HW#13 —Phys624—Fall 2017 Prof. Ted Jacobson
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1. S16.4 (*Landau pole with standard model fermions*) (Note: To keep this simple, define the renormalized charges at $m_{\rm top}^2$, and run up the energy scale starting from $m_{\rm top}$, using the $|p^2| \gg m_{\rm top}^2$ approximation in the vacuum polarization graphs. The fine structure constant at $m_{\rm top}$ is $e_R^2(m_{\rm top})/4\pi \simeq 1/127$.

2. *Bianchi identity and (non-abelian) Faraday's law*
Maxwell's equations can be separated into the ones sourced by charge and current, and the ones with no sources: $\vec{\nabla}\cdot\vec{B} = 0$ and Faraday's law, $\vec{\nabla}\times\vec{E} = -\partial\vec{B}/\partial t$.

   (a) Show that the sourceless equations follow from the identity $\partial_{[\alpha}F_{\beta\gamma]} \equiv 0$, which follows from the fact that $F_{\beta\gamma} = 2\partial_{[\beta}A_{\gamma]}$, where the square brackets are antisymmetrizers.

   (b) Establish the non-abelian generalization of this identity, the "Bianchi identity", $D_{[\alpha}G_{\beta\gamma]} \equiv 0$, where $G_{\beta\gamma} = 2(\partial_{[\beta}A_{\gamma]} + A_{[\beta}A_{\gamma]})$ is the field strength tensor, and $D_\alpha G_{\beta\gamma} \equiv \partial_\alpha G_{\beta\gamma} + [A_\alpha, G_{\beta\gamma}]$ is the gauge-covariant derivative of the field strength. You may crank this out, but there is a slick method: use the fact that the gauge-covariant derivative is gauge-covariant, together with the fact that you can always choose a gauge in which the gauge potential vanishes at a given point.

   (c) Show that $D_\alpha G_{\beta\gamma}$ is gauge-covariant.

   (d) Show that you can always choose a gauge in which the gauge potential vanishes at any one given point x_0.

3. *Does the field strength determine the potential up to gauge?*
(a) Show that in Maxwell theory, if two gauge potentials give the same field strength, they are necessarily related by a gauge transformation. (b) Show that the same is not true in non-abelian gauge theory. You may use any method or example you like. For a straightforward approach, consider the following two potentials in the xy plane, for the SU(2) gauge theory:

$$(A_x = -iy\sigma^3,\, A_y = ix\sigma^3), \quad \text{and} \quad (A'_x = i\sigma^1,\, A'_y = -i\sigma^2).$$

   Show that these have the same field strength tensor, and show by computing their gauge-covariant derivatives that they are not related by a gauge transformation.

   [Although the same nonzero field strength can thus arise from gauge-inequivalent potentials, a zero field strength arises only from potentials that are gauge-equivalent to zero, at least in a region that is contractible to a point. (If there are non-contractible closed loops int the space, then there can be gauge potentials whose field strength vanishes everywhere but which are not globally gauge-equivalent to zero.)]