Final Exam —Phys622—Spring 1998
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1. Give the definition of a unitary operator and prove that the eigenvalues of a unitary operator have unit modulus. [10 *points*]

2. What state results when a $\pi/2$ rotation about the y-axis is applied to the spin triplet state $|1,0\rangle = (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}$? [10 *points*]

3. Beryllium (^{9}Be) is the fourth element in the periodic table. Is a neutral Beryllium atom a fermion or a boson? How about a Beryllium ion ^{9}Be$^+$? Explain your reasoning. [10 *points*]

4. Consider a particle of charge e and mass m confined to a circular ring of radius R. Suppose the ring surrounds a magnetic flux Φ, but the magnetic field vanishes everywhere on and near the ring. Find the energy levels of the particle on the ring, and for each level find the corresponding electric current. Treat the motion of the particle as a one-dimensional problem, including only the angular degree of freedom. [20 *points*]

5. Two (non-identical) particles of mass m_1 and m_2 interact through a potential given by

$$V(\vec{r}_1, \vec{r}_2) = \begin{cases} -V_0 & \text{if } |\vec{r}_1 - \vec{r}_2| < d \\ 0 & \text{if } |\vec{r}_1 - \vec{r}_2| \geq d \end{cases} \tag{1}$$

(a) Prove that if there is only one bound state then it has zero angular momentum in the center of mass frame. [10 *points*]

(b) Find the condition on V_0 and d that ensures there is at least one bound state. [10 *points*]

(c) Assume there is one bound state. Write explicitly the complete two-particle wavefunction for this state assuming the total linear momentum is $\vec{P}$ and the total energy is E. (Don't worry about overall normalization.) [10 *points*]

6. An infinite chain of equally spaced identical potential wells with impenetrable barriers between them has an infinite degeneracy of each energy level. This degeneracy is lifted if there is some tunneling between the wells. Let us focus on the lifting of the degeneracy of the ground state. Let $|n\rangle$ denote the ground state of a particle in the nth well with impenetrable barriers in place. Let us approximate the hamiltonian including tunneling as

$$H = V_0 \sum_{n=-\infty}^{\infty} \Big(|n+1\rangle\langle n| + |n\rangle\langle n+1| \Big) \tag{2}$$

where V_0 is a positive real quantity. For simplicity we have set the ground state energy of each well to zero.

(a) Let T be the translation operator that shifts one well to the next. Thus $T|n\rangle = |n+1\rangle$. Use this definition of T to show that T is unitary acting on the space spanned by the states $\{|n\rangle\}$. [5 *points*]

(b) Since T commutes with H, these two operators (being unitary and hermitian respectively) can be simultaneously diagonalized. Using this fact, find the energy eigenstates and eigenvalues. Show that the tunneling lifts the ground state degeneracy into a continuous a band. What are the minimum and maximum energies in this band? [15 *points*]

Optional bonus problem: Now let us replace the infinite chain by a finite ring of N potential wells. The above calculations can be mostly taken over by identifying the $(N+1)$st well with the 1st well. Into how many levels does the ground state now split, and what are the corresponding energies? [10 *points*]