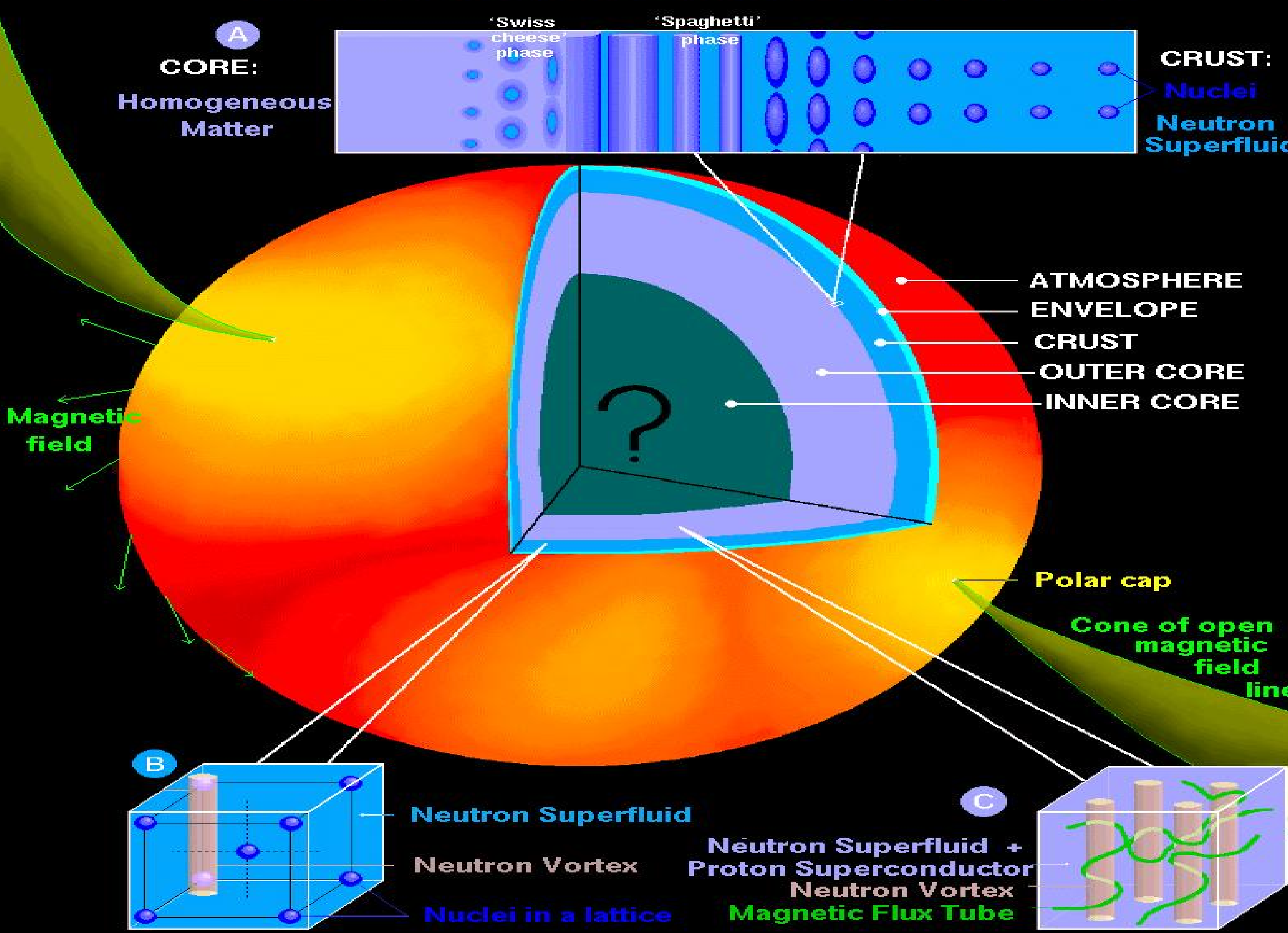
Magnetar Oscillations:

Shedding Light on the Neutron Star Equation of State

Stephen R. Schwartz

A NEUTRON STAR: SURFACE and INTERIOR



State of Nuclei Below Neutron Drip

EQUILIBRIUM NUCLEI BELOW NEUTRON DRIP

Nucleus	b (MeV)	Z/A	ρ_{max} (g cm^{-3})	μ_e (MeV)	$\Delta\rho/\rho$ (%)
^{56}Fe........	8.7905	0.4643	8.1×10^{6}	0.95	2.9
^{62}Ni........	8.7947	0.4516	2.7×10^{8}	2.6	3.1
^{64}Ni........	8.7777	0.4375	1.2×10^{9}	4.2	7.9
^{84}Se........	8.6797	0.4048	8.2×10^{9}	7.7	3.5
^{82}Ge.......	8.5964	0.3902	2.2×10^{10}	10.6	3.8
^{80}Zn........	8.4675	0.3750	4.8×10^{10}	13.6	4.1
^{78}Ni........	8.2873	0.3590	1.6×10^{11}	20.0	4.6
^{76}Fe........	7.9967	0.3421	1.8×10^{11}	20.2	2.2
^{124}Mo......	7.8577	0.3387	1.9×10^{11}	20.5	3.1
^{122}Zr.......	7.6705	0.3279	2.7×10^{11}	22.9	3.3
^{120}Sr.......	7.4522	0.3166	3.7×10^{11}	25.2	3.5
^{118}Kr.......	7.2002	0.3051	(4.3×10^{11})	(26.2)	...

NOTE.—b is the binding energy per nucleon; ρ_{max} is the maximum density at which the nuclide is present; μ_e is the electron chemical potential at that density, and $\Delta\rho/\rho$ is the fractional increase in the mass density in the transition to the next nuclide. The value of $\rho_{max} = 4.3 \times 10^{11}$ g cm^{-3} is the density at which neutron drip begins.

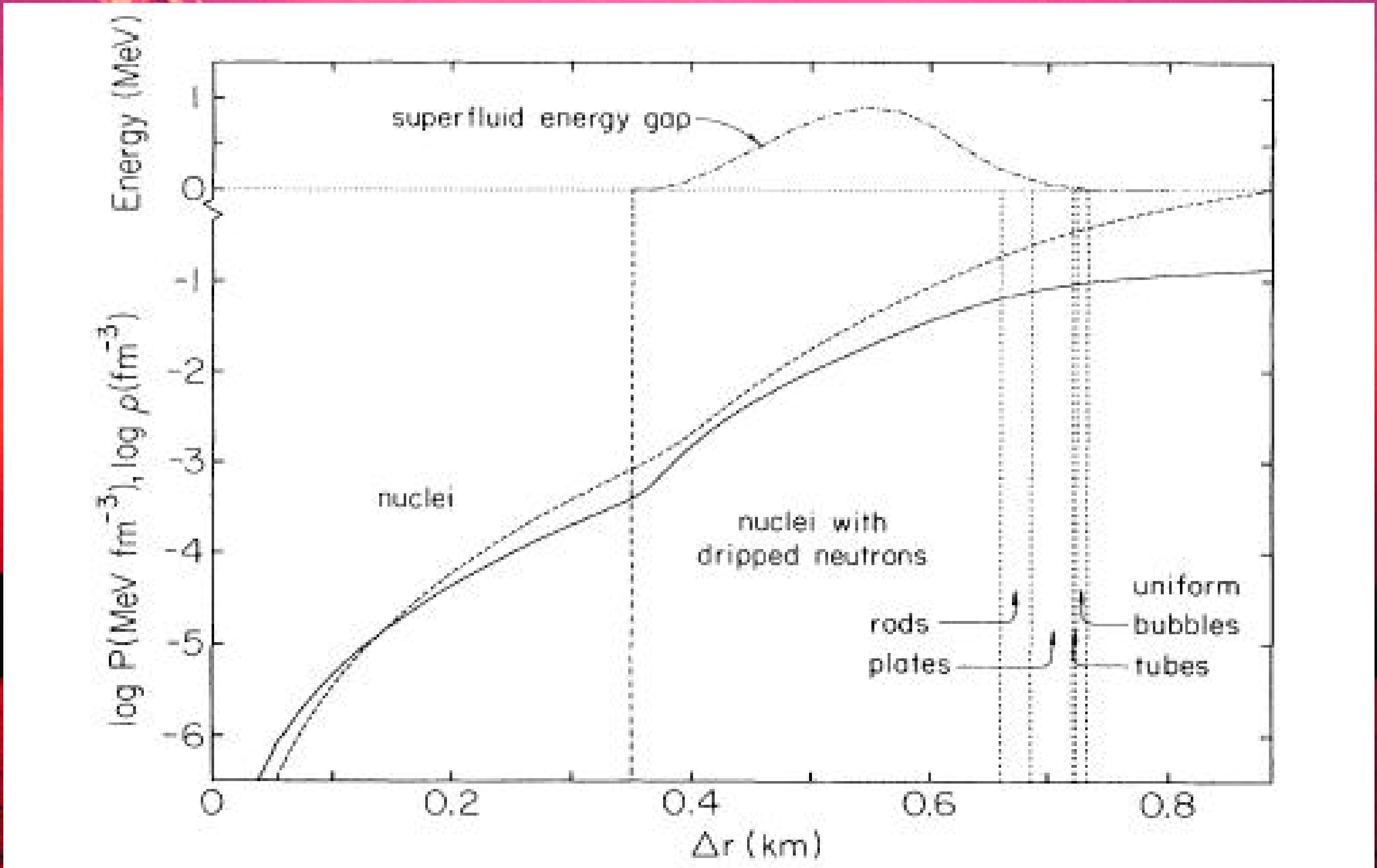
BPS (1971)

State of Nuclei Above Neutron Drip

μ_n [MeV]	μ_p [MeV]	n_b [cm^{-3}]	Element	Z	N
0.2	-26.8	2.79×10^{35}	^{180}Zr	40	140
0.3	-29.4	4×10^{35}	^{200}Zr	40	160
0.6	-29.5	6×10^{35}	^{250}Zr	40	210
1.0	-28.5	8.79×10^{35}	^{320}Zr	40	280
1.4	-29.4	1.59×10^{36}	^{500}Zr	40	460
2.6	-33.6	3.73×10^{36}	^{950}Sn	50	900
3.3	-34.5	5.77×10^{36}	^{1100}Sn	50	1050
4.2	-35.8	8.91×10^{36}	^{1350}Sn	50	1300
6.5	-43.6	2.04×10^{36}	^{1800}Sn	50	1750
10.9	-54.0	4.75×10^{37}	^{1500}Zr	40	1460
15	-68.3	7.89×10^{37}	^{980}Ge	32	950

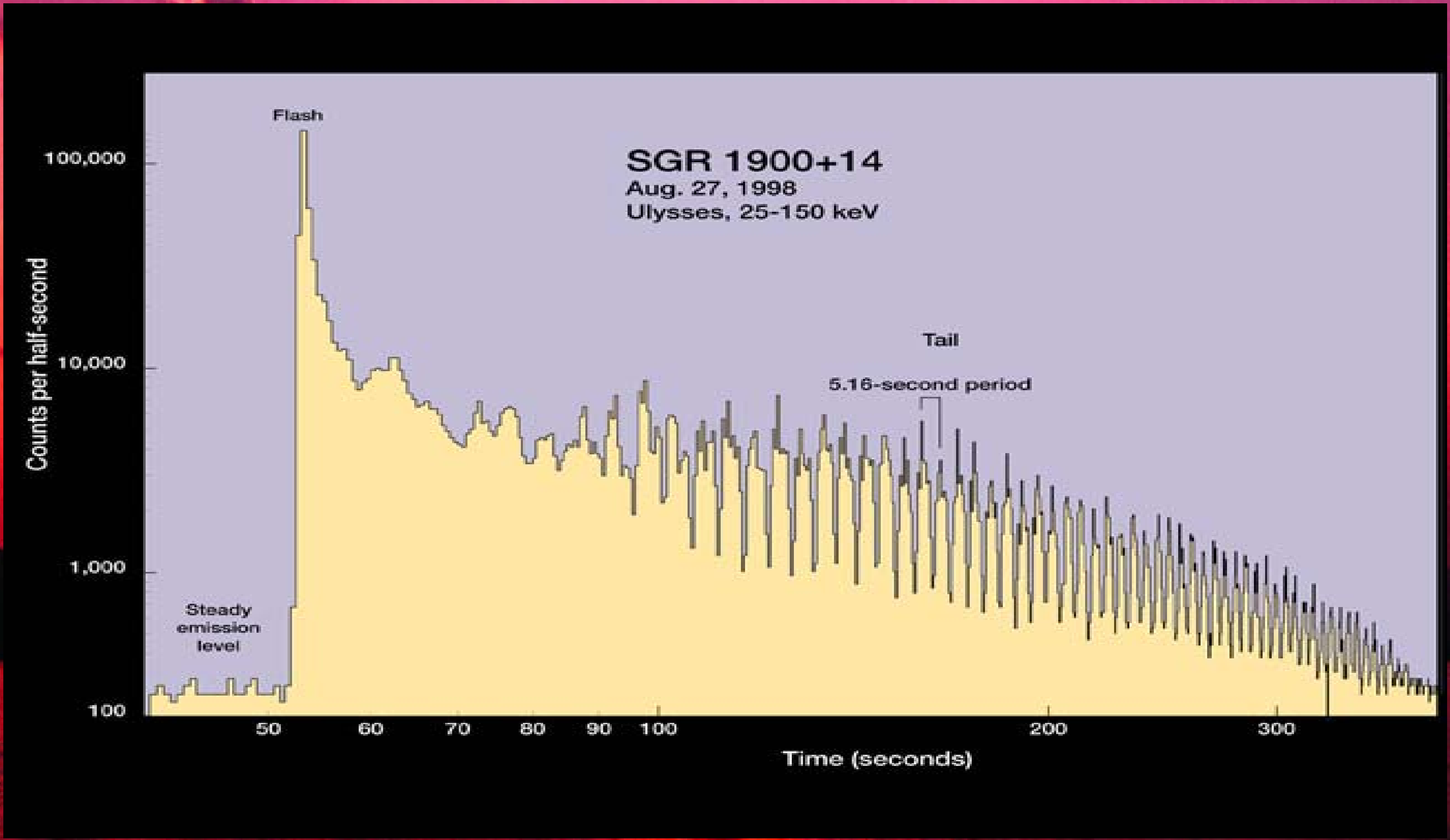
Negele (1974)

Nuclear State Within in Neutron Star Crust



LRP (1993)

γ-ray Light Curve of Magnetar Giant Flare from SGR 1900+14



Duncan (2003)

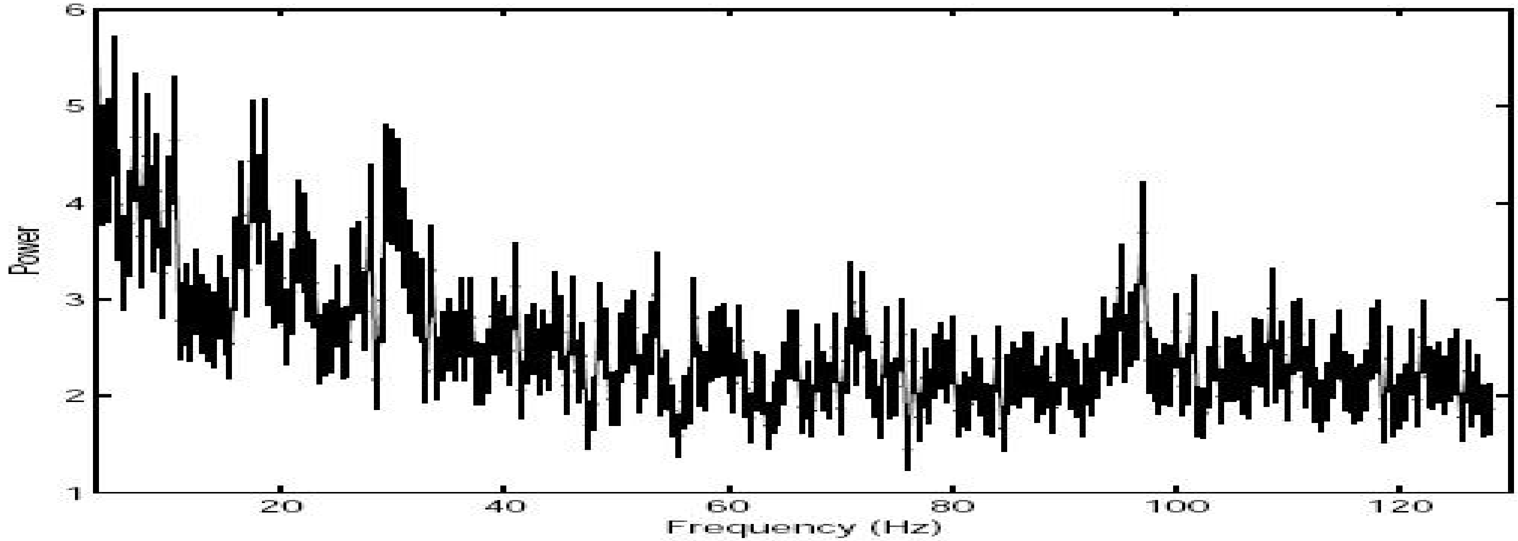
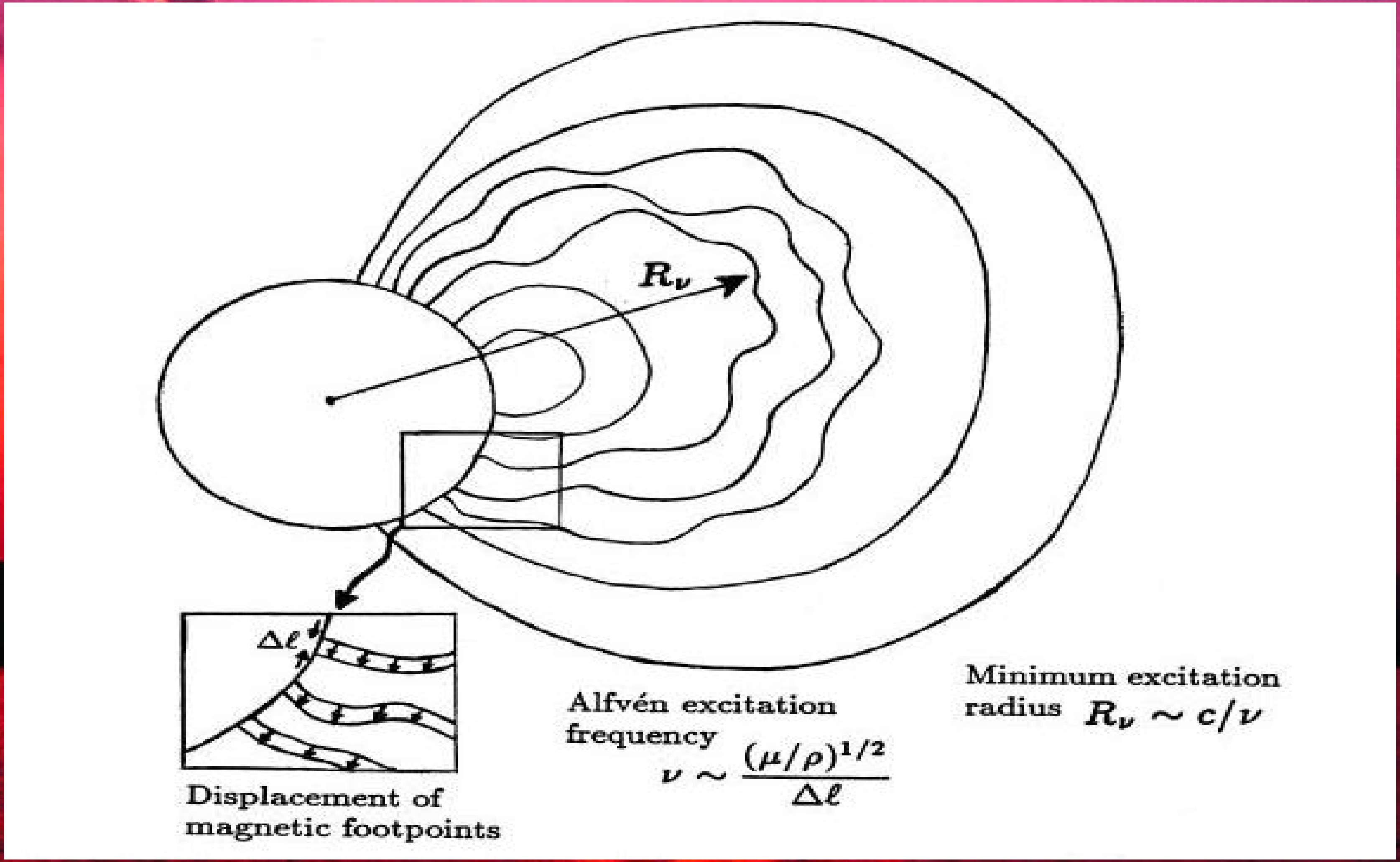


FIG. 3.— Power spectrum accumulated from data in the time interval 200–300 s.

SGR 1806-20	SGR 1900+14	Torsional shear mode identification
18*		
26*		
30*	28	$n = 0,\ l = 2$
	53	$n = 0,\ l = 4$
92*	84	$n = 0,\ l = 6$
150		$n = 0,\ l = 10$
	155	$n = 0,\ l = 11$
625*		$n = 1$
1840		$n = 3$

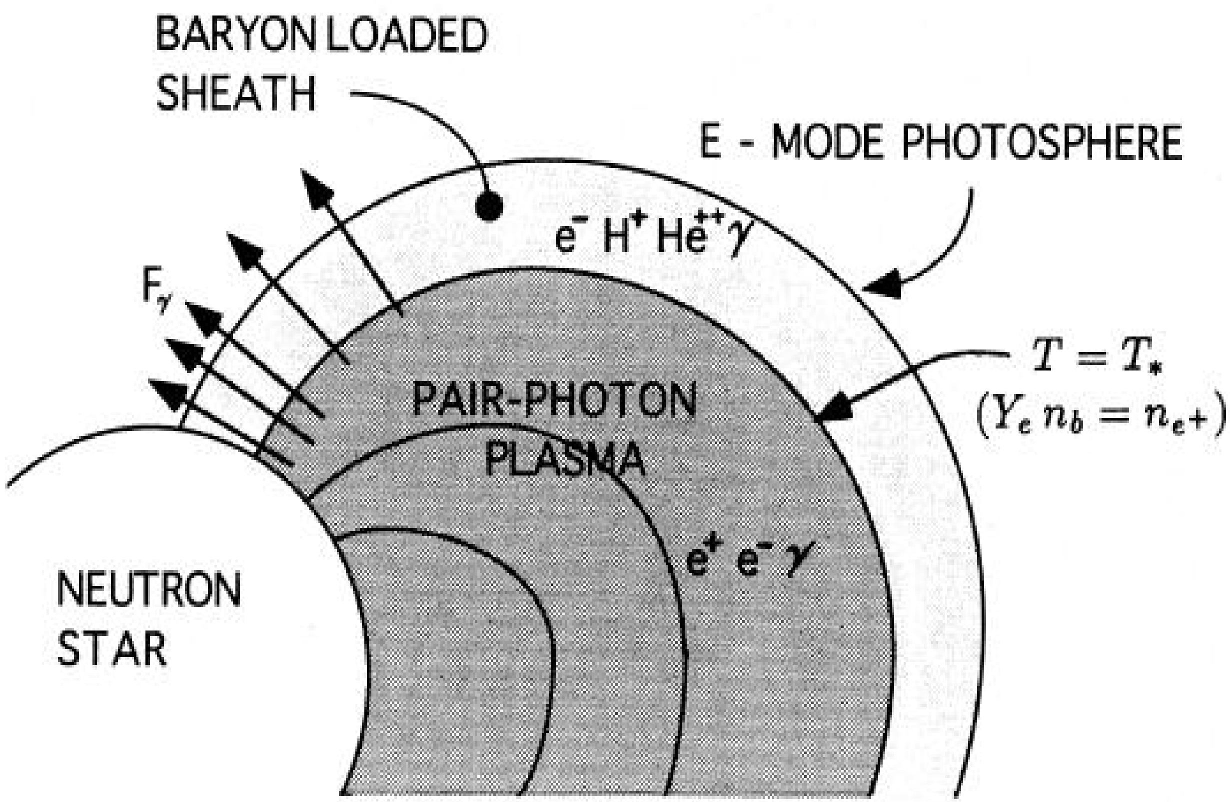
W & S (2007)

Sudden Displacement Sends Pulse Into the Surrounding Magnetosphere



Duncan (2003)

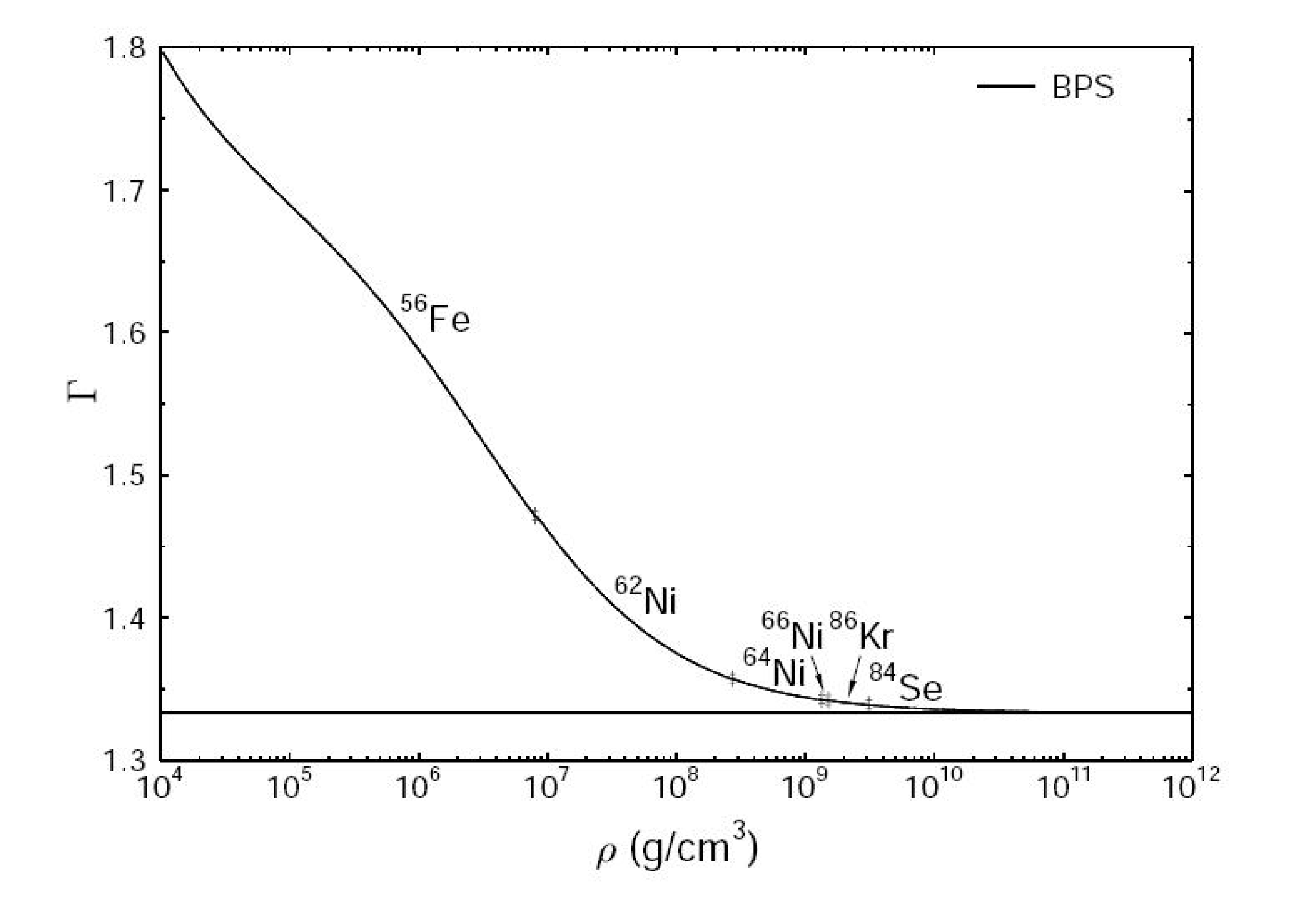
Plasma Generation in Magnetosphere



The deposition of $\geq 10^{38}$–10^{41} erg in the magnetosphere of a neutron star is sufficient to generate an optically thick photon–electron–positron plasma. The surface of this plasma is congruent with the magnetic field lines. The surface layers lose heat by radiative diffusion, and the scattering opacity in these layers is dominated by a small contaminant of ions and electrons blown off the neutron star surface.

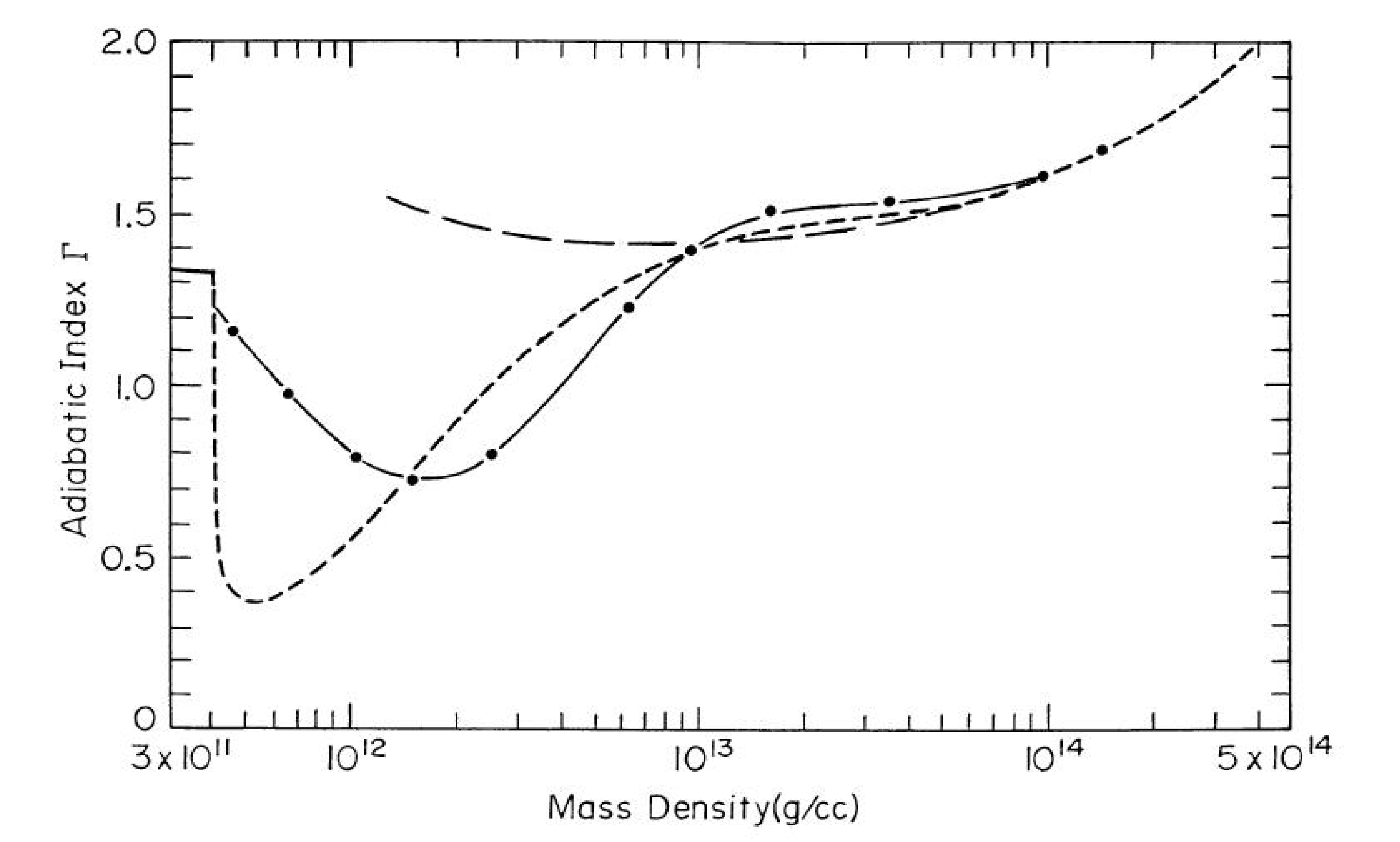
Duncan (2003)

Adiabatic Indices at Varying Densities Below Neutron Drip



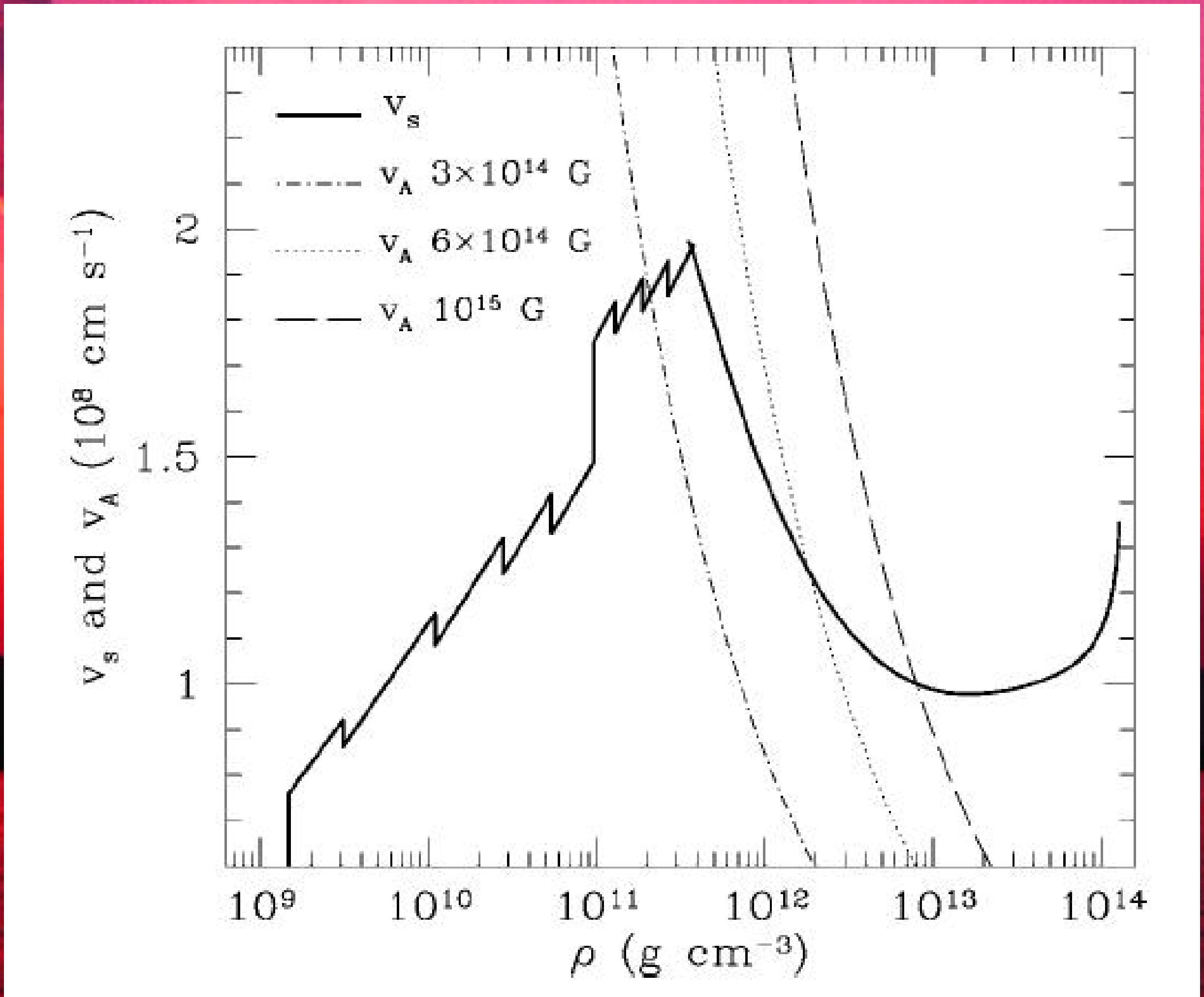
Negele (2007)

Adiabatic Indices at Varying Densities Above Neutron Drip

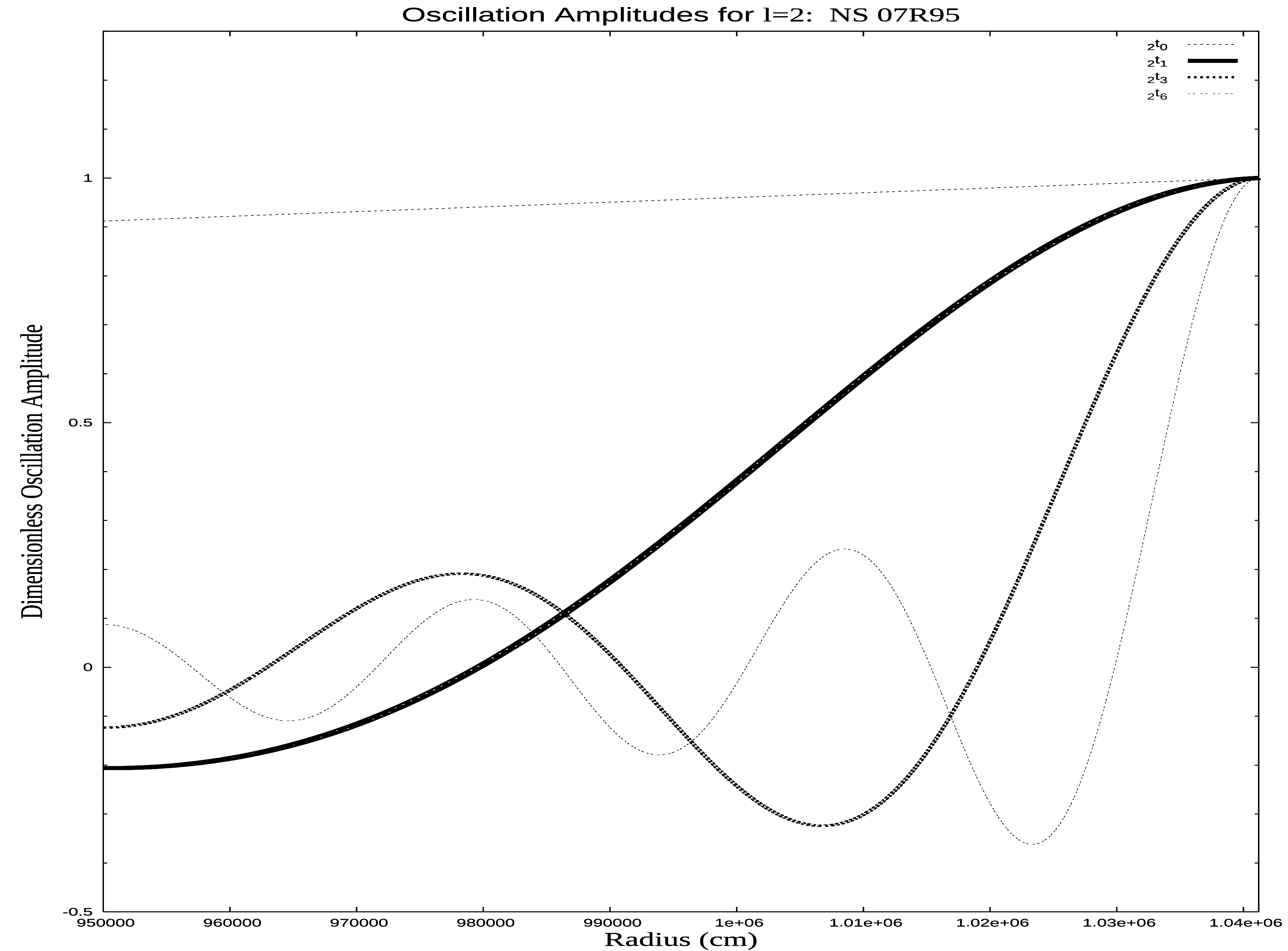


Negele (1974)

Shear Speed Nearly Constant Throughout Crust



Piro (2005)


Oscillation Amplitudes for l=2: NS 07R95
$_2t_0$
$_2t_1$
$_2t_3$
$_2t_6$
1
0.5
0
-0.5
950000
960000
970000
980000
990000
1e+06
1.01e+06
1.02e+06
1.03e+06
1.04e+06
Radius (cm)
Dimensionless Oscillation Amplitude

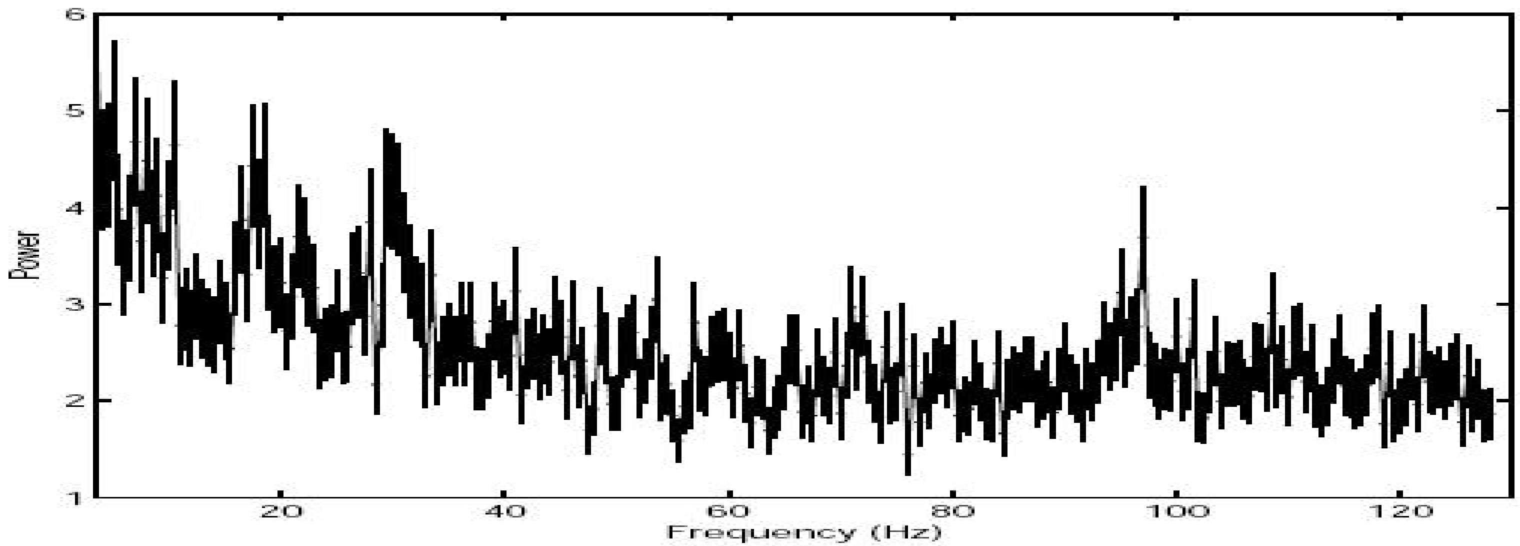
NS 11R9

Toroidal Mode Frequency

- **Thickness:** 8.24%
- **Mass Ratio:** 4.09%
- **Surface Redshift:** ~ 0.27
- **Radius Estimate:** ~ 9km

Toroidal Mode Frequencies of Model NS 11R9 (Hz)

	${}_1t_0$	${}_1t_0$	${}_1t_1$	${}_1t_1$
	Redshifted Frame	**NS Surface**	**Redshifted Frame**	**NS Surface**
${}_1t_n$	-	-	625.061772	790.591627
${}_2t_n$	29.674249	37.532631	625.746418	791.457582
${}_3t_n$	46.918967	59.344123	626.771992	792.754751
${}_4t_n$	62.948149	79.618178	628.136832	794.481029
${}_5t_n$	78.509744	99.300820	629.838746	796.633646
${}_6t_n$	93.836531	118.686471	631.875019	799.209169
${}_7t_n$	109.027436	137.900255	634.242453	802.203550
${}_8t_n$	124.132206	157.005094	636.937371	805.612141
${}_9t_n$	139.178736	176.036271	639.955659	809.429737
${}_{10}t_n$	154.183923	195.015155	643.292790	813.650612
${}_{11}t_n$	169.158612	213.955465	646.943856	818.268559
${}_{12}t_n$	184.110087	232.866414	650.903605	823.276936
	${}_1t_2$	${}_1t_2$	${}_1t_3$	${}_1t_3$
	Redshifted Frame	**NS Surface**	**Redshifted Frame**	**NS Surface**
${}_1t_n$	1150.276329	1454.894333	1670.481165	2112.860650
${}_2t_n$	1150.648802	1455.365452	1670.737735	2113.185165
${}_3t_n$	1151.207295	1456.071845	1671.122519	2113.671848
${}_4t_n$	1151.951531	1457.013172	1671.635425	2114.320583
${}_5t_n$	1152.881155	1458.188980	1672.276338	2115.131224
${}_6t_n$	1153.995717	1459.598703	1673.045110	2116.103583
${}_7t_n$	1155.294687	1461.241667	1673.941567	2117.237441
${}_8t_n$	1156.777442	1463.117089	1674.965501	2118.532535
${}_9t_n$	1158.443282	1465.224079	1676.116684	2119.988576
${}_{10}t_n$	1160.291420	1467.561643	1677.394852	2121.605230
${}_{11}t_n$	1162.320989	1470.128687	1678.799715	2123.382131
${}_{12}t_n$	1164.531045	1472.924012	1680.330958	2125.318881



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W & S (2007)

NS 24R115

Toroidal Mode Frequency

- Redshift neglected model yields NS with unrealistically high densities
- Information on crust thickness still informative

	${}_1t_0$	${}_1t_1$
${}_1t_n$	-	623.549219
${}_2t_n$	29.308363	624.218862
${}_3t_n$	46.340456	625.221987
${}_4t_n$	62.172008	626.557002
${}_5t_n$	77.541746	628.221801
${}_6t_n$	92.679580	630.213783
${}_7t_n$	107.683218	632.529872
${}_8t_n$	122.601797	635.166543
${}_9t_n$	137.462867	638.119839
${}_{10}t_n$	152.283122	641.385414
${}_{11}t_n$	167.073274	644.958547
${}_{12}t_n$	181.840518	648.834185

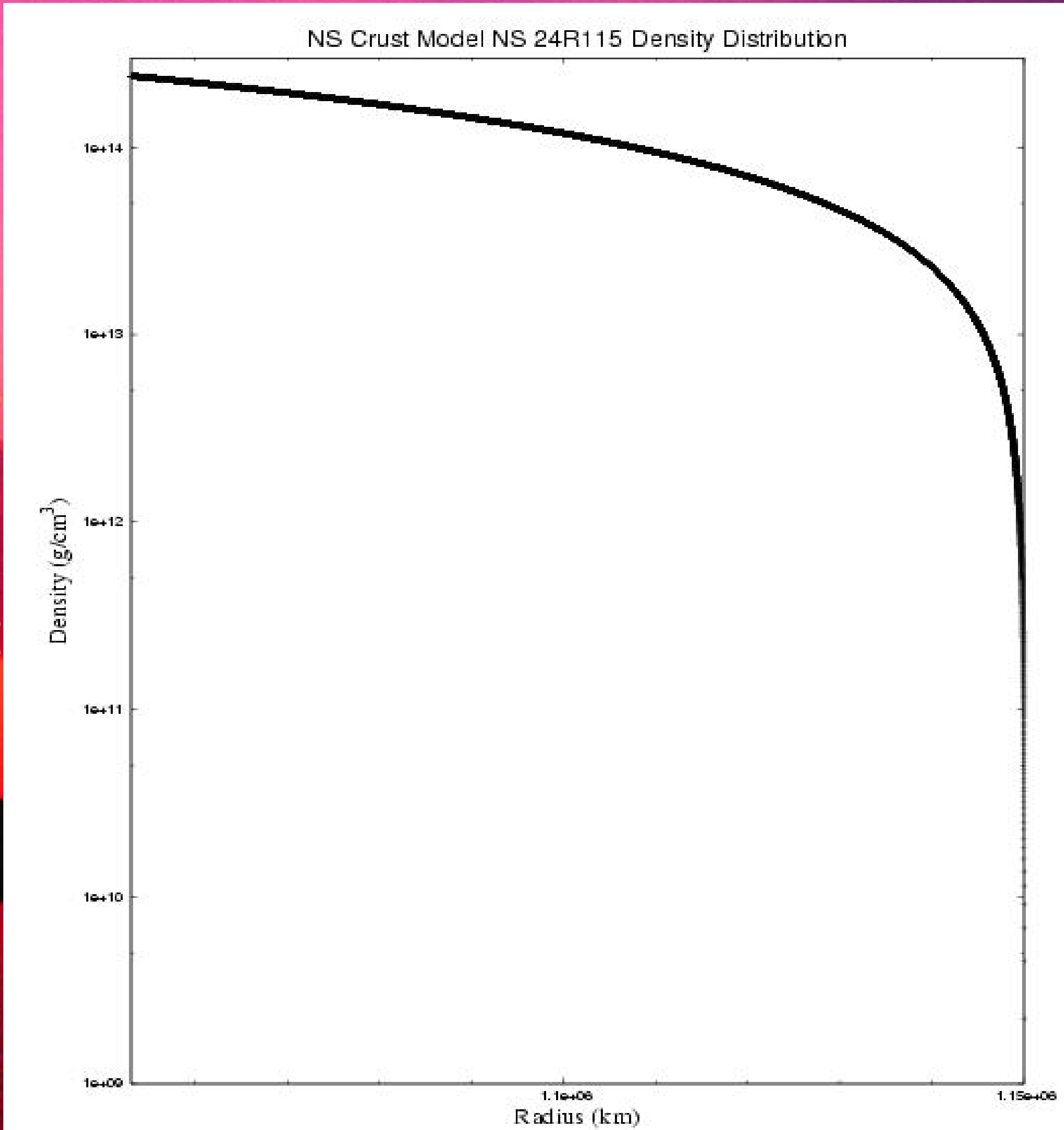
	${}_1t_2$	${}_1t_3$
${}_1t_n$	1147.44418	1666.34735
${}_2t_n$	1147.80851	1666.59831
${}_3t_n$	1148.35479	1666.97468
${}_4t_n$	1149.08276	1667.47638
${}_5t_n$	1149.99208	1668.10329
${}_6t_n$	1151.08231	1668.85527
${}_7t_n$	1152.35295	1669.73215
${}_8t_n$	1153.80341	1670.73374
${}_9t_n$	1155.43300	1671.85982
${}_{10}t_n$	1157.24097	1673.11013
${}_{11}t_n$	1159.22650	1674.48439
${}_{12}t_n$	1161.38867	1675.98231

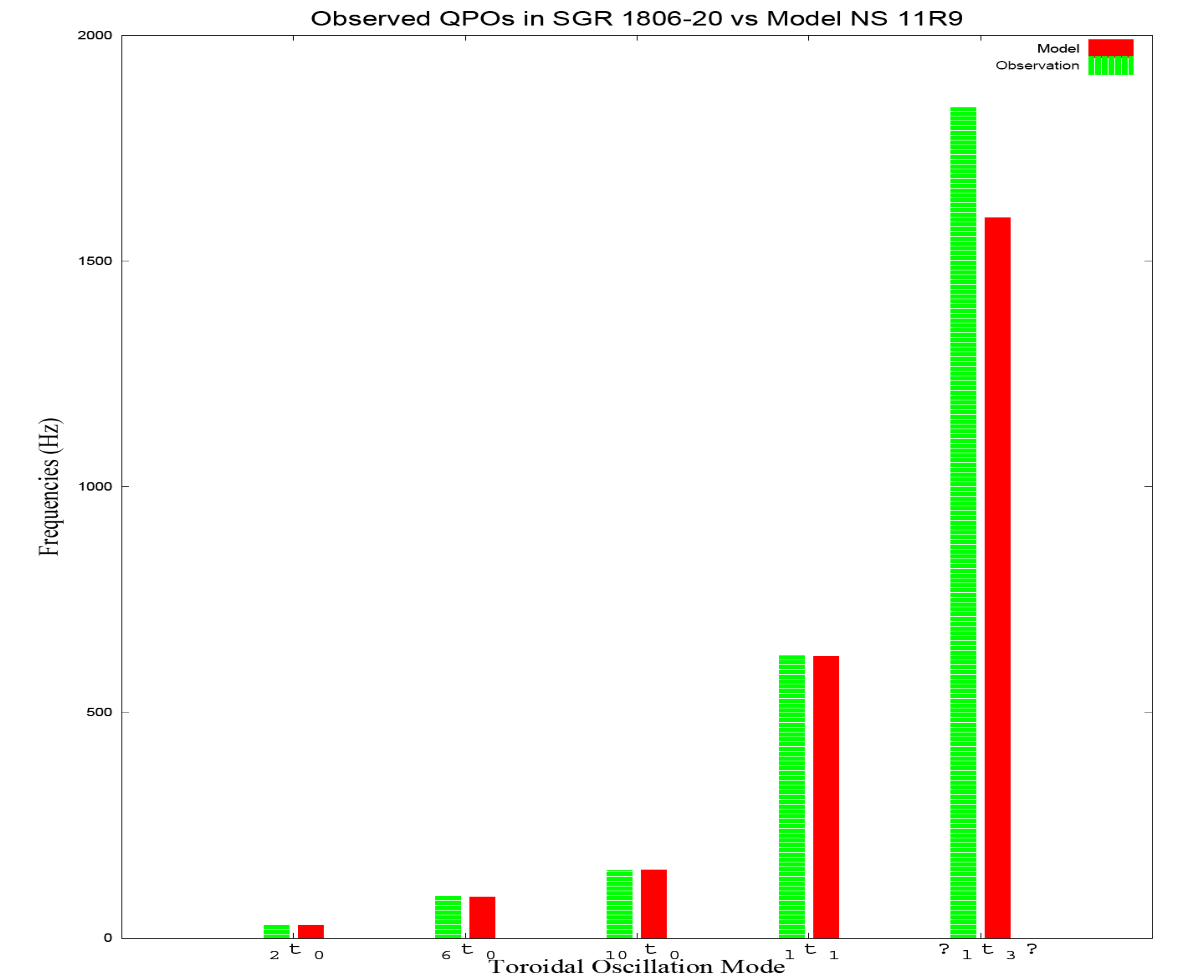
- **Thickness:**

  8.43%

- **Mass Ratio:**

  3.49%





Observed QPOs in SGR 1806-20 vs Model NS 11R9
2000
1500
1000
500
0
Frequencies (Hz)
Model
Observation
2 t 0
6 t 0
10 t 0
1 t 1
? 1 t 3 ?
Toroidal Oscillation Mode

Some Thoughts

- Crust turned out to be about half as thick as expected
- Would expect that this could be slightly mitigated by adding the effects of the magnetic field into the pulsation equation
- Waiting for more flares!

References

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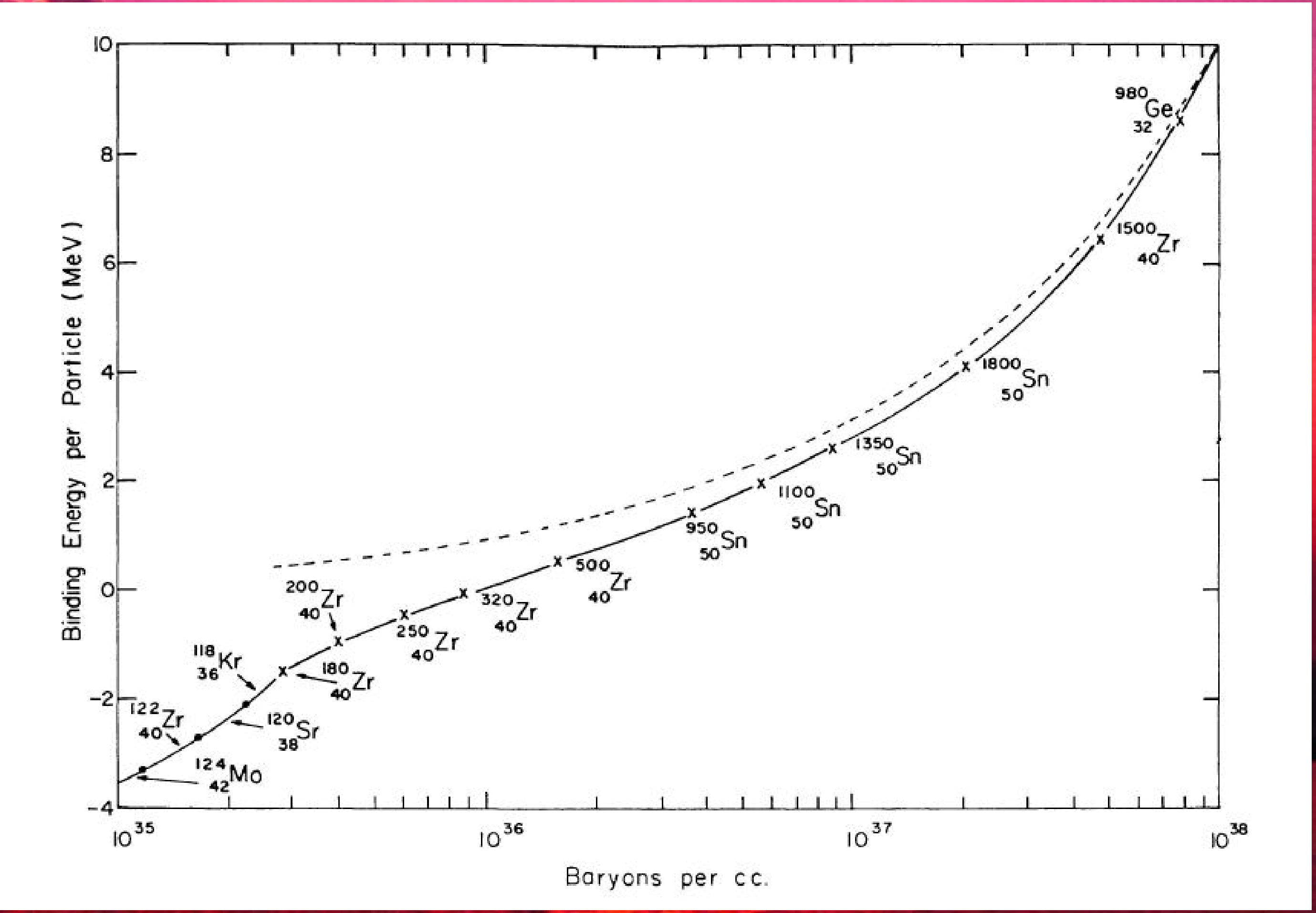
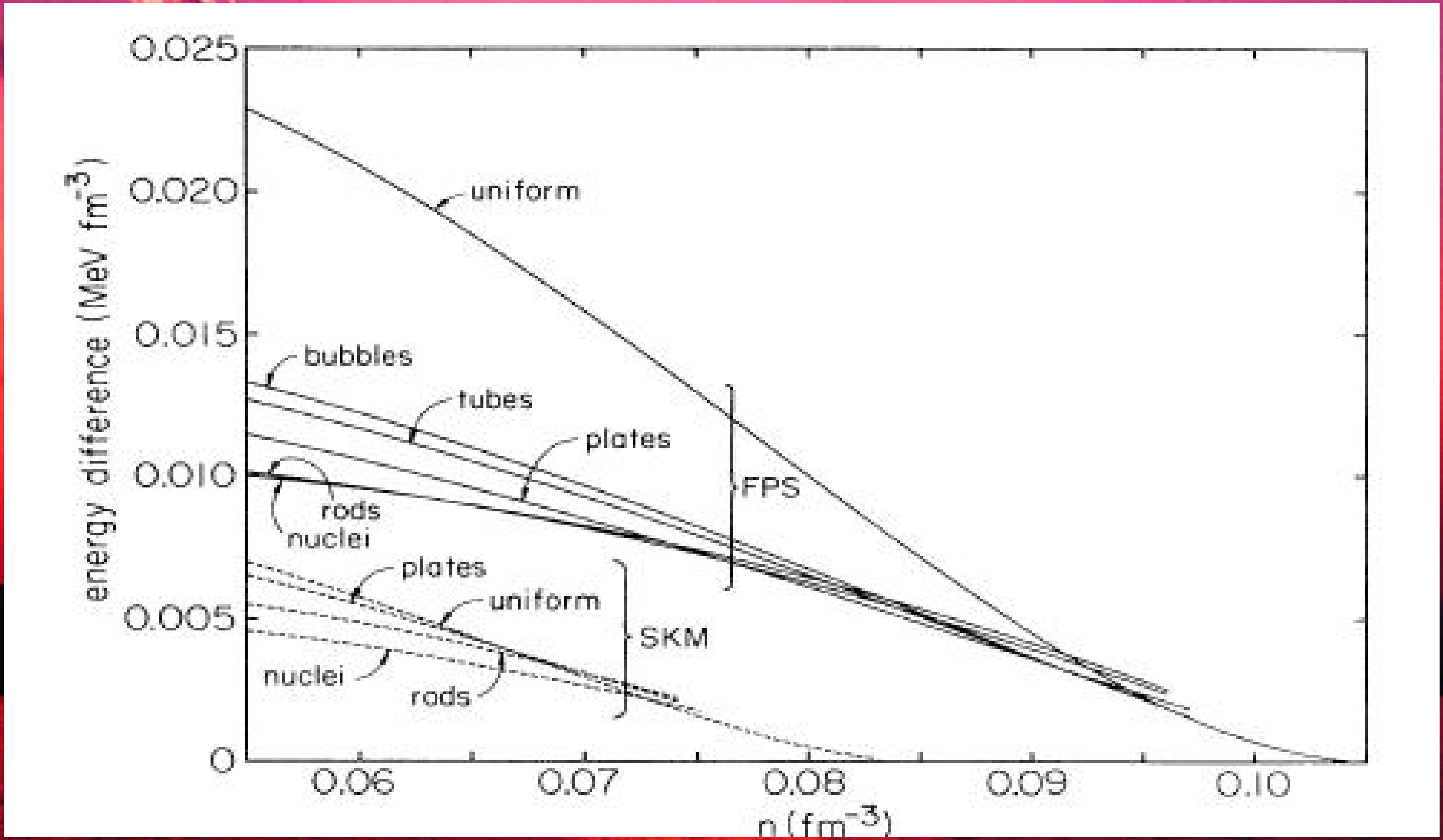
Binding Energy of Extremely Large Nuclei



Negele (1974)

Nuclear State Within Neutron Star Crust



LRP (1993)