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GOLDEN OLDIE EDITORIAL

# Editorial note to: Georges Lemaître, The beginning of the world from the point of view of quantum theory

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The year 1931 can undoubtedly be called Georges Lemaître's *annus mirabilis*. Indeed, major contributions to relativistic cosmology by the Belgian physicist and priest appeared within a few months:

(a) *A homogeneous universe of constant mass and increasing radius accounting for the radial velocity of extra-galactic nebulae* [1] in the March 7 issue of the *Monthly Notices of the Royal Astronomical Society*, as an English translation of the article published four years earlier in French [2], in which Lemaître was the first to interpret the astronomical data about the galaxy redshifts by a positively curved space model in which the universe slowly expanded from an equilibrium Einstein state at $t = -\infty$,

(b) *The expanding universe* [3],[1] just following the previous one in the same *M.N.R.A.S.* issue, in which Lemaître calculated that the expansion of space could be induced by a preceding phase of "stagnation" taking place about $10^{10}$ years in the past,

[1] Not to be confused with *L'Univers en expansion*, reproduced as a Golden Oldie as *The expanding Universe* [4].

This Golden Oldie Editorial forms a unit with the Golden Oldie republication of a paper by G. Lemaître that can be found in this issue following the editorial note and online via doi:10.1007/s10714-011-1214-6.

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(c) The short note *The beginning of the world from the point of view of quantum theory*, published in the March 21 issue of *Nature* and reproduced here as a Golden Oldie,
(d) *Contribution to a discussion about "The question of the relation of the physical universe to life and mind"* [5], published in the October 24 issue of *Supplement to Nature*, in which Lemaître advocated an abrupt beginning of the universe from an initial, superdense concentration of nuclear matter called the "primeval atom",
(e) *L'expansion de l'espace* [6], a quantitative account of c) and d) published in French[2] in the November 20 issue of a Belgian scientific journal, where the author developed his major cosmological ideas about the primeval atom hypothesis in an extraordinary literary style,

and, since Lemaître was also fascinated by the brand new theory of quantum mechanics, one should not forget to mention

(f) *L'indétermination de la loi de Coulomb*[3] [8] in the August 8 issue of the *Annales de la Société Scientifique de Bruxelles*, in which he applied Heisenberg's uncertainty principle to the Coulomb law, and
(f) *Sur l'interprétation d'Eddington de l'équation de Dirac*[4] [9], in the same issue, in which he investigated the mathematical structure of quantum electrodynamics by using the formalism of quaternions.

In the middle of this string of pearls, the smallest (457 words) but the brightest contribution (c) can be considered to be the true "Charter" of the modern Big Bang theory.[5] To understand why, it must be explained and enlightened by a larger *corpus* of cosmological papers by Lemaître and other leading cosmologists of the time during the crucial years 1930–1932. That is the reason why the present editorial note, although devoted to a single-page article, will take unusual proportions. It can also be seen as a celebration of the 80th anniversary of Georges Lemaître's momentous ideas about the birth and evolution of our universe.

## 1 Recession of galaxies and expanding universe

Contrary to Friedmann (whose cosmological works were republished as a Golden Oldie, see [10]), who came to astronomy only in 1921–1922, that is to say three years before his premature death only, Lemaître was closely related to astronomy all his life. He always felt the absolute need for confronting the observational facts and the general relativity theory (adding considerations from quantum mechanics). He was, for example, much more aware than most of his contemporaries of the experimental status of relativity theory, and that as early as in his years of training [11]. Lemaître was

[2] An English translation was published later in [7].

[3] *Indeterminacy of Coulomb law.*

[4] *On Eddington's interpretation of Dirac's equation.*

[5] Due to its potential public impact and its non-technical character, the note was reprinted almost *in extenso* in the New York Times issue of May 19, 1931.

no less a remarkable mathematician, in the domain of fundamental mathematics (see his works on the quaternions or Störmer's problem) as well as in numerical analysis.

In short, the cosmological work of Lemaître was built in two phases. Initially, he found independently of Friedmann that the Einstein field equations of general relativity admitted non-static cosmological solutions. At the same time, he took into account the observations on the recession velocity of galaxies, to which he gave a physical meaning by interpreting them as an experimental proof of an expanding space. In a next phase, Lemaître dared an even more provocative assumption, which was however partly a logical prolongation of the theory of the expanding universe: if the universe is today expanding, in the past it was much smaller and denser; one remote day, it was thus condensed into a "primeval atom", whose successive fragmentations due to quantum processes made it such as it is now. Reviewed and improved during the following decades, Lemaître's primeval atom hypothesis has become the standard Big Bang model. Let us now follow in more details the evolution of Lemaître's cosmological insights.

In his 1927 article *Un univers homogène de masse constante et de rayon croissant, rendant compte de la vitesse radiale des nébuleuses extragalactiques* [2], Lemaître calculated the exact solutions of Einstein's equations by assuming a positively curved space (with elliptic topology), time varying matter density and pressure, and a non-zero cosmological constant. He obtained a model with perpetual accelerated expansion, in which he adjusted the value of the cosmological constant so that the radius of the hyperspherical space $R(t)$ constantly increased from the radius of Einstein's static hypersphere $R_E$ at $t = -\infty$. Therefore there was no past singularity and no "age problem". The great novelty was that Lemaître provided the first interpretation of cosmological redshifts in terms of space expansion, instead of a real motion of galaxies: space was constantly expanding and consequently increased the apparent separations between galaxies. This idea proved to be one of the most significant discoveries of the century.

Using the available astronomical data, Lemaître provided the explicit relation of proportionality between the apparent recession velocity and the distance: "Utilisant les 42 nébuleuses extra-galactiques figurant dans les listes de Hubble et de Strömberg[6], et tenant compte de la vitesse propre du Soleil, on trouve une distance moyenne de 0,95 millions de parsecs et une vitesse radiale de 600 km/s, soit 625 km/s à $10^6$ parsecs. Nous adopterons donc $R'/R = v/rc = 0,68 \times 10^{-27}\,\mathrm{cm}^{-1}$ (Eq. 24)". Eq. 24 is exactly what would be called later the Hubble law.

The significance of Lemaître's work remained unnoticed. Eddington, his former PhD mentor to whom Lemaître had sent a copy, did not react. When Lemaître met Einstein for the first time at the 1927 Solvay Conference, the famous physicist made favorable technical remarks, but concluded by saying that "from the physical point of view, that appeared completely abominable" [13]. In 1929, Hubble [14] published new experimental data on the spectral redshifts of extra-galactic nebulae, suggesting

[6] Lemaître combined the redshifts published by Strömberg—who relied himself on redshifts published earlier by Slipher—and Hubble' distances via magnitudes; in his book *The Mathematical Theory of Relativity*, Eddington had also published a redshift table, quoting data from Slipher who prepared that table for him; see [12] for all references.

the linear velocity-distance relation $v = Hr$ with $H = 600\,\mathrm{km/s/Mpc}$. This law was strictly identical to Lemaître's Eq.24, with the same proportionality factor, but Hubble did not make the link with expanding universe models. In fact Hubble never read Lemaître's paper; he interpreted the galaxy redshifts as a pure Doppler effect (due to a proper radial velocity of galaxies), instead of as an effect of space expansion.

A new opportunity for the recognition of Lemaître's model arose early in 1930. In January, in London, a discussion between Eddington and De Sitter took place at a meeting of the Royal Astronomical Society. They did not know how to interpret the data on the recession velocities of galaxies. Eddington suggested that the problem could be due to the fact that only static models of the universe were hitherto considered, and called for new searches in order to explain the recession velocities in terms of dynamical space models. Having read a report of the meeting of London [15], Lemaître understood that Eddington and De Sitter posed a problem which he had solved three years earlier. He thus wrote to Eddington to point out his communication of 1927 and requested him to transmit a copy to de Sitter. This time, Eddington, who had not read the paper at the right time, made apologies and reacted. He sent the note to de Sitter, who answered very favorably in a letter to Lemaître, dated March 25, 1930.[7]

On his side, Eddington reorganized his communication to the following meeting of the Royal Astronomical Society in May, to introduce Lemaître's ideas on dynamical universes [17]. Then he published an important article [18] in which he reexamined the Einstein static model and discovered that, like a pen balanced on its point, it was unstable: any slight disturbance in the equilibrium would start the increase of the radius of the hypersphere; then he adopted Lemaître's model of the expanding universe—which will be henceforward referred to as the Eddington–Lemaître model—and calculated that the original size of the Einstein universe was about 1200 million light years, of the same order of magnitude as that estimated by Lemaître. Interestingly enough, Eddington also considered the possibility of an initial universe with a mass $M$ greater or smaller than the mass $M_E$ of the Einstein model, but he rejected the two solutions, arguing that, for $M > M_E$, "it seems to require a sudden and peculiar beginning of things", whereas for $M < M_E$, "the date of the beginning of the universe is uncomfortably recent".

Next, Eddington carried out an English translation of the 1927 Lemaître article for publication in the *Monthly Notices of the Royal Astronomical Society* [1]. Here took place a curious episode: for an unexplained reason, Eddington replaced the important paragraph quoted above (where Lemaître gave the relation of proportionality between the recession velocity and the distance) by a single sentence: "From a discussion of available data, we adopt $R'/R = 0,68 \times 10^{-27}\,\mathrm{cm}^{-1}$ (Eq. 24)". Thus, due to Eddington's (deliberate?) blunder, Lemaître will never be recognized on the same footing as Edwin Hubble for being the discoverer of the expansion of the universe.

Just following his translated article in the issue of *M.N.R.A.S.*, Lemaître published a technical paper entitled *The expanding universe* [3], also communicated by Eddington, in which he studied the mechanism of the initial expansion (see [19] for a detailed

[7] Reproduced in [16], pp. 104–105.

analysis). By dividing the Einstein universe into cells in which matter condensed toward the centre of the cell, Lemaître calculated that a diminution of the pressure on the edge of the cells would induce a global expansion of space. He interpreted such a diminution of the pressure as a diminution of the exchange of kinetic energy between distant parts of the universe; in other words, the kinetic energy would remain stagnant near the centre of the cells. Thus he introduced the phenomenon of "stagnation" as the cause of the expansion of the universe: "If, in a universe of equilibrium, the pressure begins to vary, the radius of the universe varies in the opposite sense. Therefore, stagnation processes induce expansion". Another original idea of this article was to generalize the Birkhoff theorem of general relativity to describe the stagnation phenomenon within the framework of a homogeneously expanding universe. Lemaître visualized the initial Einstein universe as something like the dilute primeval gas appearing in the Kant–Laplace nebular hypothesis. At the end of his paper, he considered the effect of a sudden stagnation process in which the pressure dropped instantaneously to zero, and found that the epoch of the rupture of equilibrium would have taken place some $10^{10}$ to $10^{11}$ years ago, depending on the ratio between the pressure and the density.

## 2 Quantum birth of the universe

Thus, at the beginning of 1931, the expansion of space appeared to be the only coherent explanation to account for the astronomical observations. But the same year when his vision of a dynamic universe was to be accepted by the scientific community, including Eddington, de Sitter and Einstein, Lemaître dared to make a much more outrageous assumption: if the universe is expanding now, must it not have been much smaller and denser at some time in the past? Instead of considering the static Einstein world as an initial stage from which the dynamic model started, is it not more logical to think the universe as starting its expansion from an extremely small and condensed state, governed by quantum processes?

One of the reasons of this momentous idea was that, like many other physicists, Lemaître was impressed by the new theory of quantum mechanics. Another reason was to reply firmly to a communication delivered by Arthur Eddington at the British Mathematical Association on January 5th, 1931 and published in the March 9 issue of *Nature* [20]. The British astrophysicist initially paid tribute to Lemaître while declaring "We recently learned, mainly thanks to the work of Prof. Lemaître, that spherical space is expanding somewhat fast". Dealing with the role of entropy as an arrow of time, he considered that, following time backwards, one would find more and more organization in the world, up to a state of minimum entropy. But, for philosophical reasons, Eddington refused to go back further in time up to the concept of singularity, otherwise "we have come to an abrupt end of space-time—only we generally call it the 'beginning' ". For him like for most others, this question laid outside the range of science, and he added that "philosophically, the notion of a beginning of the present order of Nature is repugnant to me".

In the Golden Oldie reproduced here, Lemaître argued that the world had come into existence a finite time ago in an explosive event, which he likened to a giant radioactive flash. Just like Eddington, he supposed that time and its arrow are connected

to the growth of the entropy. In the direction of increasing time, the universe evolves to a state of infinite entropy, i.e. of complete disorganization. In the direction of the past, the universe would have proceeded from a state of zero entropy. Eddington had wondered whether the moment of zero entropy could mark the beginning of the world, a concept that he had personal reasons to discard. Lemaître disagreed and pointed out that entropy is a measurement of proper time, and not of the time-coordinate; consequently, Eddington was wrong to believe that the moment of minimal entropy separated "before-creation" from "after-creation" on an axis of universal time. It should be seen, on the contrary, like an essential singularity, where the concepts of space and time even lose their meaning. In order that spacetime can exist within the framework of general relativity, one needs a tensor of matter-energy, due to the identification of geometry and matter. The state of matter with zero entropy constitutes a singularity of the matter-energy tensor in the right-hand side of the field equations, which is equivalent to a singularity in the curvature tensor in the left-hand side. There was no time nor space prior to the state of condensation at zero entropy. It was the initial singularity which created the space-time. Thus, the plurality and the diversity of the physical world appeared to come from "something" physical, coinciding with the $R = 0$ singularity of relativistic cosmological models. The atom-universe exploded and plurality emerged. The entropy became nonzero, time and its arrow also appeared.

The radical innovation introduced by Lemaître thus consisted in linking the structure of the universe at large scales with the intimate nature of the atoms, in other words in relating the early universe to quantum mechanics.[8] Lemaître used the term "single quantum" and took care to stress that at this stage, the laws of physics such as we know had no meaning anymore because the concepts of space and time were not defined. It is the frontier of science such as Lemaître conceived it, and in the present-day quantum cosmology nothing clearly indicates that this frontier of physical knowledge, called the Planck era, can be crossed.

Let us analyze in more detail Lemaître's argumentation. He begins by stating that the number of distinct quanta is ever increasing in the course of time. He will explain better this assertion in the semi-popular paper published later in the same year [6] (see also below). Let two volumes $V_1$ and $V_2$ contain heat radiation at temperatures $T_1$ and $T_2$, and let $T$ be the equilibrium temperature of the total volume $V_1 + V_2$. From the law of energy conservation it follows that $V_1 {T_1}^4 + V_2 {T_2}^4 = (V_1 + V_2)\, T^4$, and the number of photons will increase proportionally to $(V_1 + V_2)\, T^3 - V_1 {T_1}^3 - V_2 {T_2}^3$, which is always positive. The demonstration is valid only for a gas of photons and not for material particles, but Lemaître generalizes it by assuming in an intuitive way that the number of particles sharing a given amount of energy is constantly increasing.

Next, Lemaître continues his short text by following closely an argument by Niels Bohr published a few months before [22], according to which the concepts of space and time in quantum mechanics have only statistical validity. As a consequence, when the number of quanta was reduced to a single one, as assumed to be the case at the beginning of the world, the notions of space and time failed. They got meaning only

[8] He was not the first to suggest a connection between cosmology and quantum theory. As early as 1925, Cornelius Lanczos introduced quantum mechanics in a cosmological model, concluding that 'The solutions of the quantum secrets are hidden in the spatial and temporal closedness of the world' [21].

when the original quantum began to disintegrate. Therefore, the beginning of the world (namely the single quantum) happened "a little before" the beginning of space and time. The phrasing is equivocal, since "a little before" seems to imply a temporal sense, which would be contradictory with the idea that time did not yet exist. Lemaître wanted to say that space and time emerged from the original quantum in a logical sense.

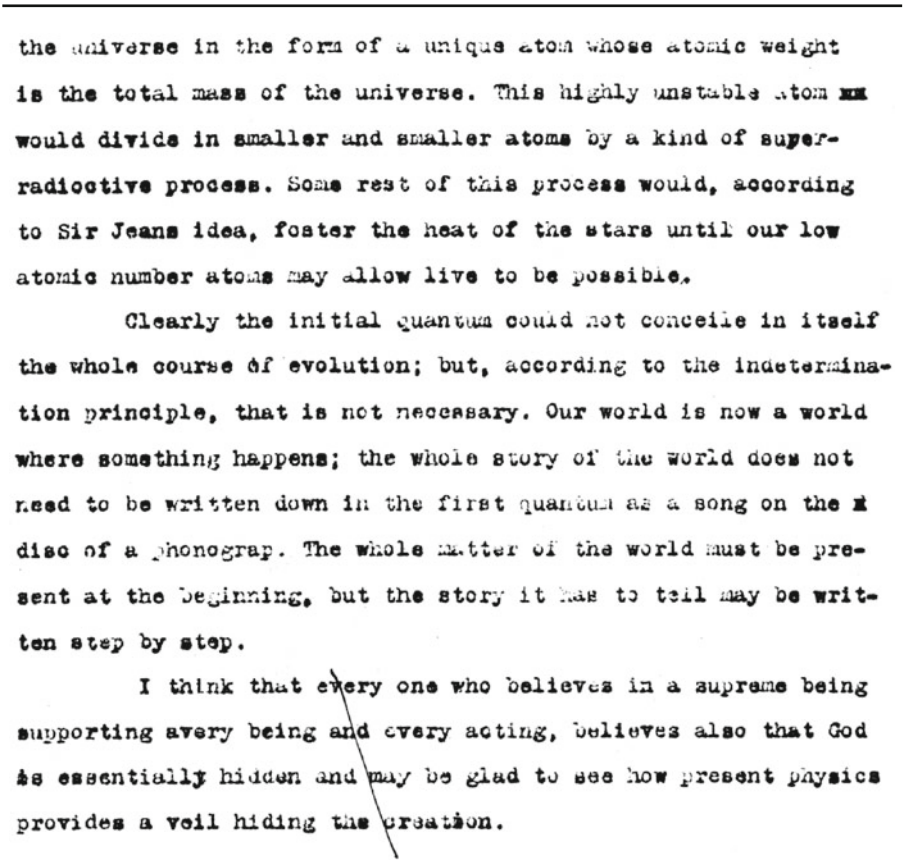
Now, what did he consider the original quantum to be made of? Lemaître suggested that it might be a huge atomic nucleus, with an extremely large atomic number corresponding to the total mass of the universe, and acting like a quantum number. In 1931, nuclear physics was still in its infancy and the neutron had not yet been discovered; but Lemaître knew about radioactive processes, and he hypothesized that a huge atom would be unstable and explosively decay into a large number of quanta. As he explained later, the word "atom" had to be taken in the Greek sense, as something completely undifferentiated and deprived of physical properties.

In the final paragraph, Lemaître appealed to Heisenberg's uncertainty principle to express the idea that the whole course of cosmic evolution was not written down in the first quantum.

As underlined in [19], Lemaître's note was not an ordinary scientific communication, but rather "a visionary piece of cosmo-poetry that was meant to open the eyes of the readers rather than convince them". He wanted to make his own view concerning the beginning of the world publicly known and understood by everyone, thus he did not introduce any equation. In addition, he chose to sign his communication as a private person, namely "G. Lemaître, 40 rue Namur, Louvain", and not as a distinguished physicist and cosmologist, professor at the University of Louvain.

It is time to recall that Lemaître was also a Catholic priest, and since the creation of the universe a finite time ago is a dogma in Christian thought, it might be tempting to jump to the conclusion that the explosive universe was motivated by the aim to reconcile relativistic cosmology with religious belief. It is interesting to point out that the manuscript (typed) version of Lemaître's article, preserved in the Archives Lemaître at the Université of Louvain, ended with a sentence crossed out by Lemaître himself and which, therefore, was never published. Lemaître initially intended to conclude his letter to *Nature* by "I think that every one who believes in a supreme being supporting every being and every acting, believes also that God is essentially hidden and may be glad to see how present physics provides a veil hiding the creation" (see Fig. 1). This well reflected his deep theological view of a hidden God, not to be found as the Creator in the beginning of the universe. But before sending his paper to *Nature*, Lemaître probably realized that such a reference to God could mislead the readers and make them think that his hypothesis gave support to the Christian notion of God.

As well analyzed in [23], Lemaître will preserve all his life the conception of a supreme and inaccessible God, enabling him to keep the natural origin of the world within the strict limits of physics, without mixing it with a supernatural creation. As a priest just like a scholar in theology, Lemaître was very conscious of the potential conflict—or, on the contrary, of the concordance—between the Christian dogma of a world created by God and the scientific theory of a universe formed approximately ten billion years ago. However, Lemaître never confused science and religion. Contrary to some other Christian cosmologists, he took care not to use one of these two "ways


the universe in the form of a unique atom whose atomic weight is the total mass of the universe. This highly unstable atom would divide in smaller and smaller atoms by a kind of super-radioctive process. Some rest of this process would, according to Sir Jeans idea, foster the heat of the stars until our low atomic number atoms may allow live to be possible.

Clearly the initial quantum could not conceile in itself the whole course of evolution; but, according to the indetermination principle, that is not necessary. Our world is now a world where something happens; the whole story of the world does not need to be written down in the first quantum as a song on the disc of a phonograp. The whole matter of the world must be present at the beginning, but the story it has to tell may be written step by step.

I think that every one who believes in a supreme being supporting avery being and every acting, believes also that God is essentially hidden and may be glad to see how present physics provides a veil hiding the creation.

**Fig. 1** A copy of the original Lemaître typescript, with the last paragraph crossed by the author

of knowledge" as a legitimisation of the other. He took, for example, great care to distinguish between the "beginning" and the "creation" of the world, and never spoke about the initial state of the universe in terms of "creation" (contrary to Friedmann, a fervent orthodox Christian, who eventually appears more "concordist" than the Belgian priest). Lemaître was convinced that science and theology dealt with two separate worlds.

## 3 The primeval atom

Lemaître had to convince himself that his model of an explosive universe with finite age was physically realistic. He thus prepared a quantitative article to be published in the fall of 1931. In the meantime, he accepted the invitation of the British Association for Science to take part in its centenary meeting, to be held in London on September 29, including a session on cosmology devoted to "The Question of the Relation of the Physical Universe to Life and Mind". Jeans, Eddington, Milne, de Sitter and Millikan made also scientific contributions [24]. Among other questions, they had to deal with the problem of the age of the universe, which, when deduced from the Hubble constant known at the time, gave a value about 1.8 billion years,

conspicuously smaller than the time required for stellar evolution, as emphasised by Jeans and others.

Lemaître [5] argued (without any explicit calculation) that the problem could be solved by making use of the stagnation process he had previously introduced in the context of the Eddington–Lemaître model [3]. But he went much further by pushing forward his suggestion of an abrupt beginning of the universe. As he said, "a complete revision of our cosmological hypothesis is necessary, the primary condition being the test of rapidity. We want a 'fireworks' theory of evolution [...] It is quite possible to have a variation of the radius of the universe going on, expanding from zero to the actual value."

The singular creation of the universe had been briefly hypothesized by Friedmann, but completely ignored by the scientific community. Lemaître refined the argument and introduced for the first time (as far as we know) the expression of *primeval atom*: "I would picture the evolution as follows: at the origin, all the mass of the universe would exist in the form of a unique atom; the radius of the universe, although not strictly zero, being relatively small. The whole universe would be produced by the disintegration of this primeval atom. It can be shown that the radius of space must increase. [...] Whether this is wild imagination or physical hypotheses cannot be said at present, but we may hope that the question will not wait too long to be solved."

Lemaître also suggested that the cosmic rays, which had been recently discovered, were the fossils of the original explosion, as "ashes and smoke of bright but very rapid fireworks [...] We are led to the conclusion that the stars were born some ten thousand million years ago without atmospheres, and that the cosmic rays are outstanding features of the formation of a star". The origin of the cosmic rays was thought to be important evidence for the primeval atom cosmology, but no trace of the idea has been found in Lemaître's writings before this fall of 1931.

Eventually, the model of the primeval atom was quantitatively developed in *L'expansion de l'espace*, published in French in November 1931 [6]. Lemaître assumed a positively curved space (with elliptic topology), time-varying matter density and pressure, and a cosmological constant such that, starting from a singularity, the Universe first expanded, then passed through a phase of "stagnation" during which its radius coasted that of the Einstein's static solution, then started again in accelerated expansion.

The style of Lemaître contrasts drastically with that of Friedmann [10], in the scientific argumentation as well as in the form. In the argumentation, the approach of Friedmann was as axiomatic, as that of Lemaître—himself a remarkable mathematician—was physical. As for the form, very literary (adapted to that of the public conferences that Lemaître frequently gave), it is a model of mixed rigor and lyricism, readable by almost everyone and which testifies to the years of studies of Lemaître in graeco-latin humanities. We reproduce below broad extracts of this extraordinary text in its English translation [7], where the technical developments are omitted.

> Following the Laplace and Kant cosmogonies, we became accustomed to taking a very diffuse nebula as the starting point for evolution, a nebula filling all space and becoming more and more condensed by splitting into partial nebulae and finally into stars.
>
> This very old idea has been adapted to the recent progress of astronomy. It has been recently expanded in that fine book [*The Universe Around Us*] which Sir James Jeans dedicated to the

study of the universe. We are now in a position to estimate the density of the primeval nebula by evaluating the masses and the distances of less large condensations of stars called the extra-galactic nebulae, which enclose all that we know of the universe. If the actual mass of the stars was supposed to be distributed uniformly throughout the whole space that they occupy, one would find that the primeval nebula must have been more rarefied than the highest vacuum which our physicists can hope to achieve in the laboratories. The density of the universe reduces to $10^{-30}$ gram per cubic centimeter, a figure which is generally considered to be reliable within a factor of one hundred.

The idea of a primeval nebula has to meet a very serious difficulty which can be removed only with the help of the theory of relativity and of non-Euclidian geometries: the different parts of the nebula are pulled together by gravity, and it seems as though they should have to collapse toward their center of gravity. A first element of solution is brought about by the possibility that real space was not Euclidian but should obey the laws of Riemann's elliptic geometry. Then there is no longer a center of gravity.

[...]

All points of the nebula remain uniformly distributed in space; the distance between any two of them is always the same fraction of the total length of the closest straight line on which they lie; but this length, equal to $\pi R$, varies with the radius $R$; every distance varies in the same ratio as the variation of the radius of space.

To study in detail the variation of the radius of space, it is necessary to appeal for the equations of general relativity. It is possible, however, to illustrate the result of relativistic computations by elementary considerations involving the laws of classical mechanics. This is possible because laws of relativity are reduced to a limit to the laws of Newton, when they are applied to an infinitely small volume.

[...]

These equations account for the dynamics of the universe; they accustom us to thinking of the radius of the universe as a physical quantity, able to vary. The manner in which these equations have been obtained must not be regarded as a rigorous demonstration. A demonstration which is not open to criticism can be deduced only from the general equations of relativity. Nevertheless, the elementary considerations evolved hitherto may allow us, in some degree, to grasp the physical significance of results involving more abstract methods. Now we must explain what change must be made in these equations, in order to account for the equilibrium of the Laplace nebula, and to show how this change can be justified.

*The cosmological constant*

One of the most important achievements of the theory of relativity is the identification of the idea of mass with that of energy. Energy is essentially a quantity which is defined, except for an additive constant; mass, on the contrary, insofar as it affects the law of universal gravity, does not involve any arbitrary constant.

The identification of mass and energy, therefore, admits of a choice of the constant of energy, or, inversely, of the introduction of an arbitrary constant to the expression of the gravitational mass. The theory of relativity teaches us the manner in which this arbitrary constant must be introduced. The equations of gravity are obtained by integration of equations which express both the conservation of energy and momentum. This integration naturally introduces a constant of integration. But this constant of integration is not added to the energy or to the total mass; it is added to the density. In other words, the necessary adjustment between energy and gravitational mass is made, not on the total mass, but on the density. This arbitrary constant, which is introduced in the equations, has been called the "cosmological constant", because it has no importance except in problems involving the whole universe.

[...]

The interpretation of the cosmological term is straightforward. It means that an elastic force, which tends to increase the radius, is superimposed on the Newtonian force, which tends to diminish it. A value of the radius exists, called the equilibrium radius, for which these two forces neutralize one another. The nebula of Laplace will last, provided that the value of the radius be suitably adjusted to the value of the total mass of the nebula.

Thus we have succeeded in making the Laplace nebula maintain equilibrium. Let us not rejoice too soon, because we shall have to realize that this equilibrium is quite precarious.

[...]

We can therefore conclude that the formation of local condensations in the Laplace nebula in equilibrium must have upset this equilibrium and initiated the universe.

*Expansion of the universe*

The hypothesis of Laplace has, therefore, as its consequence, the expansion of space. Does this expansion take place, and with what speed it is produced?

In a space with increasing radius, the material points, the great extra-galactic nebulae, for example, remain uniformly distributed in space. Nevertheless, their mutual distances increase, all in the same ratio. Thus, if we observe the extra-galactic nebulae, we shall be able to state that their distances increase while remaining proportional to one another and therefore that all extra-galactic nebulae have velocities of recession proportional to their distance. The velocities of stars or of nebulae are observed through the displacement of their spectral lines, known by the name of the Doppler–Fizeau effect. The spectrum of distant nebulae shows displacement toward the red, corresponding to velocities of recession up to 10,000 km per second; and insofar as it is possible to judge their distances, these velocities are quite proportional to this distance. Up to now, we possess about fifty measurements of velocity,[9] and, as a consequence of all these measurements, we can estimate that a nebula, located at a distance of one hundred million light-years (a distance at which it is still possible to photograph the nebula) has a velocity of recession equal to one-twentieth of the speed of light, that is, about 15,000 km per second.

This result permits us to estimate the size which the nebula of Laplace would have had originally, and it determines the initial radius of equilibrium of space at about one billion light-years. The present value of the radius depends on the estimate of the density of matter. In utilizing the value which we indicated at the beginning of this section, we find that it is equal to a dozen times the initial radius.

The present state of the expansion enables us to get some idea, not only of the primeval nebula, but also of the epoch in which the local condensations were formed while initiating the expansion of space.

[...]

One finds that, if the world began as a Laplace nebula in equilibrium, the first general condensation of any importance which took place in it, and which therefore initiated the expansion of space, could not have occurred at an epoch dating back more than one hundred billion years.

*The time-scale*

To realize the importance of this result, one must not forget that the cosmogony of Laplace-Jeans is a slow cosmogony. The primeval, gaseous masses are condensed as a result of small inequalities in their initial distribution and form the first condensations: the extra-galactic nebulae. As we have just seen, this event dates back only hundred billion years, at a maximum. These nebulae were still gaseous at that time. Weak condensations then formed, by chance, and, as Jeans has shown, they must tend to increase provided that they be of sufficient dimension, comparable to the mutual distances of the stars. But how much time is necessary for these vast condensations to have the opportunity to be formed and to be able to be concentrated in a sphere whose diameter is a hundred thousand times smaller than their initial diameter?

Jeans asks one hundred thousand billions years for this evolution and I am not sure that he has proved whether this is enough; we can only give him one- thousandth of this time.

One hundred billion years is, at the most, fifty times the age attributed to the earth. It is one hundred times the amount of time necessary for the lunar tides to brake the rotation of our satellite and force it to turn the same face constantly toward the Earth. It is only a thousand times the amount of time which it takes light to come to us from nebula which have been photographed by our telescopes. Did evolution really take place according to Laplace's theory, starting with extreme diffuseness and reaching the present state of matter: stellar condensations dispersed in a virtual

[9] Editor's footnote: In his 1929 article, Hubble displayed the data for 46 radial velocities; four of them were negative,—for the Andromeda galaxy M31, its two satellites M32 and NGC 205, and for the Triangulum galaxy M 33 –, all the other were positive, from which he deduced the velocity-distance relation of proportionality.

vacuum? Light would not require one minute to cross the Sun, and it would need four years to reach the nearest star. The stellar world, like the atomic world, seems to be extraordinarily empty.

A really complete cosmogony should explain atoms as suns, and certainly atoms cannot have extreme diffuseness as their origin.

*Radioactivity*

In the atomic field, we know about a spontaneous transformation which can give us some idea of the direction of natural evolution; it is the transformation of radioactive bodies. Disregarding photons and electrons whose mass is nil or very small, an atom of uranium is ultimately transformed into an atom of lead and eight atoms of helium. This is a transformation from a state of greater condensation to one of lesser condensation. On the average, uranium can remain extant only four or five billion years before making its transformation. Thorium behaves in an analogous manner.

If we had appeared on Earth one hundred billion years later, there would have been no appreciable amounts of radioactive substances, and we would doubtless have ended our table of elements at bismuth and lead. Does the table of elements really end with uranium?[10] Have we not come too late to know heavier elements which were almost completely disintegrated before our birth? Are not radioactive transformations a faint residue of the original evolution of the world and did they not take place, on the stellar scale, several billion years ago?

[...]

Our universe bears the marks of youth[11] and we can hope to reconstruct its story. The documents at our disposal are not buried in the piles of bricks carved by the Babylonians; our library does not risk being destroyed by fire; it is in space, admirably empty, where light waves are preserved better than sound is conserved on the wax of phonograph discs. The telescope is an instrument which looks far into space, but it is, above all, an instrument which looks far into the past. The light of nebulae tells us the history of hundred million years ago, and all the events in the evolution of the world are at our disposal, written on fast waves in internebular ether.

[...]

*The primeval atom*

The world has proceeded from the condensed to the diffuse. The increase of entropy which characterizes the direction of evolution is the progressive fragmentation of the energy which existed at the origin in a single unit. The atom-world was broken into fragments, each fragment into still smaller pieces. To simplify the matter, supposing that this fragmentation occurred in equal pieces, two hundred sixty generations would have been needed to reach the present pulverization of matter in our poor little atoms, almost too small to be broken again.

The evolution of the world can be compared to a display of fireworks that has just ended: some few red wisps, ashes and smoke. Standing on a well-chilled cinder, we see the slow fading of the suns, and we try to recall the vanishing brilliance of the origin of the worlds.

The sun-atom splinters into fragments held together by universal attraction, fragments which splinter in their turn, hurling into the vacuum particles which are fast enough to escape the attraction of the entirety, sparks escaping from the burning crucible where the atom became a star. Rays travel in a straight line in the still-increasing desert of space, until they encounter a lost oasis, our galaxy, a chilled seed, our Earth, and discharge an electrometer, proving the formation of the suns.

Primeval nebula or primeval atom? Slow cosmogony or fast cosmogony? Gaseous cosmogony or radioactive cosmogony? How far must the old ideas be preserved? Was the Earth ejected in the atomic state by the sun-atom, or was it separated from it in the gaseous phase? What are the properties of giant atoms and the laws which govern their disintegration? It would be premature to try to answer these questions.

In concluding, we must indicate the manner in which the theory of the expansion of the universe is adapted to the idea of the primeval atom. We can conceive of space beginning with the primeval atom and the beginning of space being marked by the beginning of time. The radius of space began

[10] Editor's footnote: As we know, the answer is yes concerning the natural elements. The first element beyond uranium, the neptunium—atomic weight 93—was synthesised in 1940.

[11] Editor's footnote: A first reference to a "young" universe is found in the famous poem of Lucretius *De Natura Rerum* (first century BC), the kind of classical latin literature that Lemaître had read.

at zero; the first stages of the expansion consisted of a rapid expansion determined by the mass of the initial atom, almost equal to the present mass of the universe. If this mass is sufficient, and the estimates which we can make indicate that it is indeed so, the initial expansion was able to permit the radius to exceed the value of the equilibrium radius. The expansion thus took place in three phases; a first period of rapid expansion in which the atom-universe was broken into atom-stars, a period of slowing-up, followed by a third period of accelerated expansion.[12] It is doubtless in the third period that we find ourselves today, and the acceleration of space which has followed the period of slow expansion could well be responsible for the separation of stars into extra-galactic nebulae.

It is not completely proven that we are not in the first period of expansion, and in this case, that the present expansion might not be capable of making us exceed the equilibrium radius, which would therefore be quite large. After having continued their movement of expansion for several billion years, the nebulae would stop, then fall back toward one another, and finally collide with one another, putting an end to the history of the world, with final fireworks, after which the radius of space would again be reduced to zero.

This hypothesis was proposed by Friedmann in 1922, and revived recently by Einstein. Against it, there are the present estimates of density, but these are not quite certain. Moreover, we can reassure ourselves by stating that space is still extending and that, even if the world must finish in this manner, we are living in a period that is closer to the beginning than to the end of the world.

But it is quite possible that the expansion has already passed the equilibrium radius, and will not be followed by a contraction. In this case, we need not expect anything sensational; the suns will become colder, the nebulae will recede, the cinders and smoke of the original fireworks will cool off and disperse.

As can be seen, both the style and the scientific contents were of an amazing richness. Lemaître built his model from experimental data: the observation of the redshifts of remote nebulae resulted from the expansion of space, but the existence even of these nebulae imposed that, in its past, the universe underwent local processes of condensation which gave them birth. For Lemaître, the expansion of space and the condensation of matter were the demonstrations of imbalances between two opposite cosmic forces: the gravity, attractive, and the cosmological constant, repulsive. In addition, the observational results constrained the evolution of the world to a short duration and implied a fast cosmogony. According to Hubble measurements indeed, the expansion rate was equal to 540 km/s/Mpc. With such a fast growth rate and without a cosmological constant, the present universe should have some 2 billion years of existence. However it was already known, by the study of radioactive elements, that the age of the Earth was at least 4 billion years. Obviously the Earth could not be older than the universe. Lemaître thus needed the cosmological constant both to get an age of the universe compatible with that of the Earth, and to leave enough time for galactic condensations to be formed.

Lemaître's model (cf. Fig. 2) divided the evolution of the universe into three distinct phases: two fast expansion phases separated by a period of deceleration. The first phase was an expansion of explosive type, resulting from radioactive decay of an atom-universe. The initial expansion was determined by the mass of the primeval atom, "almost equal to the present mass of the universe". The "almost" presumably referred to his early picture of the primeval atom as a huge condensation of nuclei. It was known from nuclear physics that an atomic nucleus is lighter than the sum of its constituent particles by an amount known as the mass defect. Likewise, the primeval atom would be somewhat lighter than the galaxies resulting from its explosion. For this

[12] Editor's footnote: This is very close to the time evolution of the so-called "standard" Big Bang model of 2011!

phase, Lemaître used the image of a "fireworks" which, if poetic, is not less pedagogically debatable: it caused constant misunderstanding—repeated by popular accounts—presenting the beginning of the universe like an explosion of matter localized in outer space.

The second phase of Lemaître's model corresponded to a quasi-equilibrium between the density of matter and the cosmological constant, resulting in a practically constant radius during a period of stagnation; the attractive effects of gravitation being dominating at small scales, it was during this phase that the density fluctuations were formed, which condensed later on to give rise to the large scale structures of the universe, with stars grouped into galaxies and galaxies into clusters. The formation of local condensations disturbed the equilibrium conditions, which made the cosmological constant predominant and started again the process of expansion. According to Lemaître, the universe was presently in the third stage.

Technically, the solution was obtained starting from the relativistic equations by supposing space with positive curvature and a cosmological constant $\lambda$ slightly higher than the Einsteinian value $\lambda_E = 1/{R_E}^2 = 2GM/\pi c^2 {R_E}^3$, where $R_E$ was the equilibrium radius of the 1917 Einstein universe model. As to the age of the universe, Lemaître mentioned as a possible value ten billion years, but it could be considerably higher as it depended on the value of the cosmological constant. The duration of the stagnation phase depended essentially on $\lambda = \lambda_E(1+\varepsilon)$, being arbitrarily large when $\varepsilon$ tended to zero. For this reason, Lemaître's model was sometimes called "hesitating universe".

The reasoning of Lemaître was based on the will to use the new knowledge of atomic physics and to link nebulae to the atoms, as he wrote it. Compared to his model of 1927, which had a slow evolution, Lemaître proposed from now a fast cosmology with an explosive origin, which, starting from the simplest, generated the complex.

The Belgian physicist ended his paper with a brief discussion of the eschatological aspects of his world model, arguing that in the far future, the universe would inescapably end in heat death. Of course, all life would irreversibly disappear...

## 4 Conclusion

As a result of Lemaître's choice to publish his primeval-atom model in French and in a semi-popular journal unknown to most physicists and astronomers, it took some time until his article was noticed. When it became known, it was poorly received by the majority. The fact that Lemaître was a mathematician more than an astronomer, allied to his religious convictions, no doubt added to a natural resistance towards cosmological revolutionary ideas.[13] Eddington never accepted the primeval-atom hypothesis or other ideas of the universe having an abrupt beginning a few billion years ago. Like most other scientists, he felt uneasy about a created universe, and this attitude was

[13] One can also wonder whether the literary quality of his work did not harm the credibility of its scientific contents, in a community little accustomed to such a flowery way of writing science. Still today, many scientists quickly and pejoratively brand as "popular" a text raising the quality of the form to the same level as that of the contents.

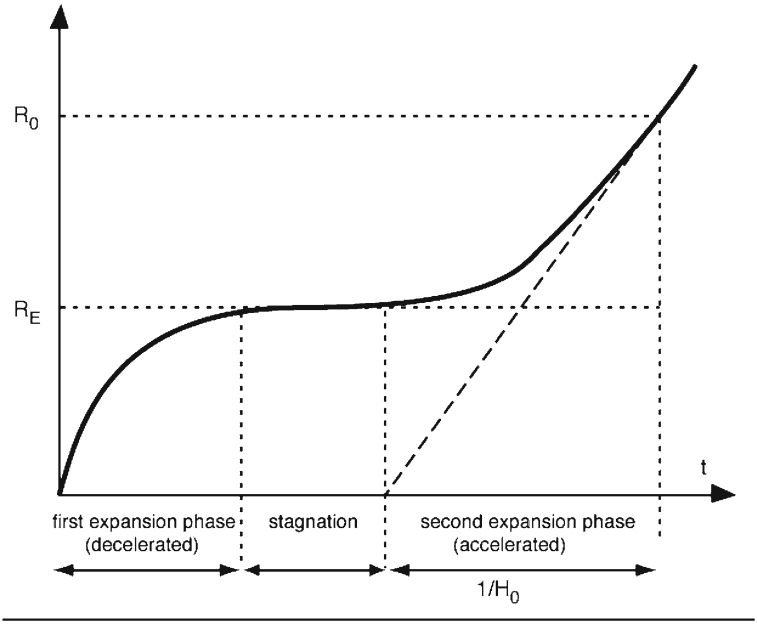



**Fig. 2** Lemaître's "hesitating universe". The tangent to the expansion curve measured today (oblique in *dotted lines*), i.e. the current expansion rate, gives the Hubble time, often considered as an estimator of the age of the universe. One sees clearly how the introduction of a cosmological constant and a stagnation phase invalidates this estimate

shared by the large majority of physicists and astronomers in the 1930s. As we have mentioned above, this was an unfair prejudice, because for Lemaître, as he expressed it several times, the physical beginning of the world was quite different from the metaphysical notion of creation, and for the priest-physicist, science and religion corresponded to separate levels of understanding.

Therefore, even if Lemaître's hypothesis was mentioned in cosmological reviews of the time by Tolman, de Sitter, Robertson and some others, it was not assigned a physical reality. There were however a few exceptions. For instance, the Harvard astronomer Donald Menzel wrote a quite enthusiastic article in a popular science journal, beginning with "Out of a single, bursting atom came all the suns and planets of our universe! That is the sensational theory advanced by the famous Abbé G. Lemaître, Belgian mathematician. It has aroused the interest of astronomers throughout the world because, startling as the hypothesis is, it explains many observed and puzzling facts." [25]. Also, the quantum physicist Pascual Jordan supported Lemaître's model in a book of 1936 [26]. The attitude of Einstein was less clear. As early as 1931 and probably unaware of Lemaître's hypothesis, he derived from the Friedmann equations a cyclic cosmological model in which the universe started expanding from $R = 0$ and contracted into a "big crunch" [27], but he considered the appearance of the singularity $R = 0$ to have no physical significance. Later on, when he learnt about the primeval atom hypothesis, he considered it as inspired by the Christian dogma of creation and totally unjustified from the physical point of view. Einstein had also a great prejudice

against the cosmological constant he had originally introduced in his static model of 1917, and that he considered as the "greatest blunder" of his life. It is probably the reason why, in the new relativistic model he proposed in 1932 with de Sitter [28]—a space with zero curvature and uniform density that expanded eternally—, the term disappeared. Their model too belonged to the class of universes with a singular beginning, so far that $R = 0$ for $t = 0$, but the authors did not mention this feature, and did not even refer to either Friedmann or Lemaître. After that, Einstein gave up research in cosmology.

Unfortunately, due to Einstein's authority, this over-simplified solution became the "standard model" of cosmology for the next 60 years. However Lemaître kept his original views. In 1933 he published another fundamental article about cosmology, galaxy formation, gravitational collapse and singularities in the *Annales de la Société Scientifique de Bruxelles*, translated to English and reproduced as a *Golden Oldie* more than ten years ago [4]. In that paper of 1933, Lemaître found a new solution of Einstein's equations, known as the "Lemaître–Tolman" or "Lemaître–Tolman–Bondi" model, which is more and more frequently used today for considering structure formation and evolution in the real Universe within the exact (i.e. non perturbative) Einstein theory. In the less known *Evolution of the expanding universe* published in 1934 [29], he had a first intuition of a cosmic background temperature at a few Kelvins: "If all the atoms of the stars were equally distributed through space there would be about one atom per cubic yard, or the total energy would be that of an equilibrium radiation at the temperature of liquid hydrogen." He also interpreted for the first time the cosmological constant as vacuum energy: "The theory of relativity suggests that, when we identify gravitational mass and energy, we have to introduce a constant. Everything happens as though the energy in vacuo would be different from zero. In order that motion relative to vacuum may not be detected, we must associate a pressure $p = -\rho c^2$ to the density of energy $\rho c^2$ of vacuum. This is essentially the meaning of the cosmological constant $\lambda$ which corresponds to a negative density of vacuum $\rho_0$ according to $\rho_0 = \lambda c^2/4\pi G \sim 10^{-27}$ gr./cm$^3$". Such a result would be rediscovered only in 1967 by Sakharov (the article has been republished as a *Golden Oldie*, see [30]) and Zel'dovich [31] on the basis of quantum field theory; it is now considered as one of the major solutions to the "dark energy problem".

To conclude, the following list summarizes the cosmological questions[14] discussed by Lemaître in the period 1927–1934:

- Expansion of space starting from an initial singularity
- Dominating role of the cosmological constant in cosmic dynamics
- Importance of the pressure of radiation in the early universe
- Role of quantum theory at the origin of the universe
- Problem of the age of the universe solved with a cosmological constant
- Interpretation of the cosmological constant as the energy of the quantum vacuum
- Possibility of phoenix universes
- Existence of relics of the early universe (cosmic residual temperature, ultra-high energy cosmic rays)

[14] Not all of them were discussed here, for a complete survey see [16].

– Formation of galaxies due to random fluctuations of density
– Topology of the universe.

On all these questions, Georges Lemaître showed an astonishing perspicacity. This is the reason why the astronomer William McCrea, although an adept of Milne's Newtonian cosmology, could declare in an article judiciously entitled *Some lessons for the future* [32]: "Lemaître was a scientist of superbly robust common sense. All of us who knew him must ever have wished we had paid attention to his ideas. [...] Einstein, Eddington and Milne may have been greater scientists than Lemaître, and more famous in their day. But on the subject of cosmology and its importance for astronomy, Lemaître had more to impart. He talked better sense."

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A brief biography of Georges Lemaître was printed together with another Golden Oldie in *Gen. Relativ. Gravit.* **29**, n°5, 639 (1997), doi:10.1023/A:1018803604510

## References


1. Lemaître, G.: A homogeneous universe of constant mass and increasing radius accounting for the radial velocity of extra-galactic nebulae. M.N.R.A.S. **41**, 483–490 (1931)
2. Lemaître, G.: Un univers homogène de masse constante et de rayon croissant, rendant compte de la vitesse radiale des nébuleuses extra-galactiques. Annales de la Société Scientifique de Bruxelles, série A, **47**, 49–59 (1927)
3. Lemaître, G.: The expanding universe. M.N.R.A.S. **41**, 491–501 (1931)
4. Lemaître, G.: L'univers en expansion. Annales de la Société Scientifique de Bruxelles A **53**, 51 (1933), reproduced as a Golden Oldie as The expanding universe. (translated by M. A. H. MacCallum), Gen. Relativ. Gravit. **29**, n°5, 641 (1997). Editorial note by A. Krasiński, p. 637
5. Lemaître, G.: Untitled contribution to a discussion on "The question of the relation of the physical universe to life and mind". Supplement to Nature **127**, 704–706 (1931)
6. Lemaître, G.: L'expansion de l'espace. Rev. Quest. Sci. **20**, 391–410 (1931)
7. Lemaître, G.: The primeval atom: an essay on cosmogony (English translation by B.H. and S.A. Korff). Van Nostrand Company, Princeton (1950)
8. Lemaître, G.: L'indétermination de la loi de Coulomb. Annales de la Société Scientifique de Bruxelles, série B, **51**, I$^{\text{ère}}$ partie, pp. 12–16 (1931)
9. Lemaître, G.: Sur l'interprétation d'Eddington de l'équation de Dirac. Annales de la Société Scientifique de Bruxelles, série B, **51**, I$^{\text{ère}}$ partie, pp. 89–99 (1931)
10. Friedmann, A.: On the curvature of space (translated by G. F. R. Ellis and H. van Elst) Gen. Relativ. Gravit. **31**(12), 1991 (1999) (b) Friedmann, A.: On the possibility of a world with constant negative curvature of space (translated by G. F. R. Ellis and H. van Elst). Gen. Relativ. Gravit. **31**(12), 2001 (1999); Editorial note by A. Krasiński and G. F. R. Ellis, p. 1985; biography by A. Krasiński, p. 1989; see also Addendum in Gen. Relativ. Gravit. **32**, 1937 (2000)
11. Lemaître, G.: La théorie de la relativité et l'expérience. Rev. Quest. Sci. 4$^{\text{e}}$série **9**, 346–374 (1926)
12. Strömberg, G.: Analysis of radial velocities of globular clusters and non-galactic nebulae. Astrophys. J. **61**, 353 (1925); Slipher, V.M.: Nebulae. Proc. Am. Philos. Soc. **56**, 403 (1917); Hubble, E.: Extra-galactic nebulae. Astrophys. J. **64**, 321 (1926); Eddington, A.: The Mathematical Theory of Relativity. Cambridge University Press (1923), p. 162 (reprinted 1960)
13. Lemaître, G.: Rencontres avec Einstein. Rev. Quest. Sci. t. 79 5$^{\text{e}}$série **19**, 129–132 (1958)
14. Hubble, E.: A relation between distance and radial velocity among extra-galactic nebulae. In: Proceedings of the National Academy of Sciences vol. 15, March 15, Number 3 (1929)
15. Eddington, A.S.: Remarks at the meeting of the Royal Astronomical Society. The Observatory **53**, 39–40 (1930)
16. Luminet, J.-P.: L'invention du big bang. Le Seuil, Paris (2004)
17. Eddington, A.S.: The Observatory **53**, 162–164 (1930)

18. Eddington, A.S.: On the instability of Einstein's spherical world. M.N.R.A.S. **90**, 668–678 (1930)
19. Kragh, H.S., Lambert, D.: The context of discovery: Lemaître and the origin of the primeval-atom universe. Ann. Sci. **64**, 445–470 (2007)
20. Eddington, A.: The end of the world from the standpoint of mathematical physics. Nature **127**, 447–453 (1931)
21. Lanczos, C.: Über eine zeitlich periodische Welt und eine neue Behandlung des problems der Atherstrahlung. Zeitschrift für Physik **32**, 56–80 (1925)
22. Bohr, N.: The use of the concepts of space and time in atomic theory. Nature **127**, 43 (1931)
23. Lambert, D.: Monseigneur Georges Lemaître et le débat entre la cosmologie et la foi. Revue théologique de Louvain **28**, 28–53 (1997); Lambert, D.: L'itinéraire Spirituel de Georges Lemaître suivi de "Univers et Atome" (inédit de G. Lemaître). Bruxelles, Lessius (2008)
24. Jeans, J. et al.: The Question of the Relation of the Physical Universe to Life and Mind. Supplement to Nature **128**, 700–722 (1931)
25. Menzel, D.H.: Blast of giant atom created our universe. Pop. Sci. Monthly **121**, 28–30 (Dec. 1932)
26. Jordan, P.: Die Physik des 20. Jahrhunderts, Braunschweig, 1936; English translation: Physics of the 20th Century, New York (1944)
27. Einstein, A.: Zum kosmologischen Problem der allgemeinen Relativitätstheorie. Sitzungberichte der Preussischen Akademie der Wissenschaften vol **XII**, SB Phys.-math. 235–237 (1931)
28. Einstein, A., de Sitter, W.: On the relation between the expansion and the mean density of the universe. Proc. Natl. Acad. Sci. **18**, 213–214 (1932)
29. Lemaître, G.: Evolution of the expanding universe. Proc. Natl. Acad. Sci. **20**, 12 (1934)
30. Sakharov, A.D.: Doklady Akademii Nauk SSSR. **177**, 70 (1967). Golden oldie: Vacuum quantum fluctuations in curved space and the theory of gravitation (translated by Consultants Bureau). Gen. Relativ. Gravit. **32** n° 2, 365 (2000). Editorial note by H. J. Schmidt, p. 361
31. Y.B. Zel'dovich Cosmological constant and elementary particles. Sov. Phys. JETP Lett. **6**, 316–317 (1967); see also Zeldovich, Y.B.: The Cosmological Constant and the Theory of Elementary Particles. Sov. Phys. Uspekhi **11**, 381–393 (1968), republished as a Golden Oldie in Gen. Relativ. Gravit. **40** n°7, 1562 (2008)
32. McCrea, W.: Personal recollections: some lessons for the future. In: Bertotti B., et al (eds.) Modern Cosmology in Retrospect. pp. 197–220 Cambridge University Press (1990)