Homework: Lectures 6, 7 and 8

Solving exercises is the most effective way of learning physics. Although only one third of the final grades for this course will be based on the homeworks, you should take them very seriously.

Recommended readings:

1. Chapter 9 in B. Schutz, " A first course in general relativity".
2. GW energy-momentum tensor: Sec. 10.3 in S. Weinberg, " Gravitation and Cosmology"; Secs. 35.13, 35.14 and 35.15 in C. Misner, K.S. Thorne and A.J. Wheeler, "Gravitation"; D. Brill and M. Hartle, Phys. Rev. 135, b271 (1964); R.A. Isaacson, Phys. Rev. 166, 1272 (1967), 1263 (1968); P. Anderson, Phys. Rev. D 55, 3440 (1997).
3. Generation of GWs: Sec. 10.4 in S. Weinberg, " Gravitation and Cosmology".
4. K.S. Thorne, Multipole expansions of gravitational radiation, Rev. Mod. Phys. 52 (1980) 299.

Assignement to be turned in at the beginning of the class on Thursday, March 9 by students registered to the course:

- State what of the above readings you have done
- Work the three exercises below

Exercises:

1. **Mass octupole radiation from a binary star [M. Maggiore (2006)] (3.5 points)**

   Let us consider a binary system of reduced mass μ whose center-of-mass coordinate moves along the circular orbit

   $$x(t) = R\cos\omega_s t \qquad y(t) = R\cos\iota\sin\omega_s t \qquad z(t) = R\sin\iota\sin\omega_s t\,,$$

   where $R^2 = x^2 + y^2 + z^2$. Let us set the observer's direction $\hat{n}$ along the z direction. [Henceforth, the (x, y, z) axes are also labeled as $(1, 2, 3)$.] As derived in class, the octupole (oct) gravitational radiation is

   $$(h_{ij}^{\rm TT})_{\rm oct} = \frac{1}{r}\frac{2G}{3c^5}\,\Lambda_{ij,kl}(\hat{n})\,\dddot{M}_{kl3}\,,$$

   where r is the distance of the observer from the binary and

   $$M^{klm} = \mu\, x^k\, x^l\, x^m \qquad \Lambda_{ij,kl} = P_{ik}\,P_{jl} - P_{ij}\,P_{kl}/2 \qquad P_{ij} = \delta_{ij} - \hat{n}_i\,\hat{n}_j\,.$$

   Evaluate the $(h_+)_{\rm oct}$ and $(h_\times)_{\rm oct}$ components. At which frequencies the octupole gravitational radiation is emitted?

2. **Current quadrupole radiation from a binary star [M. Maggiore (2006)] (3.5 points)**

   Evaluate the current-quadrupole (cq) radiation, notably the components $(h_+)_{\rm cq}$ and $(h_\times)_{\rm cq}$, for the binary of Problem 1. To simplify the problem, evaluate first $(h_+)_{\rm oct+cq}$ and $(h_\times)_{\rm oct+cq}$, and then use the results of Problem 1 to get the current-quadrupole part. As derived in class

   $$(h_{ij}^{\rm TT})_{\rm oct+cq} = \frac{1}{r}\frac{4G}{c^5}\,\Lambda_{ij,kl}(\hat{n})\,\dot{S}_{kl,3}\,,$$

   where $S^{kl,m} = \mu\,\dot{x}^k\,\dot{x}^l\,x^m$.

3. **Power radiated [M. Maggiore (2006)] (3 points)**

   Using results of Problem 2 determine the power radiated using the following formula derived in class,

$$P_{\rm oct+cq} = \frac{r^2\, c^3}{16\pi G}\, 2\pi \int_{-1}^{1} d\cos\iota < \dot{h}_+^2 + \dot{h}_\times^2 > .$$

   Express the ratios $P_{\rm oct+cq}(\omega_s)/P_{\rm quad}(2\omega_s)$ and $P_{\rm oct+cq}(3\omega_s)/P_{\rm quad}(2\omega_s)$, where $P_{\rm quad} = 32G\mu^2 R^4\omega_s^6/(5c^5)$, in terms of v/c using the Newtonian relation between R and ω_s, and estimate those ratios for $v/c = 10^{-2}$.