HW#6 —Phys622—Spring 1998
Due Friday, March 13, 1998
Turn in at or before beginning of class.

Prof. Ted Jacobson
Room 4117, (301)405-6020
jacobson@physics.umd.edu

1. Prove that an attractive delta function potential has no bound states in two or three spatial dimensions. (It may be convenient to use the momentum representation). [5 *points*]

2. Problem 3.17, Schwabl. (*...every energy allowed*) [3 *points*]

3. Problem 3.13, Schwabl. (*Over-barrier transmission*) Do not assume $V_0 > 0$. Compare the $V_0 > 0$ and $V_0 < 0$ cases. [10 *points*]

4. Consider a particle of mass m and charge e in a constant electric field $\vec{E} = E\hat{x}$. [1+3+3+3=10 *points*]

(a) Write the Hamiltonian.

(b) Solve for the energy eigenfunctions $\psi(p_x, p_y, p_z)$ in the momentum representation. (Don't worry about normalization.)

(c) Show that the current density for your energy eigenfunctions vanishes. (*Suggestion*: Transform your eigenfunctions to the position representation, but do *not* evaluate explicitly the integral. (Let me know if you find a different way to do this.))

(d) How can the current density vanish in the presence of a uniform force? Interpret your result for the current in terms of the classical orbits in the electric field.

5. Let us consider one-dimensional scattering on an arbitrary, real potential $V(x)$ localized within a finite interval $-a < x < a$. In the general case, a wave function of the energy $E = \hbar^2 k^2/2$ has the following asymptotic form:

$$\psi(x) = \begin{cases} Ae^{ikx} + Be^{-ikx}, & x \ll a, \\ Fe^{ikx} + Ge^{-ikx}, & x \gg a. \end{cases} \tag{1}$$

The coefficients A, B, C, and D are related by the scattering matrix:

$$\begin{pmatrix} B \\ F \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} A \\ G \end{pmatrix}. \tag{2}$$

In this problem we discuss general limitations on the structure of the matrices imposed by symmetry. [3×7=21 *points*]

(a) Express the reflection and transmission coefficients r and t for scattering from the left in terms of the S-matrix elements.

(b) Taking into account conservation of probability show that the matrix S must be *unitary*:

$$S^\dagger S = I. \qquad \textit{unitarity} \tag{3}$$

Hint: What is the probability current?

(c) The scattering matrix depends on k. Show that in general

$$S(-k) = S(k)^\dagger. \tag{4}$$

(d) Show that time reversal symmetry implies that S must be *symmetric*:

$$S^T = S. \qquad \textit{time reversal symmetry} \tag{5}$$

Hint: Time reversal is complex conjugation.

(e) Show that if the potential is reflection symmetric ($V(-x) = V(x)$) then

$$S_{11} = S_{22}, \qquad S_{12} = S_{21} \qquad \textit{reflection symmetry} \tag{6}$$

(f) Show that if the potential is reflection symmetric then S can be parametrized in terms of two real parameters α, β as:

$$S = e^{i\beta}\begin{pmatrix} \cos\alpha & i\sin\alpha \\ i\sin\alpha & \cos\alpha \end{pmatrix}, \tag{7}$$

(g) *Adapted from Physics Qualifier, Spring 1967, Problem II-1.* Consider scattering on a real one-dimensional potential $V(x)$. Is the coefficient of transmission for a particle incident from the left the same as that for an identical particle incident from the right with the same energy? Prove your answer. Do *not* assume that the potential $V(x)$ has a certain reflection symmetry (parity).

6. Consider scattering from a delta function potential $aV_0\delta(x)$ with $a > 0$ but V_0 arbitrary. [5+1+3+3=12 *points*]

(a) Find the S-matrix for positive values of the energy. Check that the general properties of S found in the previous problem hold.

(b) Find the reflection and transmission coefficients r and t for the delta function potential.

(c) Show that the δ-function limit of the square barrier in problem 3.13 and section 3.3.1 of Schwabl agrees with your results for r and t calculated in this problem.

(d) Analytically continue the S-matrix elements to a negative energy E of the incoming particle. Is there any (negative) value of E such that the S-matrix elements become infinite? What is the physical meaning of this energy? Does this effect exist for both positive and negative values of the coefficient V_0 that stands in front of the delta-function?