HW#4 —Phys622—Spring 1998
Due Friday, Feb. 27, 1998
Turn in at or before beginning of class.

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At this point you should have read in Schwabl chapters 1,2,4, and 8 (except for section 8.6). The harmonic oscillator is treated in Section 3.1

1. Problem 4.1, Schwabl. *Minimum uncertainty wavepackets.* Note that (2.24) gives $\Delta x \Delta p$ for a free particle with Gaussian wavepacket. On part (*b*) explain why, even though the wavepacket is Gaussian, it is not a minimum uncertainty wavepacket. [3+2=5 *points*]

2. In this problem we study the spreading of the wavepacket for a free particle in the Heisenberg picture. [3+3+4=10 *points*]

(a) Solve the Heisenberg equations of motion for a free particle in one dimension.

(b) Use your result from the previous part to show that, in *any* state,

$$(\Delta x_t)^2 = (\Delta x_0)^2 + \frac{t^2}{m^2}(\Delta p_0)^2 + \frac{t}{m}\langle\{\tilde{x}_0, \tilde{p}_0\}\rangle, \tag{1}$$

where we have used the notation $A_t \equiv A(t)$ and $\widetilde{A} \equiv A - \langle A \rangle$ for any operator A.

(c) Show that if the state is a minimum uncertainty state for x_0 and p_0, then (1) implies

$$\Delta x_t = \Delta x_0 \sqrt{1 + (t\hbar/2m(\Delta x_0)^2)^2}. \tag{2}$$

Note that (2) agrees with (2.16) in Schwabl, which was computed explicitly with a Gaussian wavepacket. Your result is more general since it applies to *all* minimum uncertainty wavepackets. *Hint*: For part (*c*), what do you know about $\langle\{\tilde{x}_0, \tilde{p}_0\}\rangle$ for a minimum uncertainty state?

3. Consider a harmonic oscillator of mass m and frequency ω in two different initial states: (*i*) the number eigenstate $|25\rangle$, and (*ii*) the coherent state $|5e^{i\theta}\rangle$. For both these states compute (*a*) $\langle x \rangle$ and Δx initially, and (*b*) $\langle x \rangle$ after a time t has passed. *Hint*: Use ladder operators, and *do not* go into the position representation. [5+5=10 *points*]

4. *Adapted from Physics Qualifier, Spring 1980, Problem II-2.*

   A harmonic oscillator is in its ground state $|0\rangle$. Suddenly, the origin (the equilibrium point) of the oscillator is displaced by the distance l. The wave function does not change during the displacement.

   (a) Calculate the probability to find the oscillator in an energy eigenstate $|n\rangle$ defined with respect to the new origin.

   (b) Calculate the change of the average energy of the system.

   Hint: The initial state is a coherent state for the displaced potential. Use ladder operators, and *do not* go into the position representation. [3+3=6 *points*]

5. A harmonic oscillator of frequency ω is subject to a time dependent force $F(t)$. [2+2+4+2=10 *points*]

   (a) Write the Hamiltonian in terms of ladder operators.

   (b) Write the Heisenberg equation of motion for the lowering operator $a(t)$.

   (c) Show that the solution is given by

$$a(t) = e^{-i\omega t}\left[a(0) + \frac{ix_0}{\hbar\sqrt{2}}\int_0^t e^{i\omega t'}F(t')dt'\right] \tag{3}$$

   (d) We will show elsewhere that the Schrödinger state remains a coherent state $|\alpha(t)\rangle$ under time evolution, with a time-dependent parameter $\alpha(t)$. Determine $\alpha(t)$ using the result of the previous part.

6. One can write a resolution of the identity using coherent states, even though they are not mutually orthogonal. Show that

$$\frac{1}{\pi}\int d^2z\; |z\rangle\langle z| = I. \tag{4}$$

   Hint: One way is to express the coherent states as a sum of number eigenstates, and use polar coordinates to compute the integral. [4 *points*]