on the floor, as shown during demonstrations in the classroom. When working with students on this material in the classroom, I have had some students state that the mathematics should be consistent with their observations (though we find that many students are unable to operationally carry out this general principle). The correct application of the deep principle that the mathematics and physics should be consistent (which we should encourage students to develop) may lead to student difficulties in this situation. Thus, the interpretation students use is strengthened by the fact that they consider the result to be obvious, i.e. consistent with their observations.

Second, students do not describe the possible physical reasons for the decreased amplitude in their explanations. Instead, they often cite the equation and the effect of a change in x on the exponent. Students fit the mathematics to the situation they observe by using the archetypal example based on a classroom demonstration, and in the process, they give (nonphysics) explanations which incorrectly use the mathematics. We observe that students are trying to interpret the equation and make sense of the equation (again, a skill which we should encourage them to develop), but that they have difficulties knowing how to make sense of the mathematics.

Evidence from other areas of wave physics show that students seem to be applying this primitive to more than the wave-math problem. In wave superposition questions, students who were asked to sketch the shape of the string when two asymmetric wavepulses partially overlapped often sketched the shape of each individual pulse without adding displacements at the appropriate points. (See, for example, Figure 4-13c). Those students again appeared to be simplifying an extended region of displacement down to one point. Students often use of the word “amplitude” to describe this point. A student who drew a sketch like the one in Figure 4-13c explained, “The waves only add when the amplitudes meet.” Unless the two points of the wavepulses which the student considers relevant overlap, these students assume there is no summation of displacements (superposition) in the region where the wavepulses do overlap. Interviewed students who gave an explanation like the one just described merely asserted that the shape was just as they had sketched it. They were unable to give a more detailed explanation, other than to say that there was no addition until the peaks overlapped. When asked about the other displaced regions, students often had no explanation as to how they would interact. Many students often are unable to explain through more than an assertion. It seemed that the assertion itself was sufficient as an explanation for these students. This "non-dissociability" of an explanation is a common characteristic of cognitive primitives.[1]

We also see an application of the simplification of a wavepulse to a single point in student descriptions of how to change wave propagation speed. A more detailed description of student explanations for changes to wave propagation speed will be given below. At this point, it is sufficient to say that students seem to make an analogy between the wavepulse and an object like a ball. By thinking of the wavepulse as a single point, students can apply ideas to wave propagation based on analogies to the motion of a point particle. Furthermore, a student who states "You flick [your hand] harder...you put a greater force in your hand, so it goes faster," gives an example of the heuristic principle which states that students will use simple body motions as part of their explanations. In interviews, students often make the hand motion of flicking their wrist up and down slowly to describe slow pulses and quickly to describe fast pulses. They use their body to help describe the base vocabulary of their reasoning, again consistent with diSessa's heuristic principles.

Another example of student simplification of waves to single points comes from the research into student understanding of sound waves. In the interview quoted at length in chapter 3, Alex described the sound wave as exerting a force on the particle. He sketched the wave as a series of pulses and described the pulse exerting a force on the dust particle as a "kick" or a "hit." During the interview, he had simplified the repeating sinusoidal wave to a succession of pulses, and then described each pulse as a point which could exert a force, kick, or hit the dust particle which it encountered in only one direction.

The point primitive is characterized by the description of a large, global object or wave in terms of a single point. In the case of wave physics, it seems to function as an interface between the shape of a wave (in some given or assumed representation) and the manner in which the wave can be influenced or influences its surroundings. We have frequently encountered the point primitive in student responses to questions in all areas of wave physics investigated for this dissertation.

Beyond the difficulties discussed in chapter 3, we have found additional evidence of its use in student descriptions of wave reflection. Students drawing a wavepulse on a string attached to a wall state that the wavepulse will not be reflected until the peak of the pulse has reached the wall. These students have difficulties in deciding on the shape of the string or pulse when the front of the wavepulse has reached the wall but the peak hasn't; they want to preserve pulse shape, but they also know that the string remains attached at all times.

The point primitive is not necessarily problematic. Instead, it is a perfectly reasonable and useful reasoning method when quickly analyzing certain physics problems. For example, when solving simple trajectory problems in Newtonian physics, the community consensus is to immediately simplify the object traveling along the trajectory to a point particle. Especially in situations where the rotation of an object is unimportant and there are no collisions, we treat the center of mass as this point, and ignore all other points. The analysis by which the point primitive is applied to the rigid body can be quite complicated. Finding the center of mass of a non-symmetric body involves complex integration and is today typically only briefly discussed in upper-division graduate mechanics or advanced engineering courses.[2] The source of difficulties in the use of the point primitive lies in how it is used in wave physics, not to its existence in the student's repertoire of reasoning tricks.

Common primitives in wave physics

Rather than again describing each of diSessa's primitives (see chapter 4) as they apply to wave physics, I describe how they are used in the context of four student's difficulties. Table 5-1 gives a summary of the primitives used by each student. Table 5-2 gives a brief description of each of the primitives first described in chapter 4 and the wave physics topics in which students applied it. (Note that Table 5-2 has been split into three sections due to its size.) For Table 5-1, some categories of Table 5-2 have been combined into one due to their similar nature in the context of waves. The reader is asked to refer to these tables during the discussion below.

In S96, we carried out a set of pretest interviews with four students over the course of several weeks. Each week, students were interviewed about their responses to questions asked on the pretest given in preparation for that week's tutorial.[3] Five weeks of interviews were carried out, where three addressed issues discussed in this dissertation. The four students were asked to answer the pretest questions while an interviewer probed their responses and investigated whether their written and interview comments were similar. Certain issues were probed more deeply during the interview than had been possible in the pretest.

In S97, 20 students from two different instructional settings participated in a diagnostic interview. Fifteen students had participated in early versions of tutorials designed to address student difficulties with wave physics. Five students participated after traditional instruction in a class with recitations. Most of the 20 students answered 18 questions which dealt with wave propagation speed, superposition, the physics of sound, wave reflection, and wave mathematics. Two subjects, wave propagation and wave mathematics, were investigated with both FR and MCMR questions. The sound question was asked in MCMR format only. A copy of the final version of the S97 interview diagnostic test is given in Appendix D-1. It is discussed in more detail in chapter 7.

Table 5-1

Students **Primitives**	**Ford**	**David**	**Kyle**	**Ted**
Object as point	X	X	X	X
Force and Motion (Ohm's, actuating agency, smaller is faster, working harder)	X	X	X	X
Collision (bouncing, Canceling, overcoming)	X	X	X	X
Dying away (possible interpretation of point **primitive)**		X		
Guiding			X	X

Brief summary of primitives used by students. For a more complete description of each primitive, see Table 4-1, chapter 4.

Not all students answered all questions for a variety of reasons. Due to time limitations, some students did not finish the diagnostic test. Also, during the course of the interviews, I made changes in the protocol based on student feedback and responses. Therefore, during the course of the 20 interviews, some questions were rephrased, some dropped, others added. The development of a diagnostic test to investigate student understanding of wave physics will be described in more detail in chapter 7.

Table 5-2a)

Primitive	Definition (wave physics specific)	Context (specific wave topic)
Object as point	An object (i.e. a wavepulse) is characterized by a single point (e.g. of maximum displacement from equilibrium); a large object is simplified by referring to just one piece of it, like the C of M …	**propagation speed:** ball-toss analogy **superposition:** a) no peak addition b) "global" addition **mathematics** a) x as peak location b) plugging in x_0 for eqn. **Sound** wave=pulse=point (exerts force)

Table 5-2b)

	Collision Primitives	
Primitive	**Definition (wave physics specific)**	**Context (specific wave topic)**
canceling	A wave can be permanently cancelled by another wave (see dying away for opposing forces).	**propagation speed:** none **superposition:** rules for partial cancellation of pulses **mathematics** none **sound** none
bouncing	A wave (as an entire unit rather than a region of displaced elements of the medium) will hit another wave and bounce off it.	**propagation speed:** none **superposition:** rules for partial cancellation of pulses **mathematics** none **sound** none
overcoming	Two competing forces (i.e. waves) will interact such that one overpowers the other (see Canceling but for unequal amplitude waves).	**propagation speed:** none **superposition:** rules for partial cancellation of pulses **mathematics** none **sound** none

Table 5-2c)

Force and Motion Primitives		
Primitive	**Definition (wave physics specific)**	**Context (specific wave topic)**
Actuating Agency	A directed impetus (force) on the medium causes a displacement of the medium and/or a wave speed in the direction of the impetus.	**propagation speed:** more force into wave = faster **superposition:** none **mathematics** none **sound** force on medium in propagation direction
Ohm's primitive	An agent or impetus (force) causes an action whose effect depends on the resistance of the medium or the effort of the agent (i.e. strength of the force). Effort and resistance are in a covariant relationship with each other.	**Propagation speed:** bigger pulse feels resistance and is slow, so wave needs greater force to go faster **superposition:** rules for superposition cancellation **mathematics** none **sound** increased loudness evens out with decreased frequency.
Working harder	The mechanism by which the Ohm's primitive plays a role in the actuating agency primitive, i.e. to make pulse go faster, you have to put more effort into it (hence, larger pulses took more effort to create, and will move faster).	**propagation speed:** bigger pulse needs more force **superposition:** bigger pulse has bigger force **mathematics** none **sound** force increases with volume
Smaller objects naturally go faster	Larger waves take more effort to create because there is more resistance to their creation; i.e. smaller "quicker" pulses are faster since less force is used to overcome the resistance of the medium.	**propagation speed:** tinier, tighter pulses go faster **superposition:** none **mathematics** none **sound** high f waves move faster
Dying away	The motion of the wave must eventually die away, i.e. the amplitude of the wave decreases not due to physical reasons but due to unscientific observations. By taking away the internal "force" from an object, it loses some of its amplitude.	**propagation speed:** none **superposition:** partial cancellation of pulses **mathematics** x increases, y decreases, and pulse shrinks **sound** none
Guiding	A piece of the medium must move along the track determined by the wave, like the dust particle moving on a sinusoidal path.	**propagation speed:** none **superposition:** partial cancellation of pulses **mathematics** none **sound** none

Common primitives and their use in wave physics.

We found that some students had consistent difficulties across a broad range of wave physics topics. Many of the tutorial students did quite well on the diagnostic, indicating the effectiveness of the specially designed curriculum. (These results will be discussed in more detail in chapter 6.) Those students who had not participated in tutorials showed more profound difficulties.

By looking at how the students who performed poorly answered questions in each topic, we find suggestions that they are using a common set of primitives in conjunction with each other. Three students, "David," "Kyle," and "Ted," stand out as having similar difficulties on many of the topics in the S97 diagnostic test.[4] One student, "Ford," stands out on the S96 pretest interviews. David and Kyle had completed tutorials on wave physics roughly two months prior to the interviews. Ted did not participate in any tutorial instruction. Ford had not yet participated in tutorials at the time of the interview.

Ford

In the S96 interviews, Ford showed difficulties with the topics of wave propagation and superposition. He had no difficulty with the wave-math pretest.

Ford's most interesting comments came while describing how to change wave propagation speed and while describing the collision of overlapping, superposing pulses. We began the interview by asking him how to create a single wavepulse on a taut string. He described a quick up and down hand motion and described the force needed to create the wave, an indication of the *actuating agency* primitive. When asked how to change the speed of a wavepulse, Ford responded (Ford's comments are indicated with "F," the interviewer's with "I"):

> *F: There are two scenarios that I have to think about, and since you want me to say right now... I'd send a quicker one (He draws a much smaller pulse, both in width and in amplitude, and he does a much quicker hand motion to describe how this pulse would be created) ...*
>
> I: *By quicker you mean, you did your hand motion like this?* (repeats the motion)
>
> F: *shorter, I wouldn't go* (makes large hand motion) *I'd try to make a shorter hand motion* (makes a quick flick of the wrist) … *It would get there faster.*
>
> *Or, I would send a huge pulse, where maybe [the pulse] could cover the whole thing* (i.e. the entire string) *in one pulse and maybe get there as fast as I put my wrist back down.* (does long build-up during this, and then slams hand down hard at the end) *The odd idea is that I don't know which one would work better.*

Ford indicated two different reasoning methods in this excerpt. On the one hand, he showed with his body how to make a smaller pulse and indicated that the smaller pulse would move faster. He did not distinguish between transverse velocity and longitudinal, propagation velocity. Note that his explanation makes use of a physical motion. This suggests that students are using a primitive in their reasoning. Ford's first explanation gives evidence of the smaller is faster primitive, since he spoke of a smaller wavepulse moving faster. At the same time, upon further reflection, Ford indicated that possibly a larger pulse would move faster. This explanation seems strongly connected to his description that a larger pulse would take more area, and a pulse with more area would move more quickly. His body motion is indicative of the working harder primitives, where a larger force is needed to create a larger pulse and a faster pulse is the result. Note also that the emphasis on the downstroke of the hand motion implies that he thought specifically of the speed and force of the hand motion at this time (showing difficulties with the idea that the leading edge of the wave would already be propagating forward). Ford's difficulty and inability to resolve these two descriptions caused a conflict in his thinking. Even after further questioning during the interview, he maintained that both explanations could cause a faster pulse, but he knew that one of them must be wrong.

The following week, Ford answered questions about wave superposition. (Before the interview began, he mentioned that the previous week's tutorial had helped him resolve his dilemma; he now knew that neither of his explanations was correct and was able to correctly describe the medium changes which caused a faster pulse.) We gave him two questions concerning wave superposition, the first shown in Figure 3-12 and the second shown in Figure 3-8.

In his descriptions of superposition, Ford spoke of the wavepulses as single units which collided with each other, and, as they collided, their amplitudes would begin to add up even though the highest points of the wave were not at the same point. "When they collide," Ford stated in the interview, "they have the same base, and I just added their amplitudes." His reason for adding the points of largest displacement, even when they do not overlap, suggests that even the point primitive does not necessarily restrict an object to one point. Instead, Ford seems to use it to make the entire object into one large point, where elements of the object then overlap and interact. But full interaction does not occur until the wavepulses completely overlap. As Ford stated when describing why the amplitude was not doubled during partial coincidence of the wavepulses, "they haven't fully combined yet, they haven't fully interacted yet."

Ford used the point, actuating agency, and working harder primitives to describe changes in wave propagation, and he used the point primitive when describing superposition. The combination of point primitive and the force primitives implies that Ford had a picture of waves that was at odds with the material being taught in the class. He did not describe waves as disturbances to the medium, and did not have a clear description of how waves interacted with each other.

Though Ford showed difficulties with only two of the physics topics, his use of many primitives suggests the manner in which inappropriately applied primitives can lead students to an incorrect understanding of the physics. He used conflicting primitives but did not accept that both would be valid, suggesting that the structure of Ford's understanding involved guidelines for reasoning that are not necessarily rigid, fixed rules and that he was uncomfortable with this flexibility.

David

David participated in the S97 diagnostic test interviews approximately two months after completing tutorial instruction on waves. The details of the instruction will be given in chapter 7 when the diagnostic test is described in more detail. David had no problems with the questions dealing with sound waves and did not answer questions about wave reflection due to time limitations during the interview. His difficulties with the other topics on the diagnostic test are consistent with those described in chapter 3.

David described how one could change the speed of a wavepulse on a string by saying,

> D: *Well, I know that tension affects the wave speed, that is … the rate the pulse moves down the string. And… the amplitude would affect it.* (He shows a hand motion with a larger displacement while saying the last sentence.) *I think possibly, you see a slower … pulse if the force applied to the string is reduced, that is: the time through which the hand moves up and down [is reduced].*

In his comments, David uses the actuating agency primitive as the basis for the incorrect part of his description. His description is consistent with thinking of the wavepulse as a point particle that moves faster due to a larger force (i.e. the point and working harder primitives seem to play a role). The reader should note that he begins with the correct answer before giving other explanations.

In the question dealing with the mathematical description of waves, David describes a decaying propagating wavepulse. He states that the shape would stay the same, but the amplitude would change, and says “this is a decay function” while pointing to equation 5-1. This shows clear evidence of the dying away primitive (and again the point primitive, since he is letting the single maximum amplitude point guide his description of the whole wavepulse). David uses the variable, x, to describe the position of the peak of the pulse. Thus, he seems to use the point primitive to interpret the mathematics with which he has difficulties.

David had two difficulties with superposition. In one question, he described two pulses meeting on a string as bouncing off each other, reversing sides on the string, and returning from whence they came. The collision analogy came quickly and spontaneously. With equal sized pulses, nothing cancelled out, and the pulses bounced off each other. The idea of one pulse overcoming another was probably not considered because the pulse sizes and shapes were equal. As soon as the interviewer asked him to explain how he arrived at the answer, he changed his mind, and gave a different description. Rather than using the bouncing primitive, he suddenly gave the correct answer that the wavepulses would pass through each other with no permanent effect. David’s easy change of mind is consistent with our research results that, after any form of instruction, very few students give explanations which are consistent with the collision analogy.[5] David’s changing explanation also supports diSessa’s comment that many primitives related to collisions (overcoming, bouncing, and collision) are generally weakly supported by students.[6]

When describing superposition of two pulses whose peaks had not yet coincided but whose bases overlapped, David also showed difficulties consistent with the use of the point primitive. He gave answers which described no superposition in the region between the peaks of the pulses. The highest point of the string due to either pulse determined the shape of the string.

David showed evidence of a variety of primitives in his explanations during the diagnostic interview. As with Ford, we see evidence for three classes of primitives. David used the point primitive, some of the force primitives (actuating agency, working harder), and one of the collision primitives (bouncing). Furthermore, he applied the point primitive to the mathematics of waves and made use of the dying away primitive in the process. David's interpretation of wave physics seems based on reasoning that is inconsistent with actual observations. Instead, it seems to rely on analogies and reasoning based on previous studies in mechanics.

Kyle

Like David, Kyle participated in the S97 diagnostic test interviews two months after having completed tutorial instruction in waves. The details of the instruction will be given in chapter 7 when the diagnostic test is described in more detail. During the interview, Kyle inappropriately used many of the primitives described above. Only on the wave-math problem did he not use one of the previously described primitives, but he was unable to write an accurate equation for the wavepulse at a later time. Instead, he gave the same equation as the original one, when the pulse's peak is located at the origin. Though a profound difficulty, I had no further evidence to indicate the source of Kyle's answer. The lack of a temporal element to his equation possibly implies that Kyle was using the given equation to describe the entire wavepulse shape rather than the mapping of coordinates into each other (this would imply use of the point primitive to interpret the mathematics), but I am unable to support or refute this speculation from statements made in the interview.

When describing changes to wave propagation speed on the FR question, Kyle gave a variety of answers, not including the correct one. Kyle stated that one could make a slower pulse by "moving your hand slower," or "[moving] your hand higher, increasing the amplitude." In this description, a larger wavepulse would move more slowly through the medium, as if there were more resistance to its motion. This answer is indicative of the Ohm's primitive. Also, his response shows evidence of the smaller is faster response (since larger must be slower).

Later, when answering the same question a second time (in its MCMR format, rather than its FR format), Kyle seemed to reverse his reasoning. He gave the correct responses in addition to the responses he had given earlier. Both Kyle and David used inconsistent reasoning depending on the question asked of them. In this case, Kyle gave a correct response and stated that a lower tension would cause a slower wave. But, Kyle used faulty reasoning to arrive at his answer. He stated that a lower tension would create a smaller wavepulse, "the wave will be smaller, because you have less tension for it, [and that will make it slower]." Note that his response contradicts his earlier explanation that a larger pulse would be slower. This provides an excellent example for the student variation in student responses on material they have not mastered.

When describing wave superposition, Kyle spoke of colliding waves, and he showed no indication of wave superposition unless the peaks of the wavepulses overlapped. Much like Ford, Kyle spoke of collisions but never indicated that the pulses would cancel permanently. Describing the moment before the wave peaks coincided and the wavepulses partially overlapped, he said that the waves "were about to add," suggesting that he thought of the wave as only the peak of the pulse and not the entire displaced region. This shows evidence of the point primitive.

Kyle also had difficulties with the physics of waves reflecting from a fixed boundary (in the case of a wavepulse on a string firmly attached to a pole reaching the pole). He spoke of the pulse being absorbed into the pole to which the string was firmly attached, though he later changed his answer to state correctly that the pulse would be reflected and no energy would be lost in an ideal situation. As with Ford's description of wave propagation and David's description of wave superposition, Kyle switched from one explanation to another. He gave no clear reason for the switch. My asking him to explain how he arrived at his answer may have led to an evaluation of his answer which he did not verbalize. Also, my questions may have led to a change in the type of reasoning that he used, from a "gut instinct" to a "classroom style" answer.

Kyle later described reflection from a free end in terms of energy absorption (the pole would absorb the energy that was in the wavepulse). He said that the ability of the end of the string to move along the pulse meant that nothing would be reflected back, stating that "the movement of the string takes away the movement of the pulse." Though he did not explicitly use reasoning based on the overcoming primitive, we can describe his answers in those terms. With this primitive, one could say the pole is unable to be moved and must therefore exert a large force on the incoming wavepulse. That large force then manages to cancel out the incoming pulse.

On the sound questions, Kyle described sound waves pushing dust particles in a sinusoidal path away from a loudspeaker. As he stated, (his words are indicated by a "K" while the interviewer's words are indicated by an "I")

> K: *The dust particle will move up and down and the dust particle will be pushed away from the speaker.*
>
> I: *So the dust particle is going to move in a path away from the speaker* (indicates a sinusoidal path with a hand motion in front of Kyle)
>
> K: *Yeah.*
>
> I: *Why is that? If you could explain…*
>
> K: *Since the speaker is* (mumble, incomprehensible) *it will push the dust particle sideways. Since the dust particle is affected by the frequency, as long as the frequency is constant, it will move in a constant path.*

Later, when asked the effect of a change in the frequency, Kyle explained that

> K: *[The dust particle] will move faster because the frequency is higher which means that – since frequency is the one that affects the dust particle, if the frequency increases, the speed of the particle should increase, also.*

Though on the surface this indicates a correct response (a higher frequency will cause a faster average speed, since the particle is oscillating more rapidly), Kyle seemed to refer to the motion of the particle away from the speaker. Kyle often used the phrase “the frequency affects the dust particle,” and when asked what he meant by the phrase, he stated

> K: *Frequency produces a sound wave, and the sound wave, somehow it will…* (He did not finish the statement, even with prodding from the interviewer).

In this description, Kyle is not using the term frequency to describe a property of the sound wave, rather, he states that the frequency *causes* the sound wave. This seems to be consistent with a description where the effect of a higher frequency is to make the particle move faster. Thus, Kyle’s explanation seems consistent with the explanation given by Alex which was described in chapter 3.

In Kyle’s explanation, the sound wave seems to guide the particle along the sinusoidal path. Kyle showed evidence of the guiding primitive (where the dust particle must move along the path determined by the sinusoidal sound waves) and the Ohm’s primitive in his explanations of sound waves (where higher frequency has a proportional effect on wave speed).

Consistently, we see Kyle having difficulties with wave physics in nearly all areas that were investigated. He made consistent use of the point primitive, either directly as in wave superposition or implicitly as in wave propagation or sound waves (where the point primitive is a necessary step for the actuating agency primitive). He also made use of force primitives and collision primitives. Based on Kyle’s responses, it is possible to interpret the guiding primitive as related to the force primitives, since it describes the relationship between force and motion.

Ted

Unlike David and Kyle, Ted did not have any tutorial instruction during the S97 semester. His class had traditional, TA-led recitations. In the diagnostic test interview, Ted answered questions from an early version of the diagnostic test which did not include any questions on wave-math. Ted showed profound difficulties with all other areas investigated in the diagnostic.

On the wave propagation question, Ted first stated on the FR question that only a larger amplitude would slow the pulse down. The larger pulse would move with more difficulty, he implied, because the pulse would “have to move more distance in the same amount of time.”[7] The movement of the wavepulse <u>along</u> the curved string (i.e. with a larger curve, the length of the path to be traveled increases) implies the existence of the guiding primitive in his reasoning. It may also imply the existence of the smaller is faster (since larger is slower) primitive. When Ted came to the MCMR question on the diagnostic, he used far more explanations in his reasoning. He kept the larger amplitude response, but added that a slower hand motion would create a slower pulse. The slower hand motion would put less force into the wave (an example of working harder or Ohm's primitives, as explained above with respect to Kyle). In addition, Ted stated that changes in the medium, both tension and mass density, would affect the speed of the wavepulse.

Ted's description of superposition did not include the addition of displacement between peaks when the peaks did not overlap. For a situation where the waves (but not their peaks) overlapped, he stated that "the pulses haven't quite overlapped, so there's no reason [the amplitude] should jump up until they meet." He seemed to use the point primitive to consider only the peak of the pulse.

In his description of a wave reflecting from a boundary, Ted stated that the pulse would not be reflected from a free end. He spoke of the energy being absorbed into the pole, so that "nothing is left." This idea of absorption is consistent with the *overcoming* primitive, as described with Kyle above. The energy of the wavepulse is not sufficient to affect the pole, so the pole absorbs energy and does not transmit any back to the string. Thus, the wave does not return.

Ted's description of sound waves was also similar to Kyle's. He spoke of the dust particle moving away from the loudspeaker along a sinusoidal path, suggesting the use of the *guiding* primitive in this context (recall his previous use of it in the context of wave propagation). Furthermore, when the frequency of the wave changed, the speed of the dust particle being pushed away from the speaker changed. Thus, Ted seemed to use the *Ohm's* primitive in which proportional changes in frequency and speed occurred. Frequency was associated with the force of the wave on the dust particle, as Alex described in the interview excerpts given in chapter 3. Ted differed from Alex in that he explained that the volume of the sound wave affected its amplitude and not the force that it exerted on the dust particle. He maintained consistency with his description of transverse motion while moving away from the loudspeaker by stating that the speed of the particle away from the speaker would be the same as with a lower volume, but that the amplitude of its motion would change.

Ted had profound difficulties with all the topics of wave physics investigated in the version of the diagnostic which he took. He used both the point and *guiding* primitives in more than one context. Additionally, he used many of the force and collision primitives that we also found evidence for in the other students.

Summary of Common Student Used Primitives in Wave Physics

These four students consistently used a small set of primitives when incorrectly describing parts of wave physics. The four students all use the point primitive to simplify the shape of a wavepulse to a single point. In addition, they all make use of at least one of the force primitives. The collision primitives are also common to all students.

The primitives that these students applied to wave physics seem to be more strongly connected to Newtonian particle mechanics than they are to wave physics. The force primitives all seem related to difficulties students have in understanding the relationship between force and motion in classical mechanics (see, for example, the discussion in chapter 2 and references cited there). Since the collision primitives all seem related to collisions between hard objects (such as billiard balls), the students' difficulties appear to come from the incorrect application of ideas that may be appropriate in other areas of physics. In addition, some of the students used other primitives which suggest that they are not thinking of waves when they answer questions, or that when they think of waves, they have a model unlike the one that we would like them to have.

The Particle Pulses Pattern of Association

When students have consistent difficulties in one area of physics, it gives us the opportunity to organize their difficulties in a way that is productive and relevant for the development of instruction and investigations in other areas of physics. The four students described above all used a common set of primitives to describe wave physics. The primitives are often closely related to each other, such as the bouncing and overcoming primitives in descriptions of superposition or reflection from a boundary or the force primitives when describing sound waves and the physics of wave propagation. Because students use these primitives in conjunction with each other, we conclude that students are associating certain primitives in the context of wave physics. We find a consistent inappropriate pattern of association in the students' responses. We also find that students are not coherent in their use of this pattern of association and they are not consistent in its application. Thus, we cannot necessarily say that they are using a mental model in their reasoning. The topic of wave physics can serve as a context in which we show the value of using primitives, patterns of association, and mental models to describe many different student difficulties with physics.

In chapter 4, I discussed the idea of patterns of association and mental models. Both can be thought of as sets of primitives that are consistently applied to a situation and may serve as guiding principles for reasoning. One can describe student reasoning as if suggested rules or analogies to guide spontaneous reasoning.[8] Thus, patterns of association and mental models serve as a type of reasoning guideline for students, but are not necessarily the only guideline. Patterns of association describe looser constructions of student reasoning that are not as coherent, rigorous, or robust as the term "model" implies.

I should note that physical models often have the same limitations as the pattern of associations and mental models I am describing here. We refer to an accepted physical model, determined through theoretical and experimental work and the agreement of the research community to be valid in certain physical realms with certain limitations as a *Community Consensus Model* (or CM). An example of a CM would be the model of wave physics that students learn in the introductory physics sequence (as described in chapter 2). Within the limitations of the small angle, non-dispersive media approximation, we can use the wave equation and certain simple rules to analyze most physics problems. In more complex situations, this model no longer holds. Furthermore, when trying to apply the model, it may lead to results inconsistent with the model. For example, two small amplitude waves that superpose may add in such a way that the small angle approximation no longer holds. Thus, the introductory physics waves CM has the limitation of being, incomplete, and inconsistent with experimental data in certain situations. The difference between a typical weakly organized naïve student pattern of association and a CM is that a physicist is (usually) aware of the limitations of the CM and knows the shortcomings of the model while students often are unaware of the consequences of the contradictions in their reasoning.

When trying to organize student difficulties described in the previous section, we find it convenient to propose the existence of a *Particle Pulses Pattern of Association* (loosely referred to as the Particle Model, or PM) of waves. Table 5-3 summarizes the different aspects of the two mental models. Typical reasoning patterns involve the incorrect use of force or energy arguments and an inability to look at local characteristics of the wave. The PM can be described as the set of common primitives discussed in the previous section. These primitives include the point primitive, some or all of the force primitives applied incorrectly, and the use of the collision primitives. Student use of these primitives has been described in the previous section. In the remainder of this section, I will summarize common student responses when using the PM to guide their reasoning in terms of analogies with Newtonian particle mechanics rather than with separate incorrectly applied primitives.

The analogy to the ball toss seems to guide student reasoning in many topics of wave physics. Exerting more force when throwing a ball makes the ball travel faster upon release. Many students can be described as if they make an analogy to the

Table 5-3

Particle Model	Particle Pulses Pattern of Association	Community Consensus Model
A harder throw implies a faster particle.	A harder flick of the wrist implies a faster wavepulse.	Wave speed depends only on medium response to disturbance.
Smaller objects are more easily thrown faster.	Smaller pulses can be created that move faster.	Size of pulse and manner of wave creation do not affect wave speed.
An object's center of mass is considered when describing its motion (e.g. trajectory).	Only the peak of the wave-pulse is considered when describing superposition.	It is necessary to consider the entire shape of a wave to describe its properties (e.g. in superposition).
Objects collide with each other and their motion changes	Wavepulses collide with each other and they cancel or bounce off each other.	Waves pass through each other with no permanent effect.
Large objects traveling on a trajectory are described as points.	Propagating wavepulses are mathematically described only by the displacement of the highest point.	The mathematics of waves describes the displacement of every point of the medium.

Newtonian particle physics analogies of the Particle Pulses Pattern of Association and correct wave physics of the Community Consensus Model. Many students use both when answering questions containing wave physics.

amount of force required to create a pulse on a taut string; greater force in the hand motion creates a greater speed. In our investigations of the physics of sound waves, we have found evidence that many of students think that waves exert a force on the medium through which they travel and push the medium in front of them like a surfer on a wave. In chapter 3, Alex gives a description of the "surfer" description when explaining the interaction of sound waves and air. He also explains that sinusoidal waves are like a succession of pulses, each exerting a force (or "kick") in only one direction on the medium through which they travel.

Other students seem to make the ball-toss analogy when using energy arguments to describe how to change wave speed. A ball with a larger kinetic energy whose mass remains constant moves faster. Similarly, a pulse with more energy whose size stays constant must move faster. The explanation that a smaller mass will move faster is consistent with this explanation, too, because a smaller mass, with energy held constant, has a larger velocity. Though students do not explicitly state the analogy between their descriptions of wave speed and a thrown ball, their descriptions in interviews are often consistent with the idea that students' patterns of associations make use of this analogy.

Students often give point-particle descriptions of wavepulses. The ball-toss analogy gives an example of this reasoning in wave propagation. Similarly, in superposition, many students give the response that the wavepulses do not add until the points of maximum displacement overlap. They treat each wavepulse as a single point and ignore all other points. Other students describe the entire wavepulse as a single point, not ignoring the non-peak displaced points, but lumping all displaced points together into one.

Many students use a collision-like description of wave superposition to describe the interaction of wavepulses. Superposing wavepulses collide with each other and either bounce off each other or cancel out, depending on the situation. The remnant wavepulse possibly moves slower, having lost energy during the collision. Here we see a clear example of the way the pattern of association we use to organize student reasoning leads to descriptions different from a physical model of the physics. We have not found that students will <u>explicitly</u> state that the waves act like colliding carts, but we find that they often give descriptions consistent with the physics of cart collisions. Because of the similar explanations for the two situations, we believe students have an associative pattern which guides their understanding of wave interactions.

The lack of explicit evidence for incorrect student patterns of association should not keep us from looking for evidence of their existence. Two reasons exist for using the PM when trying to understand student difficulties with mechanical waves. First, by trying to organize student responses in terms of a model based on analogies, we account for the manner in which students approach our classes. By building a description that accounts for a majority of student responses, we may be able to make better sense of the classroom environment in which we teach. This may provide us with the opportunity to better diagnose what individual students are doing in our classroom, and help them overcome their difficulties. Second, by gaining insight into student use of patterns of association, we may learn an approach that could aid us in our attempts to understand student difficulties in other fields of physics.

Although we might expect student reasoning in the classroom to include many weakly organized components, we have found that student responses can be categorized with only two which are reasonably coherent. We can describe students as if they used elements of either the correct community consensus model of waves or a pattern of associations based on an over-application of ideas from Newtonian particle physics. Because we see only two reasonably coherent student descriptions of the physics, we have the unique opportunity to examine and evaluate the dynamic evolution of the mix. In chapter 7, I will discuss the evolution of student reasoning in the context of the modified curriculum developed by the Physics Education Research Group. This curriculum will be described in the following chapter.

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[1] See diSessa's comments in reference 5 in chapter 4 and Hammer's comments in references 2 and 7 in chapter 4 for a more detailed discussion.

[2] For example, in my graduate mechanics course, we spent a week or two discussing moments of inertia and rotations, but did not emphasize the subject greatly. In my advanced undergraduate mechanics course, only highly symmetric objects were considered and then only briefly.

[3] For more details on the use of pretests and the implementation of tutorials, see Chapter 7.

[4] I will refer to each student by the code name given and used during the interview. Code names were chosen by the student and correctly reflect the student's gender.

[5] The data supporting this statement can be found in chapter 6, when I discuss changes in instruction that had an effect on student performance on questions such as the one David answered.

[6] See reference 5 in chapter 4.

[7] Due to an error in videotaping, Ted's original interview videotape is unavailable. Ted's comments in this section are taken from extensive notes made during the interview. Though this allows us to gain insight into his reasoning, it prevents me from presenting lengthy excerpts of Ted's own words from the interview.

[8] Redish, E. F. "Implications of Cognitive Studies for Teaching Physics" Am. J. Phys. **62**, 796-803 (1994).