Spherical Gravitational Wave Antenna

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Introduction

Gravitational waves (GW) appear as vacuum solutions to the linearized Einstein equations in the weak field limit

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$h_{\mu\nu} = h_{+}(t - \frac{z}{c}) + h_{\times}(t - \frac{z}{c})$$

To calculate order of magnitude for h,

$$h \approx \frac{G\ddot{Q}}{c^4 r} \approx \frac{G(E_{kin}^{ns})}{c^4 r}$$

$h \approx 10^{-21}$ for r~Hubble distance

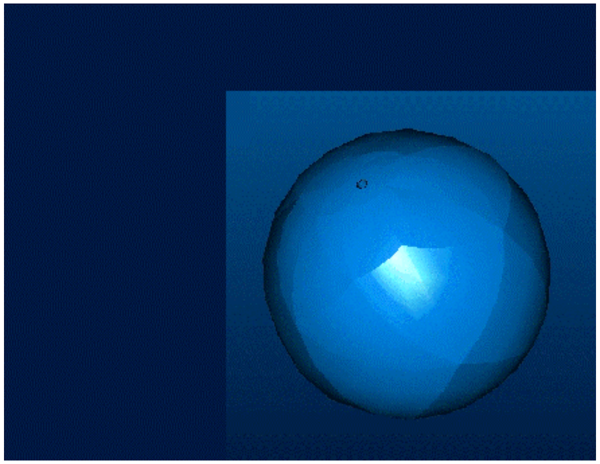
and

$h \approx 10^{-23}$ for r~Virgo cluster

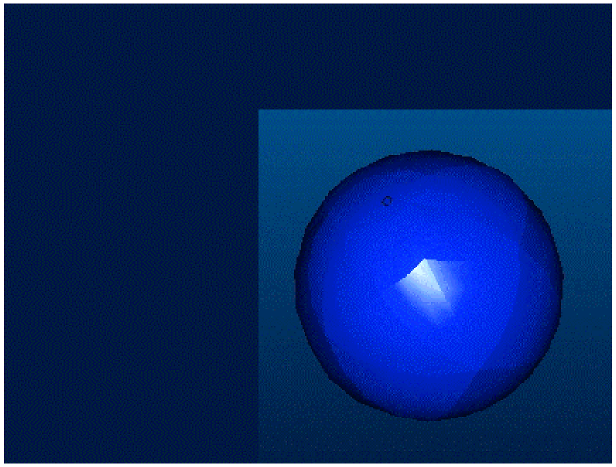
For a 3 m bar, this is a displacement of

$\Delta l = h \times l = 3 \times 10^{-21}$ meters !!

GW Polarizations



h_+



$h_\times$

Spherical Antenna

A sphere has 5 degenerate quadrupole modes.

$\Rightarrow$ Full-sky coverage with uniform cross section.

$\Rightarrow$ Can determine both source direction (θ, ϕ) and wave polarization (h_+, $h_\times$).

$\Rightarrow$ Much larger cross-section than a comparable bar antenna.

$\Rightarrow$ Due to overdetermination, non-GW disturbances can be vetoed. (Wagoner & Paik, 1976)

Mario Schenberg, Brazilian GW detector

Equation of motion for elastic sphere is

$$\rho \frac{\partial^2 \vec{s}}{\partial t^2} = (\lambda + \mu)\nabla(\nabla \cdot \vec{s}) + \mu \nabla^2 \vec{s} + \vec{F}$$

where **s** is the displacement field of the sphere.

The general solution for the spheroidal modes have the form

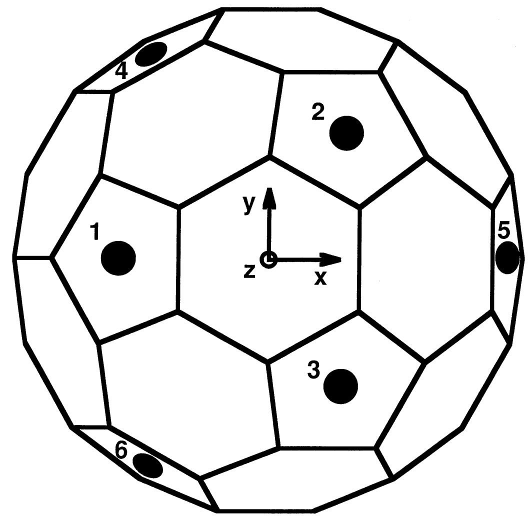
$$\vec{s}^{\,p}_{nlm}(r, \theta, \phi) = A_{nl}(r) Y_{lm}(\theta, \phi)\hat{n} - B_{nl}(r) i\, \hat{n} \times \vec{L}\, Y_{lm}(\theta, \phi)$$

Truncated Icosahedral Gravitational wave Antenna (TIGA)

Mount 6 radial transducers on face-centers of a truncated icosahedron (Johnson & Merkowitz, 1993).

$\Rightarrow$ "Spherically symmetric" detection of the sphere.

$\Rightarrow$ Signal extracted from simple combination of outputs of 6 transducers .



Mini-Grail

68 cm spherical detector made of CuAl (6%) alloy with a mass of 1400 Kg, a resonance frequency of 2.9 kHz and a bandwidth around 230 Hz.

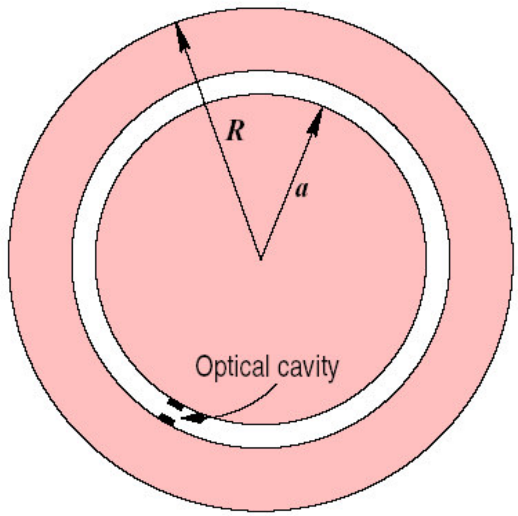
•Peak strain sensitivity of about $1.5 \times 10^{-20}\ \mathrm{Hz}^{-1/2}$.

•Sources could be non-axisymmetric instabilities in rotating single and binary neutron stars, small black-hole or neutron-star mergers

MiniGRAIL, Leiden University, Netherlands

Dual sphere detector

- Proposed by M. Cerdonio, L. Conti et. al. in 2001
- Two nested spheres
- Fabry-Perot cavity as motion sensor



Main advantages

- Wide bandwidth
- Spherical detector

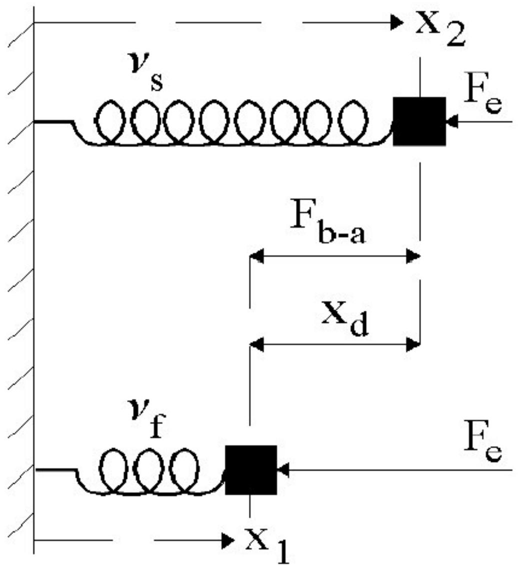
Dual sphere configuration

- Inner sphere has quadrupole mode at f
- Outer sphere at 2-3 times f

  At frequencies in between, the two spheres are driven out of phase by GW

Noise sources:

- Thermal noise
- Back-action noise
- Photon counting noise



Noise spectral density for each sensor

$$S_{uu}^{[th+ba]} = \sum_{nl} \frac{2l+1}{4\pi M} \left| A_{nl}(a) \right|^2 \left| L_{nl}(\omega) \right|^2 \left[\frac{2kT\omega_{nl}^2}{Q_{nl}\omega} + \frac{2l+1}{4\pi M} \left| A_{nl}(a) \right|^2 \sum_j \left| P_l(\mathbf{n} \cdot \mathbf{n}_j) \right|^2 S_{FF}^{ba} \right]$$

$$S_{FF}^{ba} = \frac{4}{c^2\pi^2} (1-\zeta)^2 h \nu_l F^2 P \frac{1}{1+\left(\frac{2FL_c\omega}{\pi c} \right)^2}$$

Total strain noise density

$$S_{hh}(\omega) = \frac{S_{uu,hollow}^{[th+ba]}(\omega) + S_{uu,solid}^{[th+ba]} + S_{uu}^{shot}(\omega)}{\left| u_{hollow}(\omega) - u_{solid}(\omega) \right|^2 \Big/ \left| \tilde{h}(\omega) \right|^2}$$

$$S_{uu}^{shot}(\omega) = 4 \times 10^{-33} \left[1 + \left(\frac{2FL_c\omega}{\pi c} \right)^2 \right] \frac{1}{F^2 P} \frac{m^2}{Hz}$$

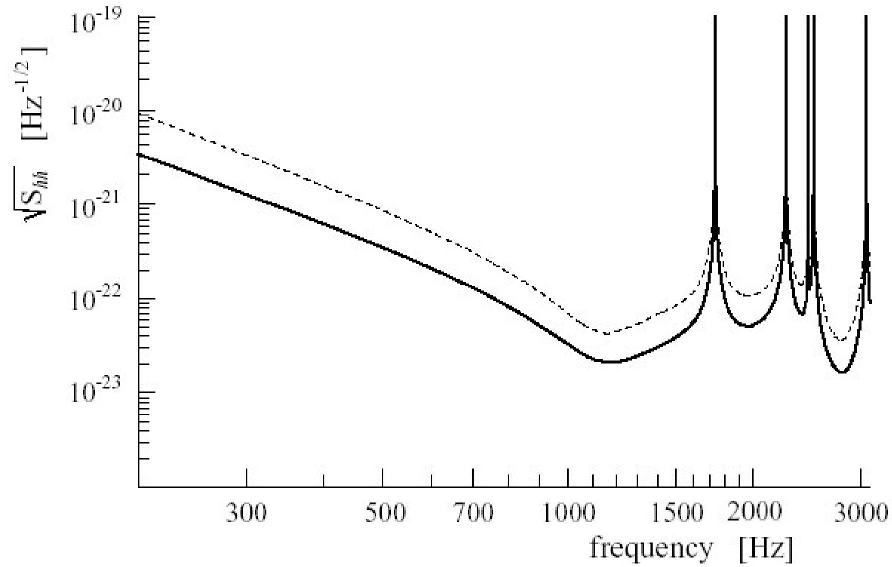
Sensitivity at Standard Quantum Limit (SQL)

Features

- $R = 0.95$ m, and $a = 0.57$ m
- Cross section proportional to ρv_s^5

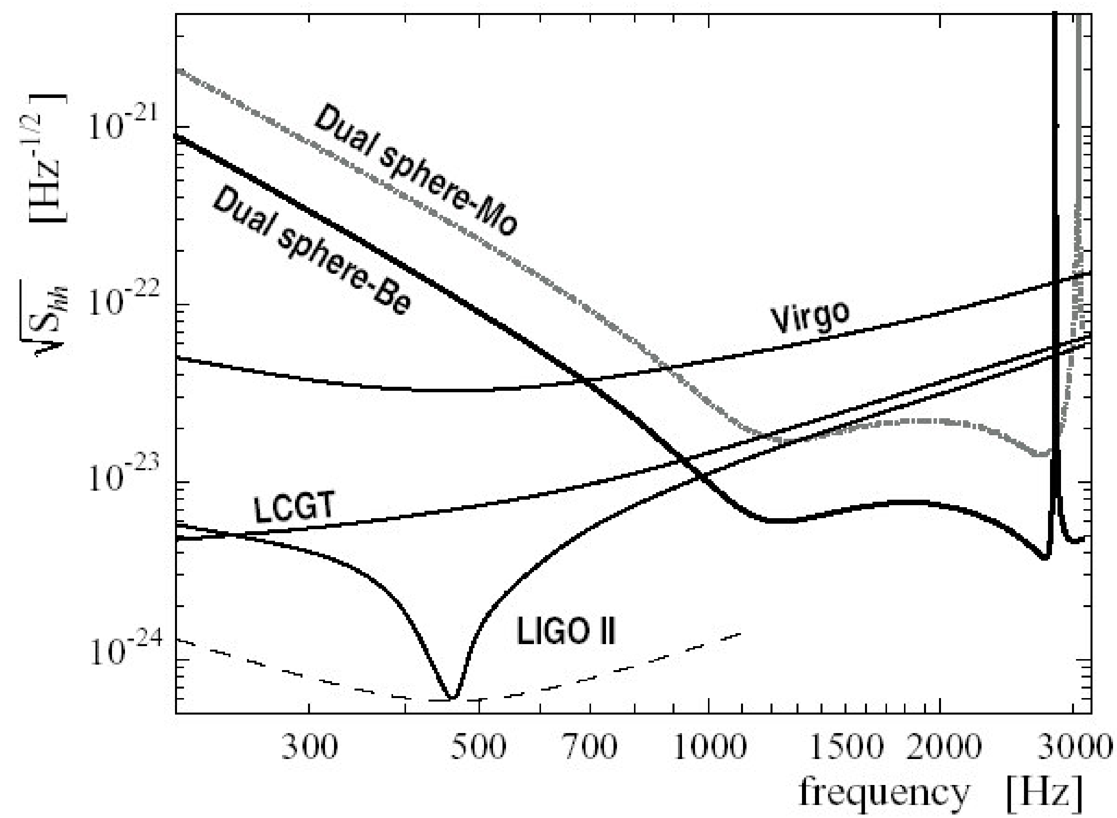
$\Rightarrow$ Molybdenum $\rho = 10000$ kg/m^3 and $v_s = 6.2$ km/s   Q ~ 20 million at $T \leq 4$ K

Input light power of 7 W , $Q/T \geq 2 \cdot 10^8$ K^{-1}



⇒ Beryllium ρ = 1900 kg/m^3 and vs= 13 km/s, Q ?
Input light power of 12 W , $Q/T \geq 2\cdot10^8$ K^{-1}

⇒ Sapphire ρ = 4000 kg/m^3 and vs= 10 km/s
$Q > 10^8$ at T < 10 K



Moon as a Spherical Detector

- Due to lack of plate tectonics, the Moon is extremely quiet seismically. The energy release per year is 10^8 times lower than the Earth.

  $\Rightarrow$ "Strong" quakes: $\sim 10^{-9}$ m Hz$^{-1/2}$ at 0.1-1 Hz, 0.5-1.3 on Richter!

  $\Rightarrow$ With the absence of ocean waves and winds, the seismic noise level between moonquakes may be extremely low.

  But how low?

- The Moon does not have atmosphere or water.

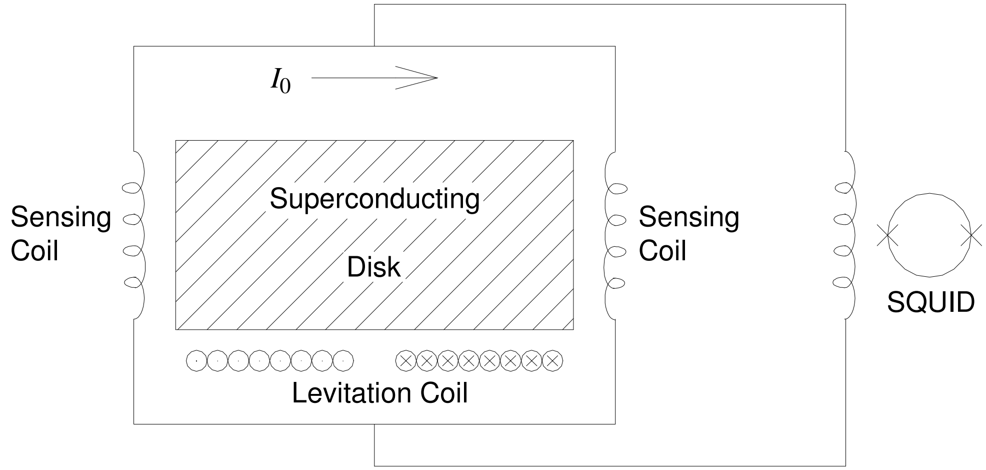
  $\Rightarrow$ The Moon is thermally quiet except at sunrise and sunset.

  $\Rightarrow$ A more stable thermal environment could be achieved by burying the instrument under the Moon dust.

- A superconducting disk is levitated magnetically.

  $\Rightarrow$ Almost free horizontally.

- Horizontal displacement is sensed in two directions with a superconducting circuit.



- Intrinsic displacement noise:

$$\boxed{S_x(f) = \frac{4}{m\omega^4}\left\{k_B T\frac{\omega_0}{Q} + E_A(f)\frac{1}{2\beta\eta\omega_0^2}\left[(\omega_0^2-\omega^2)^2 + \left(\frac{\omega_0\omega}{Q}\right)^2\right]\right\}}$$

- With $m = 100$ kg, $f_0 = 0.3$ Hz, $T = 2$ K, $Q = 10^7$, $2\beta\eta = 0.5$, $E_A(f) = 10^{-31}$ J Hz^{-1} $(1 + 0.1\ \text{Hz} / f)$,

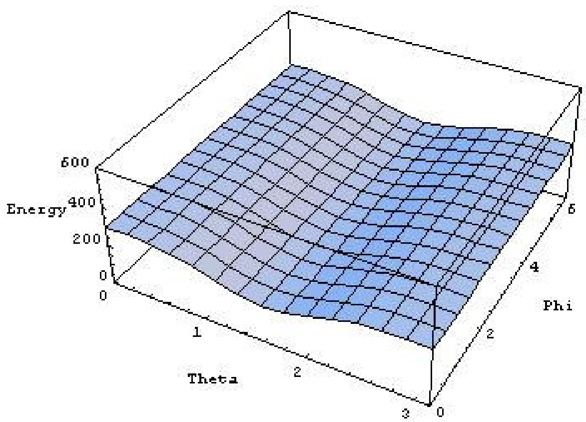
  $S_x^{1/2}(f) \approx 10^{-16}$ m Hz$^{-1/2}$, $f = 0.3$ Hz

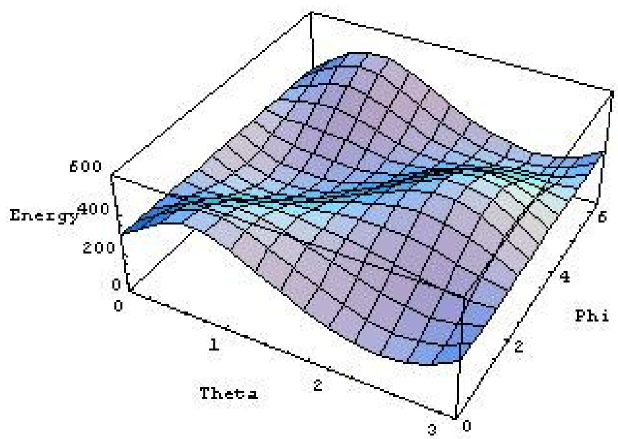
  (10^6 times more sensitive than the lunar seismometers)

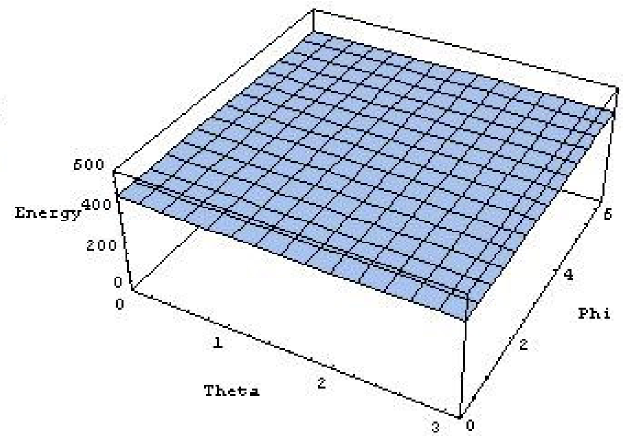
Resonant spherical detector

- Moon's quadrupole modes (0.001 to 10 Hz) are monitored.
- Directionality of various configurations:

<u>Triangle at great circle</u>   <u>Tetrahedral configuration</u>   <u>Icosahedral configuration</u>





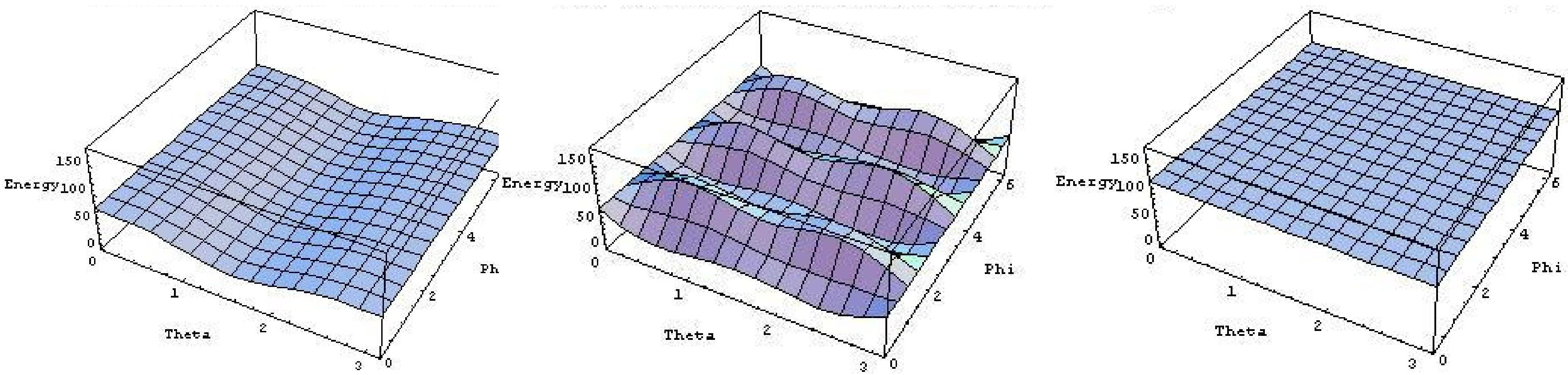


- 6 horizontal motion sensors in truncated icosahedral configuration
  - ⇒ Full-sky coverage with uniform cross section.
  - ⇒ Detection of the source direction and wave polarization.
  - ⇒ Discrimination against seismic and other disturbances.

Wideband "spherical" detector

- Wideband detection against the rigid Moon (< 0.001 Hz).
- Directionality of various configurations:

Triangle at great circle   Tetrahedral configuration   Icosahedral configuration



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  - ⇒ Discrimination against seismic and other disturbances.

Thank you!

References

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