Homework 4

due 3/27/07

1. Consider 1D quantum mechanics with a potential

$$V(x) = c\delta'(x). \tag{1}$$

Regularize the delta function using a piecewise constant potential and check that you get the same low-energy theory obtained in the text. This is a simple example of universality.

2. Consider QED, the theory of photons coupled to electrons. Write the Lagrangian as

$$\mathcal{L} = -\frac{1}{4g^2}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi, \tag{2}$$

where

$$D_\mu = \partial_\mu - iA_\mu. \tag{3}$$

This differs from the usual way of writing the Lagrangian by a field rescaling $A_\mu \to A_\mu/g$, where g is the gauge coupling.

(a) Write the Feynman rules for the theory, keeping track of the differences from the usual case from the field rescaling.

(b) Compute the 1-loop correction to the inverse photon propagator $\Pi^{\mu\nu}(p)$ in Pauli-Villars regulator. This means that you will have to introduce massive spin $\frac{1}{2}$ scalar fields whose loop contribution cancels the divergent part of the fermion loop. (You will need at least two such fields to make the loop integral finite.) Before performing any renormalization, verify that your result is gauge invariant, that is

$$p_\mu \Pi^{\mu\nu}(p) = 0. \tag{4}$$

(c) Absorb the divergent part into a redefinition of g and compute the beta function for g:

$$\beta(g) = \mu\frac{dg}{d\mu}. \tag{5}$$

Compare this with the result in Peskin and Schröder (or any other standard textbook).

3. Carry out the renormalization of the 4-point function in $\lambda\phi^4$ theory to 2 loops using a momentum space cutoff. Show by explicit calculation that the divergence is local to 2-loop order.

Note that in order to compute the beta function of the 4-point coupling to 2-loop order, you must also compute the wavefunction renormalization to two loops. This is a harder calculation; it can be done most easily using dimensional regularization. If you are interested, you can do it for "extra credit" (and bragging rights that you have done an actual 2-loop calculation). We have not discussed dimensional regularization in class, but the formulas can be found in Peskin and Schröder (and many other places). The key is to do the loop integrals one at a time in d dimensions, and expand around $d = 4$ after doing both loop integrals.