Experimental Tests of Local Lorentz Invariance

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Physics 798G

(Submitted March 15, 2007)

Lorentz invariance, fundamental to Relativity and the Standard Model, has been thoroughly investigated both theoretically and experimentally in the past 15 years. This investigation, motivated in large by attempts to unify gravity with the Standard Model through quantum gravity, has led to strict constraints on the nature and size of a violation. A historical perspective of Lorentz's work, a brief discussion of the theoretical framework, and a discussion of high-precision tests and their results are presented.

I. INTRODUCTION

In the late 19^{th} century, Albert Michelson and Edward Morley performed the most famous measurement of zero in the history of physics[1]. The prevailing theory of the day supposed a medium for light propagation, called "luminiferous ether," in a similar sense to how sound needs air to propagate. Michelson and Morley recognized that the Earth should be subject to "ether winds" that changed according to the rotation of the Earth on its axis and around the Sun. The speed of light moving with or against this wind would be different, so this should be measurable.

Michelson created an interferometer by sending a source of white light through a half-silvered mirror, then allowing the two beams to travel some distance along perpendicular arms, which were then reflected back along their paths and recombined on the far side of the splitter in an eyepiece. This formed an interference pattern, which was related to the length of the arms, or the speed of light in each arm. Since the arms were perpendicular, the "ether wind" should have a different effect on the light in each arm, causing a shift in the interference fringes. By mounting the interferometer on a marble table floating in a vat of mercury, Michelson and Morley calculated their experimental sensitivity would allow them to measure a shift of $1/100^{th}$ of a fringe. The shift was expected to be four-tenths of a fringe if the ether was stationary with respect to the Sun. They saw a shift consistent with zero.

Michelson and Morley reported their results in an 1887 American Journal of Physics article[2], stating, "*The relative velocity of the earth and the ether is probably less than one sixth the earth's orbital velocity, and certainly less than one-fourth*."

Stokes then presented a theory that supposed the ether was at rest on the earth's surface, which required a velocity potential. Michelson and Morley went on to say, "*If now it were legitimate to conclude from the present work that the ether is at rest with regard to the earth's surface, according to Lorentz there could not be a velocity potential, and his own theory also fails*."

Hendrik Lorentz[3] continued to search for an explanation of the Michelson-Morley experiment. In 1892 he proposed the idea that bodies contract in the direction of motion; length contraction. He also introduced the idea of local time, which described the relativity of simultaneity between reference frames in relative motion, and time dilation. Lorentz published in 1905 what Henri Poincaré called the "Lorentz transformations." Paul Langevin said of this publication[4], "*It is the great merit of H. A. Lorentz to have seen that the fundamental equations of electromagnetism admit a group of transformations which enables them to have the same form when one passes from one frame of reference to another; this new transformation has the most profound implications for the transformations of space and time*."

The Lorentz transformations removed contradictions between electromagnetism and classical mechanics regarding the transformation of fields, and they were the mathematical foundation of Einstein's Special Relativity. In fact, "*Until the first World War, Lorentz's and Einstein's theories were regarded as different forms of the same idea, but Lorentz, having priority and being a more established figure speaking a more familiar language, was credited with it*."[5]

While the Standard Model and General Relativity are Lorentz covariant (invariant under Lorentz transformations), they are incompatible with each other. A number of theories attempting to incorporate gravity with the three forces of the Standard Model may contain hidden or small corrections that violate Lorentz invariance and *CPT* symmetry[6]. Over the last fifteen years, Alan Kostelecky[7] has developed the Standard Model Extension (SME), which is a modification of the

Standard Model of particle physics and Einstein's theory of General Relativity. This theory provides a quantitative description of Lorentz and *CPT* violations by developing a set of coefficients that can be experimentally restricted, thus ruling out theoretical models that make predictions concerning the size of these violations.

In a way, the physics world has come full circle since Michelson and Morley's 1887 experiment. Physicists today are essentially searching for evidence of an "ether wind," for evidence of a preferred reference frame. They continue to measure zero to greater and greater precision, setting constraints on the coefficients in the SME, and restricting the size of Lorentz and *CPT* violations. These experiments cover the full field of physics, from neutrino oscillations to proton-antiproton mass measurements to atomic physics.

II. THEORETICAL BACKGROUND

Lorentz transformations consist of rotations and boosts (changes in velocity). There are three rotations, one around each spatial direction, and there are three boosts, one in each spatial direction.

A physical quantity unchanged under a Lorentz transformation is labeled Lorentz covariant. These quantities consist of scalars, four-vectors, four-tensors, and spinors. Examples of scalars include space-time interval, rest mass, and proper time. Four-velocity and four-momentum are Lorentz covariant four-vectors, as the Kronecker delta, the Minkowski metric, and the electromagnetic field tensor are examples of four-tensors. Spinors are found in Dirac's relativistic theory of spin, and examples are the Majorana and the Dirac spinors, or more simply, the Pauli matrices.

An equation is Lorentz covariant if the equation is written in Lorentz covariant quantities. The result is that if the equation is true in one inertial frame, it is true in all inertial frames. All equations where Lorentz transformations have replaced Galilean transformations are Lorentz covariant, such as electromagnetism and relativity.

Whereas Lorentz covariance is global, the addition of gravity (Special Relativity) requires that Lorentz covariance only apply locally, in an infinitesimally small region surrounding a point of space-time.

In order to understand what a Lorentz violation might be, it is important to recognize the distinction between observer and particle Lorentz transformations. An observer transformation merely states that the laws of physics do not depend on orientation. A person sitting on a bus obeys the same laws of physics as one standing at the bus stop. However, if the person on the bus stands up in anticipation of an upcoming stop, and starts walking forward on the bus, it has become a particle Lorentz transformation, since the person is moving with respect to a fixed reference frame, the bus.

Absent of Lorentz violation, the moving bus passenger is simply in a third inertial frame, and the particle and observer Lorentz transformations are the same. Yet, if there is a Lorentz violation, the laws of physics could be different for a moving observer versus a stationary one.

In Kostelecky's Standard Model Extension (SME), the observer Lorentz invariance remains valid. It is only when the particle fields are rotated or boosted relative to the vacuum expectation value that apparent violations can occur. A particle moving inside a crystal is analogous to a particle moving inside the vacuum with a spontaneous Lorentz violation. The particle's rotation and boost symmetry is broken while traveling through the crystal, not because there is a fundamental problem with theory, but because of the background fields from the crystal. Similarly, the SME extension contains all the properties of the usual Standard Model and General Relativity, except it allows for the breaking of Lorentz and *CPT* symmetry. It adds all possible coordinate-invariant operators formed by Standard Model and gravitational fields combining with couplings having Lorentz indices. It makes no prediction as to the magnitude of the coefficients on these effects, and does not identify a best test for finding Lorentz violation. Furthermore, there are different kinds of coefficients that are sensitive to different types of experiments. Therefore, precision measurements must be made in a wide range of experiments in order to set limits on the coefficients.

These tests include clock comparison experiments[8,9,10,11], QED tests in Penning traps[12,13], photon-based[14,15] experiments, Neutral-B[16], Neutral-D[17], neutrino[18], and kaon[19] oscillation measurements, spin-polarized torsion pendulum experiments[20], and muon-based[21] experiments. Each of these experiments set limits on a particular set of coefficients in the SME. I will explain one experiment in detail, and summarize the results and limits to SME coefficients of the other experiments more briefly.

III. CLOCK COMPARISON EXPERIMENT

Clock-comparison experiments are sensitive probes of rotation invariance (and therefore Lorentz invariance), mainly by observing the frequency of the clock as its orientation changes with respect to some fixed reference frame, then using that variation to set a limit on orientation effects. In order to gain precision, experimentalists typically observe two different co-located clocks' frequencies as they rotate with the Earth.

Ron Walsworth's group at Harvard uses co-located ^{129}Xe and ^{3}He masers, which both operate on nuclear spin-1/2 Zeeman transitions. By searching for variations of the clock frequency related to the rotation of the earth (sidereal movement), Walsworth is able to put a bound on the Lorentz violation coefficient of the neutron: 10^{-31} GeV.

The experiment consists of dense co-located clouds of ^{129}Xe and ^{3}He atoms. Each atom has a maser oscillation on its nuclear spin-1/2 Zeeman transition at 1.7 kHz for ^{129}Xe and 4.9 kHz for ^{3}He. These masers are stable and can be sustained as long as they want them to. The population inversion for both species is due to a spin-exchange collision with optically-pumped Rb. Both of the masers are tremendously stable; on the order of 100 nHz for measurements longer than an hour. Since both masers are so stable, they can both be used as high-precision magnetometers as well. So, Walsworth uses one maser to very accurately measure the magnetic environment, and the other free-running maser's frequency. The magnetic environment was controlled by phase-locking the ^{129}Xe-maser to a 1.7 kHz reference signal, and then feeding back to the solenoid providing the 1.5 G reference field. This effectively eliminated any systematic effects arising from stray magnetic fields, by taking advantage of the ^{129}Xe-maser as a magnetometer. These stray fields would shift the frequencies of the masers in proportion to the ratio of their magnetic moments. Once locked, the stability of the ^{129}Xe-maser was several orders of magnitude above that of the unlocked ^{3}He maser, so the ^{129}Xe Zeeman-frequency was taken as a constant.

To leading order, the possible Lorentz violating coupling of each nucleus can be thought of as a single $^{1}S_{1/2}$ valence neutron. Therefore, the magnitude and sign of the Lorentz-violating frequency shift ($\delta\nu_J^{Lorentz}$) is the same for both masers. Using the ^{129}Xe maser as the magnetometer, and the quantization axis as east-west in the Earth's reference frame, the sidereal variation ($\delta\nu_J$) of the ^{3}He is given by

$$\delta\nu_J = \delta\nu_J^{Lorentz}\,|1- \gamma_{He}/\gamma_{Xe}| \approx 1.75\,\delta\nu_J^{Lorentz} \quad (1)$$

where $\gamma_{He}/\gamma_{Xe} \approx 2.75$ is the ratio of the gyromagnetic ratios of ^{3}He and ^{129}Xe. The subscript J represents components in the sidereal reference frame orthogonal to the Earth's axis of rotation. Therefore, the sidereal frequency variation of the free-running ^{3}He maser frequency observed in the laboratory frame has the form

$$\Delta\nu_{He} = \delta\nu_X \cos(\Omega_s t) + \delta\nu_Y \sin(\Omega_s t), \quad (2)$$

where Ω_s is the angular frequency of the Earth's rotation (in Cambridge, MA where the experiment took place).

Walsworth's group measured the maser signals from the ^{3}He- and ^{129}Xe-masers using inductive pickup coils, which where then amplified and sent to lock-in detectors. The entire experiment was referenced to the same hydrogen-maser clock. Since the hydrogen-maser clock operates on the hyperfine transition, it is insensitive in first order to the Lorentz and *CPT* effect they measured.

Every 4 seconds the phase and amplitude of each maser was recorded, giving over twenty-thousand data points every 24 hours. The coefficients $\delta\nu_X$ and $\delta\nu_Y$ were then calculated, giving the size of a potential Lorentz violation signal for that day.

As is necessary in precision measurements, Walsworth's group went to great lengths to ensure there were no systematic effects they were unaware of. They recorded the temperature of the vacuum system components, the magnetization of Rb in the bulb, the broadband power emitted by the Rb repumper laser, the ambient room temperature, and the east-west component of the room's magnetic field. They added a servo loop to stabilize the Rb magnetization in the pump bulb, which stabilized the population inversion rate of the masers, and stabilized their amplitude by a factor of a thousand. They also added a servo loop to stabilize the power output of the repumper laser through temperature feedback to the laser diode. By slightly dithering the polarization of the Rb in the pump bulb, and observing the resultant modulation of resonant repumper light passing through the bulb, they produced a control signal. The control signal was fed back to a variable retarder to adjust the incident light polarization. These locks resulted in no measurable drift in the Rb magnetization over the time scale of many days.

The primary source of drift in the free-running ^{3}He-maser was noble-gas polarization-induced frequency shifts. The phase drift was in the range of 1 to 5 percent over the course of a day. Walsworth's group included a parameter in their model to fit this drift.

IV. DATA ANALYSIS

The potential Lorentz-violating effects on the phase of the ^{3}He-maser were written in terms of the $\delta\nu_J$ coefficients by integrating Eq. (2). The minimal fit model

$$\delta\varphi_{He} = \varphi_0 + 2\pi\nu_0 t + 2\pi\Omega_s^{-1} [\delta\nu_X \sin(\Omega_s t) - \delta\nu_Y \cos(\Omega_s t)], \quad (3)$$

was used to perform the initial data reduction of each one-day run. The coefficients φ_0 and ν_0 account for frequency and phase offsets from the ultra-stable hydrogen maser reference oscillator. The χ^2 coefficients of the fit model were determined, and then they added additional terms to the model, such as amplitude-correlated phase drift, if they reduced the χ^2. From the fit, the $\delta\nu_X$ and $\delta\nu_Y$ coefficients of each one-day run were extracted using a linear least-squares fitting of the model which gave the best χ^2 of the day. Figure 1 gives an example of a day's residuals.

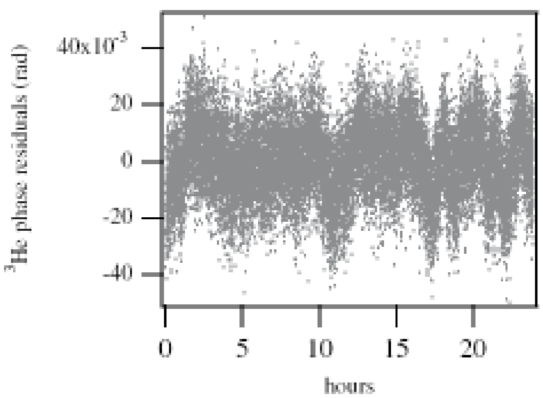

FIG. 1. Typical residuals for the ^{3}He phase data from one sidereal day, calculated using the fit model given in Eq. (3).

A final check of the fitting procedure involved writing an artificial Lorentz-violation signal of known phase and amplitude to the raw data. The data processing was repeated, and the method for the day was considered successful if the artificial signal was recovered and there was no change to the covariance matrix.

Experimental data was taken for 90 total days over 3 separate runs, each of which had a different maser cell. The quantization-axis defining magnetic field was flipped every 10 days to distinguish possible Lorentz-violating effects from diurnal systematic effects (effects that happen every 24 hours and take 24 hours to complete, say, daily changes in the ionosphere).

The potential diurnal variation of magnetic fields in the room would not average away with the field reversals. Since the masers were not perfectly co-located, a small error may occur that the ^{129}Xe magnetometer might not properly account for. Walsworth's group added large external coils that switched on and off a .5 G magnetic field in the east-west and north-south directions. A bound on the drift of the magnetic field near the apparatus was measured to result in a worst-case 8 nHz shift of the ^{3}He-maser frequency, which is far beyond the current sensitivity needed.

In order to further reduce error, the data for the two opposing magnetic field directions were analyzed separately to determine mean values and error for $\delta\nu_Y$, shown in Table 1. Figure 2 is an example of a single day values for $\delta\nu_X$.

TABLE I. Means and standard errors for $\delta\nu_X$ and $\delta\nu_Y$. Results are displayed for each of the three cells (S3, E9, and SE3) with both east (E) and west (W) orientations of the magnetic field. Two runs were performed for cell SE3.

Cell	$\delta\nu_X$ (nHz)	$\delta\nu_Y$ (nHz)
S3 E	95 ± 118	197 ± 114
S3 W	−43 ± 138	88 ± 148
E9 E	−86 ± 234	−194 ± 207
E9 W	−206 ± 186	−60 ± 134
SE3 E1	100 ± 148	9 ± 141
SE3 W1	−1 ± 88	62 ± 109
SE3 E2	−2 ± 180	68 ± 107
SE3 W2	−35 ± 118	197 ± 120

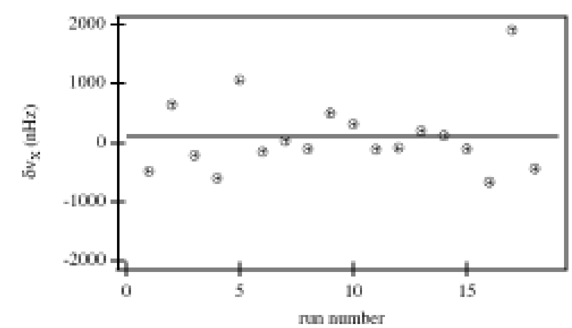

FIG. 2. Values of the Lorentz-violating parameter $\delta\nu_X$ obtained with cell SE3 in the E1 orientation. The horizontal line indicates the mean value for that data set.

The total weighted means and standard errors were combined from all the sets. These values were used to predict the rms magnitude of the correction to the ^{3}He Zeeman frequency due to Lorentz- violating couplings in the XY-plane, $R = (\delta\nu_X^2 + \delta\nu_Y^2)^{1/2}$, which gave R = 53 ± 45 nHz. This is consistent with zero.

In terms of the SME, Eq. (1) is replaced with[22]

$$2\pi \,|\, \delta\nu_J \,| = |-3.5\underline{b}_J^n + 0.012\underline{d}_J^n - 0.012 g_{D,J}^n| \quad (4)$$

Where $\underline{b}_J^n$, $\underline{d}_J^n$, and $g_{D,J}^n$ are coefficients describing the coupling of the neutron to background tensor fields that arise from spontaneous symmetry breaking in a fundamental theory. They are linear combinations of more basic quantities in the relativistic Lagrangian of the SME. The coefficients in Eq. (4) show that this experiment is most sensitive to effects associated with $\underline{b}_J^n$. Assuming the entire Lorentz violation signal is in $\underline{b}_J^n$, the above experimentally determined R corresponds to a $\underline{b}_\perp^n$ = $(6.4 \pm 5.4) \times 10^{-31}$ GeV. This was the most stringent limit on possible Lorentz and *CPT*-violating effects due to the neutron.

V. OTHER EXPERIMENTS

As this experiment demonstrates, a great deal of work goes into making a precision measurement. Furthermore, the continuous theoretical work by Kostelecky and colleagues has provided the support needed to calculate these coefficients from experiments that were not initially designed for measuring Lorentz violation. This has allowed limits to be placed on these coefficients in a broad range of the matter and photon sectors.

Other clock-based experiments have placed limits of 10^{-27} GeV on effects of the proton and electron through hydrogen maser experiments, and another experiment comparing the Zeeman frequencies of ^{199}Hg and ^{133}Cs.

QED (g-2) tests in Penning traps, measuring the relative masses of particles to their antiparticles, have set limits of 10^{-25} GeV of the electron, using electron-positron measurements, and 10^{-26} GeV of the proton, using protons and antiprotons.

A spin-polarized torsion pendulum experiment, which we talked about in class, placed a limit of 10^{-29} GeV of the electron, the best measurement to date.

The best measurement in the photon sector is done using a modern day update of the Michelson-Morley experiment, only using rotating cryogenic-temperature sapphire oscillators. The current limit is 10^{-11} GeV on 8 photon-related parameters in the SME.

Another photon-related experiment is measuring vacuum birefringence. The 1990 experiment does not relate its results to the SME, perhaps because it had not been fully developed at that time. There are current experiments in vacuum birefringence[23], recently published, that rule out small unquantized charges, a possible effect of the SME, to less than .1 eV.

Finally, there are a host of experiments using measured oscillations of Neutral-Bs, Neutral-Ds, neutrinos, and kaons. I should add that Kostelecky has proposed neutrino oscillations might not be due to a mass difference, but to Lorentz violation[24].

VI. SUMMARY

The search for a fundamental theory that includes both the Standard Model and Relativity, or some modified form of them, is perhaps the biggest question in physics today. Though physicists continue in the footsteps of Michelson and Morley, measuring zero and ruling out possible theories, it will most likely take a modern-day Lorentz to see the bigger picture. In the meantime, experimentalists will continue to search.

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