Homework: week1

Solving exercises is the most effective way of learning physics. Although only one third of the final grades for this course will be based on the homeworks, you should take them very seriously.

Recommended readings:

1. K.S. Thorne, *Gravitational waves*, `gr-qc/9506086`.
2. C. Cutler and K.S. Thorne, *An overview of gravitational-wave sources*, `gr-qc/0204090`.
3. A. Abramovici et al. *LIGO: the laser interferometer gravitational-wave observatory*, Science **256** (1992) 325.

Assignement to be turned in at the beginning of the class on Thursday, February 9 by students registered to the course:

- State what of the above readings you have done
- Work the three exercises below

Exercises:

1. **Multipolar expansion of the GW field** (2 points)

   On p 7 of the *Overview* lecture, the gravitational field is written in a multipolar expansion. Show that it is dimensionless. Show that by assuming that the gravitational field is proportional to G/r (where r is the distance between the observer and the source, and G is the Newton constant), and that each term of the expansion depends only on c and on derivatives of the multipole mass-moments I_L and current-moments J_L, the expansion, as written in slide 7, is unique.

2. **Strengths of GWs for a few astrophysical sources** (3 points)

   Using the leading non-trivial term in the multipolar expansion of p 7, i.e., the quadrupolar term $G\,\ddot{I}_2/(c^4\,r)$, evaluate how the gravitational-field h depends on the frequency in the case of (a) a rotating body, e.g., a pulsar, of characteristic scale R and (b) a binary moving along a circular orbit, for which the relation between the radial separation and the orbital angular frequency is described by the Newton law. Estimate at which distance the source should be in order to produce an $h \sim 10^{-21}$ on the Earth. In case (a) assume that the deviation from sphericity is 10^{-4}, the inertia moment is $10^{45}\,\mathrm{g\,cm}^2$, and give results for a GW frequnecy of 10 Hz and 10^3 Hz. In case (b) give the results if the GW frequency is 100 Hz, for the following binary total masses: $(1.4+1.4)M_\odot$, $(10+10)M_\odot$ and $(10^6+10^6)M_\odot$.

3. **Laser-interferometer GW detectors** (5 points)

   Consider an optical cavity (Fabry-Perot cavity) of length L composed by an input mirror with reflectivity ρ and transmissivity τ ($\rho+\tau=1$) and an end mirror with reflectivity one. Assume that a monochromatic light with angular frequency ω enters the cavity. Evaluate the amplitude and the phase of the monochromatic light coming out of the cavity. Plot the output phase as function of $\omega L/c$. For which values of $\omega L/c$ the phase changes is most sensitive to length changes?