HW#5 —Phys624—Fall 2017

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1. ϕ^3 *theory*
   (a) Write the Feynman rules for the S-matrix for a scalar field with Lagrangian density $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{3!}\lambda\phi^3$ in four spacetime dimensions.
   (b) Draw the connected diagrams at $O(\lambda^2)$ for 2-2 scattering in ϕ^3 theory, and write the corresponding amplitudes. (I think there are three diagrams.)
   (c) Do the same as in the previous part, but at $O(\lambda^4)$. (Don't forget to check for nontrivial symmetry factors.)
2. Consider a scalar field ϕ in D spacetime dimensions.
   (a) What is the mass dimension of ϕ? (Recall we use units with $\hbar = c = 1$, and thus identify the dimensions mass $\sim$ energy $\sim$ 1/length.)
   (b) What is the mass dimension of the coupling constant g in an interaction term $g\phi^n$? What does this give for the case of $D = 4$ and $n = 3$? For which values of D and n is g dimensionless?
3. Consider a free massive scalar field, with Lagrangian $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2$, and treat the mass term as an interaction, $\mathcal{L}_{\text{int}} = -\frac{1}{2}m^2\phi^2$, with no other interactions.
   (a) What are the momentum space Feynman rules for $\langle\Omega|T(\phi(x)\phi(y))|\Omega\rangle$, where $|\Omega\rangle$ is the "interacting" ground state?
   (b) Sum the infinite series and show that you recover the Feynman propagator for the massive field. (The series converges for momenta satisfying $|p^2| > m^2$. For other momenta just define the sum by analytic continuation from the convergent case.)
4. *Yukawa potential* — Problem 3.6 from Schwartz. Skip part (f). See Sec. 3.4 (Coulomb's law) for comparison.