Spherical Gravitational Wave Detectors

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Abstract

Resonant bar gravitational wave detectors have improved greatly over the last 40 years but are currently completely outperformed by laser interferometer detectors. Spherical detector configuration has several advantages over conventional bar detectors and can be considered as the next generation of resonant gravitational wave antenna. In this report, I will give a brief introduction to gravitational waves and detectors and then review the major research done on spherical detectors.

1. Introduction

1.1 Gravitational waves

Gravitational waves (GW) are a prediction of Einstein's General Relativity (GR). They are predicted to be transverse and spin 2 waves [1]. There has been indirect evidence for the existence of gravitational waves in the form certain theoretical prediction (based on GR) of inspiral velocity as measured in the binary neutron star system PSR1913. The observations match the theoretical value nearly perfectly and provide convincing evidence for the existence of gravitational waves.

However, any attempts to directly detect gravitational waves have not been successful yet.

According to GR, gravitational waves are emitted by time varying mass quadrupole moments (accelerated massive objects). The relative difference in the strengths of gravitational interactions as compared to electromagnetic interactions also ensures that gravitational waves are extremely weakly interacting while electro-magnetic waves are easily scattered and absorbed by dust and other matter between the object and the observer. Thus the detection of gravitational waves will reveal a new and different view of the universe. In particular, it might lead to new insights in cosmology, strong field gravity by observing black hole signatures, large-scale nuclear matter (neutron stars) and the inner processes of supernova explosions.

Far away from the source one can use the weak field approximation to solve the Einstein field equations in vacuum yielding a normal wave equation. Using the transverse-traceless gauge its general solutions can be written as

$$h_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & h_{+}(t-\frac{z}{c}) & h_{\times}(t-\frac{z}{c}) & 0 \\ 0 & h_{\times}(t-\frac{z}{c}) & -h_{+}(t-\frac{z}{c}) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \ldots(1)$$

where z is the direction of propagation and h_+ and $h_\times$ are the two polarizations and $h_{\mu\nu}$ is the small perturbation to the flat Minkowski metric $g_{\mu\nu}$.

A gravitational wave passing through an object, say a cylindrical rod, will exert a force on it which will cause the object to stretch and squeeze in directions perpendicular to the propagation of the wave. The strength of a gravitational wave can be expressed as a dimension-less quantity, the strain h which measures the relative length change $\Delta L / L$. Let us try to calculate an order of magnitude estimate for the strain h from typical astrophysical sources:

$$h \approx \frac{G\ddot{Q}}{c^4 r} \approx \frac{G(E_{kin}^{ns})}{c^4 r} \qquad \ldots(2)$$

If one sets the value of the non-symmetric part of the kinetic energy to be of the order of a solar mass, and r is assumed to be of the order of distance to Virgo Cluster and Hubble distance respectively, one obtains,

$$h \approx 10^{-21}$$

and

$$h \approx 10^{-23}$$

Clearly, gravitational waves are very hard to detect!

1.2 Gravitational wave detectors

The search for gravitational waves started with Joseph Weber about forty years ago, using what are now called as “Bar detectors”. Since then, various methods to detect gravitational waves have been proposed and many types of GW detectors have been built all over the world. They fall into broadly two categories:

a) Resonant mass detectors and
b) Laser interferometer detectors

Resonant mass detectors use the principle that a passing gravitational wave will deposit energy in the vibrational mode of a spring mass system, which can be detected by measuring its motion. Bar detectors use the fundamental longitudinal mode of a large cylindrical bar (usually made of Aluminum). Spherical detectors on the other hand attempt to use the quadrupole modes of a solid sphere. Transducers amplify and convert the vibrational motion into an electrical signal and can be of several types: capacitive transducers, resonant inductive transducers, optical transducers or even piezoelectric crystals.

Laser interferometer detectors like the LIGO (Laser Interferometer Gravitational wave Observatory) sense the motion of two lightly suspended mirrors at the two ends of a Michelson interferometer to detect any change in the length of the interferometer arms when a GW passes through it. They have very high strain sensitivity of the order of 10^{-23} $Hz^{-1/2}$. Also, these detectors have a wide bandwidth, as the two test masses can be considered free in the direction of the interferometer arm and are limited by noise sources, chiefly seismic noise, shot noise and radiation pressure noise.

2. Principle of a Spherical GW Detector

A sphere is a natural shape for a resonant antenna for GW as it has 5 degenerate quadrupole modes, which interact strongly with the wave [8]. It has the advantage of offering uniform cross-section and hence a full-sky coverage. By measuring the amplitude of the 5 quadrupolar modes excited, it is possible to determine both source direction and polarization of the gravitational wave. As there are 5 mode amplitudes and only 4 gravitational wave unknowns, one can use the extra information to veto non-gravitational wave disturbances [8]. Also, as the mass of the sphere would be much larger than a bar antenna of the same fundamental frequency, the sphere has a much better cross-section as compared to a bar antenna.

2.1 The TIGA Detector

When coupled with transducers in arbitrary locations on the sphere, the quadrupole modes split unequally and it becomes extremely complicated to deconvolve the signal. Johnson and Merkowitz [2] came up with the idea of locating the resonators on the face-centers of a truncated icosahedron (TIGA, **T**runcated **I**cosohedral **G**ravitational **A**ntenna). This symmetric arrangement splits the modes evenly and makes it much easier to deconvolve the GW signal. They estimate that the sensitivity in energy would be about 56 times that of an equivalent bar antenna with the same noise temperature.

The TIGA configuration is shown in Figure 1. It results from cutting off the corners of the dodecahedron, leaving a solid body in the shape of a "bucky ball" or a C60 molecule. The transducers are attached to the pentagonal faces. Johnson and Merkowitz showed that the outputs of 6 transducers could be simply combined to obtain the direction and polarization state of incoming gravitational radiation. Moreover, as should be obvious from the symmetry of the configuration, the TIGA detector is equally sensitive in all directions and to any polarization. The TIGA configuration solves one of the outstanding problems of the spherical detector, in that it has flat surfaces on which to attach transducers. In their paper they also have announced the experimental verification of these theoretical predictions.

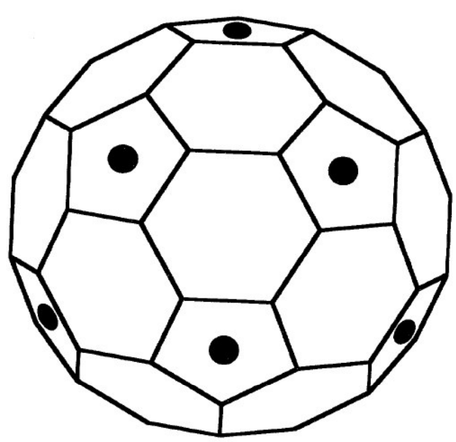

Figure 1: Truncated icosahedral arrangement of transducers

The spherical detector was analyzed in great detail by Lobo [3]. He found that the PHC (Pentagonal HexaContahedron) configuration was also a viable choice for locating the resonators. He also found that only monopole and quadrupole sphere modes could possibly be excited by a metric GW (according to GR, GW should be strictly quadrupole whereas other theories allow for monopole radiation).

The existence of so few modes which couple to GWs means that all the absorbed incoming radiation energy will be distributed amongst those few modes only, thereby making the sphere the most efficient detector, even from the sensitivity point of view. This also explains the higher energy cross section per unit mass reported for spheres

2.2 Mini-Grail

Mini-Grail is the name of a cryogenic 68 cm diameter spherical gravitational wave antenna developed at Leiden University [4, 5]. It is made of CuAl (6%) alloy with a mass of 1400 Kg, a resonance frequency of 2.9 kHz and a bandwidth around 230 Hz. In 2004, when they had their most successful run, they used 3 similar capacitive transducers with three different types of readouts: 2 Stage SQUID, single SQUID and a room temperature FET. The SQUID noise was about 1.6 $\mu\,\Phi_0$ / $Hz^{1/2}$ (~ 700$\hbar$). The peak sensitivity was about 1.5 x 10^{-20} / $Hz^{1/2}$ for the three most coupled modes. The effective temperature for a burst signal was 70 mK.

The quantum-limited strain sensitivity is expected to be ~4x10^{-21}/ $Hz^{1/2}$. The antenna will operate at a temperature of 20 mK. The possible sources for such a detector could be non-axisymmetric instabilities in rotating single and binary neutron stars, small black-hole or neutron-star mergers etc. Recently they have achieved an energy resolution of 27$\hbar$ below 250 mK with the double SQUID configuration [5].

2.3 Moon as a GW Detector

The Moon undergoes displacements under the influence of a gravitational wave, which could possibly be measured by highly sensitive displacement sensors [6]. The quadrupole modes of the moon have a frequency of about 1 mHz and it has a Q of about 2000. The amplitudes of its five degenerate quadrupole modes can be combined to determine the four unknowns: source direction (θ, ϕ) and wave polarization (h_+, $h_{\times}$), and the remaining degree of freedom can be used to discriminate against non-GW disturbances [8]. The TIGA (truncated icosahedral gravitational-wave antenna) configuration of six *radial* transducers has been shown to preserve the five-fold degeneracy and omni-directionality [2]. The same holds true for *tangential* transducers.There are two ways of operating a planetary gravitational wave detector: (1) As a *wideband* detector, by keeping the resonance frequencies of the displacement sensors to below the fundamental, where the planetary body acts as a rigid platform. (2) As a *resonant* detector, by tuning the displacement sensors to the fundamental (n = 1) or second harmonic (n = 2) quadrupole mode (l = 2), which couple strongly with GWs [3].

Due to lack of plate tectonics and its spin locked to its revolution, the Moon is very quiet seismically. Its total seismic energy release per year is estimated to be 10^9 times lower than the Earth. Moonquakes are driven mainly by tidal deformation due to the orbit eccentricity (0.05) and occur within a few days from the perigee. With the absence of ocean waves and winds, the seismic noise level between moonquakes may be extremely low.

2.4 Dual Detectors

Spherical detectors using resonant transducers suffer from the same principle drawback as bar detectors: they have limited bandwidth. To overcome this, M. Cerdonio et al [7] have thus proposed a GW detector based on a massive dual sphere system of resonators: a hollow sphere, which encloses a smaller solid sphere (see Fig. 2). They propose to use Optical transducers in the form of Fabry-Perot cavities formed by mirrors coated face to face at the inner surface of the hollow sphere and the solid sphere, in either a PHC [3] or a TIGA [2] layout.

Assuming the same material is used for both spheres, and that the inner one fills up almost completely the interior of the other external sphere, the first quadrupole resonance of the outer hollow sphere is at the lower frequency, while that of the inner solid sphere is 2-3 times higher. In the frequency region in between, the GW signal drives the hollow sphere above resonance and the solid sphere below resonance. The responses of the two resonators are then out of phase by 180 degrees and therefore the differential motion, read by the optical sensors, results in a signal enhancement. In this region only a small number of non-quadrupole resonances occur, which are not driven by GW. The pattern repeats for the two second quadrupole modes at higher frequencies and so on.

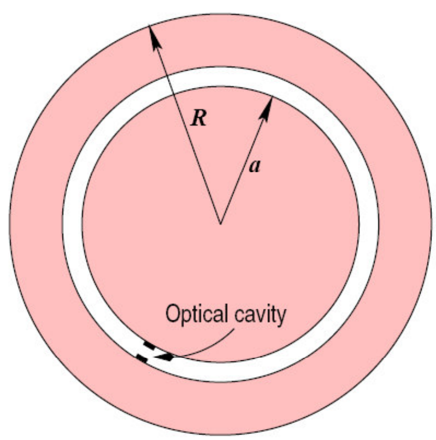

Figure 2: Dual spherical detector

To calculate the sensitivity of the system, the authors take as reference an operation at the Standard Quantum Limit (SQL). The SQL is reached at laser powers such that the shot noise and the radiation pressure equally contribute to the total noise and, at the same time, the back-action noise equals the mechanical resonator thermal noise. This procedure is also used for interferometer GW detectors. Note that, since the cavity is short, order of 1 cm, the finesse can be made very high, order of 10^6 and beyond, before loosing signal strength, and thus the SQL can be approached at laser powers of the order of 10W.

In their paper, the authors consider two choices of material for the spheres: Molybdenum and Beryllium. The sensitivities for the two cases are shown in fig.3. For Molybdenum spheres, they considered spheres of radii 0.95 m and 0.57 m, with input light power of 7 W and Q/T $\geqslant 10^8$ K^{-1}.

For Be-spheres, they use input light power of 12 W and the same Q/T.

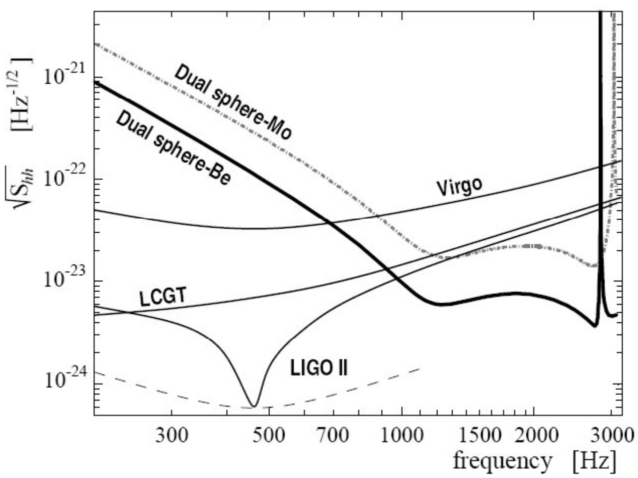

Figure 3: Plot of estimated sensitivity of dual sphere detector

3. Conclusion

The Spherical Detector has lots of interesting features, such as full sky coverage, 5 Quadrupole modes and improved cross-section (for the same mass) over bar antennae, which make it a very attractive idea for a resonant detector. The sensitivity can be comparable to that of the Laser interferometers and the dual sphere configuration even offers wideband capability. They can even be used together with Interferometer detectors and can complement and support them. The next generation of Resonant detectors are likely to be spherical detectors and if successful they could open up the exciting field of Gravitational Wave Astronomy.

References:

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