Prof. Ted Jacobson
Room PSC 3151, (301)405-6020
www.physics.umd.edu/grt/taj/624c/
jacobson@umd.edu

1. (*Massive vector vacuum correlation function*)

   (a) Verify that the massive spin-1 field operator, Eqn (8.64), satisfies the field equation coming from the Proca Lagrangian (8.23). (The polarization vectors (8.69) are orthogonal to the 3-momentum.) (b) Evaluate the vacuum correlation function $\langle 0|A_\mu(x)A_\nu(y)|0\rangle$. The sum over the three polarizations yields a two-index tensor, which can be written in a Lorentz covariant way, as $A\eta_{\mu\nu} + Bp_\mu p_\nu$ (with $p_0 = \sqrt{p^2+m^2}$). Determine A and B by finding the contraction with each of the three polarization vectors and with the 4-momentum vector. Leave the result as an integral over the momentum.

2. (*Energy positivity for the Proca and Maxwell vector fields*)

   Find the Hamiltonian density for the Proca Lagrangian (8.23), and show that the energy density is positive, apart from a total divergence term. To do this: (i) Find the momentum π^i conjugate to A_i, and the momentum π^0 conjugate to A_0. The latter should vanish identically. (ii) Express the Hamiltonian density in terms of A_i, π^i, and A_0. (For the spatial derivatives term involving A_i use the shorter notation $F_{ij}F_{ij}$.) (iii) Find the equation of motion for A_0, solve it for A_0, and substitute back into the Hamiltonian density to eliminate A_0. At this point, the result should be positive, except for a total divergence. (iv) Now consider the massless case. Show that A_0 now acts just as a Lagrange multiplier, and find the constraint that its equation of motion imposes. This is the Gauss law constraint of electromagnetism. (v) Show that if you substitute this Gauss law constraint back into the Hamiltonian density, again the Hamiltonian density is positive, and now equal to half the sum of the squares of the electric and magnetic fields, up to a total divergence.

3. (*spontaneous symmetry breaking and Higgs mechanism*)

   (i) Read the first three sections of `http://www.scholarpedia.org/article/Englert-Brout-Higgs-Guralnik-Hagen-Kibble_mechanism` (up to the section entitled Evading the Goldstone theorem). (ii) Verify equation (9). (iii) Verify that the Lagrangian (10) is invariant under the gauge transformation (11). (iv) Verify equation (12). (v) The parametrization (7) of the complex scalar field uses the real and imaginary parts of the deviation from the expectation value $\langle\phi\rangle = v/\sqrt{2}$. A more revealing choice of fields is the polar form, $\phi = \frac{1}{\sqrt{2}}(v+\rho)e^{i\theta}$. Express the gauge-invariant Lagrangian density (10), with V given by (6), in terms of the fields ρ and θ, and using this show that

   (a) the potential is independent of θ, and has a minimum at $\rho = 0$;
   (b) ρ has a mass which agrees with the mass of φ_1 in the linked article;
   (c) if θ is set to zero by a choice of gauge, the remaining term coming from $D_\mu\phi^* D^\mu\phi$ is $\frac{1}{2}(v+\rho)^2 A_\mu A^\mu$, which includes a mass term for the vector field agreeing with the one in the linked article, as well as interaction terms between ρ and A_μ. (Something like this is how the Higgs field gives a mass to the W and Z bosons, and interacts with them.)