Midterm Exam —Phys622—Spring 1998
Friday, April 10, 1998

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1. Find the momentum space matrix elements $\langle p|V|p'\rangle$ for a one-dimensional potential $V = V_0 \cos kx$ with V_0 and k constants.

2. Suppose a particle in one dimension is in a quantum state $|\psi\rangle$. (a) Write down a formula for for the probability that the particle would be found with a momentum p lying in the interval $0 < p_1 \leq p \leq p_2$? (b) Now assume the wave function $\langle x|\psi\rangle$ is constant in the interval $(-a, a)$ and zero elsewhere. Find a formula for the probability in part (a) assuming $p_2 a \ll \hbar$.

3. Let B and C be two operators, i.e.

$$\{B, C\} \equiv BC + CB = 0.$$

Let $|\psi\rangle$ be an eigenstate of both B and C. (a) What can be said about the corresponding eigenvalues? (b) For $B =$ baryon number and $C =$ charge conjugation the relations $\{B, C\} = 0$ and $C^2 = 1$ hold. What does your result imply in this case?

4. A two-state quantum mechanical system is governed by the hamiltonian

$$H = \alpha\,(|1\rangle\langle 1| + |2\rangle\langle 2|) + \beta\,(|2\rangle\langle 1| + |1\rangle\langle 2|).$$

If the system starts out in the state $|1\rangle$ at time $t = 0$, what is the probability that it will be found in the state $|1\rangle$ at a later time t? *Hint*: What are the eigenvectors of H?

5. Compute Δx, Δp and the product $\Delta x \Delta p$ for a harmonic oscillator in the number eigenstate $|n\rangle$. Use ladder operators, and give your results in terms of $x_0 \equiv \sqrt{\hbar/m\omega}$. For which values of n (if any) is $|n\rangle$ a minimum uncertainty state?

6. A harmonic oscillator of frequency ω is initially in the coherent state $|z\rangle$ with $z = \rho e^{i\phi}$. What is $\langle p\rangle$ after a time t has passed?

7. Sketch the three lowest energy eigenfunctions for the symmetric double square well potential, being careful about the sign of the curvature of the wavefunction, symmetry, etc.