Supersymmetry and a Candidate for Dark Matter

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April 20, 2007
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The Standard Model has been a powerful tool in understanding the symmetries of nature and predicting new phenomena. There are, however, some fundamental flaws in the model. The theory of supersymmetry has provided an extension to the Standard Model in an effort to resolve some of the most troubling paradoxes. This paper provides an overviewof the most basic supersymmetric models and outlines how these might provide a candidate for the mysterious dark matter permeating our universe.

1 Why a new theory?

1.1 The Standard Model

Over the last 2500 years the idea has been developed that there are indeed constituents of matter that are fundamental ingredients to all that we see and feel. The most commonly accepted theory of these fundamental particles and interactions today is the Standard Model.

According to the Standard Model there are three types of fundamental particles: leptons, quarks and force carrying particles. Figure 1 shows the groups of particles in the Standard Model. Not shown are the anti-particle partners of each of the leptons and quarks. Strong evidence for each of these particles has been seen experimentally. The quarks and leptons are fermions, half-integer intrinsic spin, while the force carriers are bosons, integer intrinsic spin. These two different natures of the particles lead to very different mathematical consequences with regard to relativistic quantum field theory. One additional particle is widely accepted as part of the Standard Model even though its existence has not yet been observed. This is the Higgs boson. The existence of the Higgs boson would provide a mechanism to explain why the $W\pm$ and Z^0 have mass, while the photon and the gluon are required to be massless.

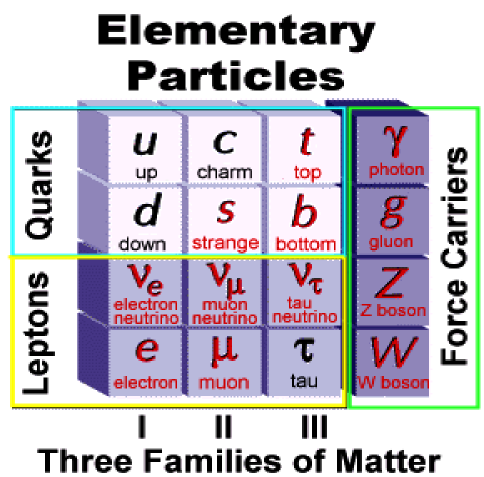

Figure 1: Simple diagram of the Standard Model

1.2 The Hierarchy Problem

In the framework of relativistic field theory one can develop a Lagrangian for this system of interacting particles. The mass of a particle is calculated using perturbation theory giving a leading order mass term and then subsequent corrections. The first oder correction (often referred to as “next to leading order” or NLO) is called a radiative correction. These involve Feynman diagrams with one loop. In order to include the correction for all possibilities of the one loop, an integral is taken over all momenta:

$$\int d^4k \frac{1}{k^2}$$

This integral is divergent, so one must choose an upper limit to integrate to, Λ, at which point new physics is expected to take over. The Standard Model does not include the force of gravity which would start to have effects of the same order as quantum mechanics at around the Planck scale, a mass of around 10^{19} GeV. If not before, the Standard Model theory must be modified at this energy, so this is an upper bound for Λ. For fermions the integral for the radiative corrections to the mass scale is only logarithmically divergent as is shown in Eq.(1) where m_f is the fermion mass and *alpha* is a coupling coefficient. Integrating up to 10^{19} GeV does not introduce significant corrections to the leading order calculation.

$$\delta m_f \propto \frac{3\alpha}{4\pi} m_f \ln\left(\frac{\Lambda^2}{m_f^2}\right) << m_f \tag{1}$$

The Higgs boson, however, would be a scalar boson. This integral is quadratically divergent as is shown in Eq. (2), so when the upper bound of the integral is set at around the Planck scale the radiative corrections become much larger than the leading order terms.

$$\delta m_{higgs}^2 \propto \frac{\alpha}{4\pi}\Lambda^2 >> m_{higgs}^2 \tag{2}$$

This instability implies that either the theory is not correct or that there is a lower mass/energy/momentum scale at which new physics must start to play a role.

1.3 Unification of forces

The relative strengths of each of the three forces included in the Standard Model have been measured at various energies around the energy of our current universe, approximately 100 GeV and below. In this region the inverses of the coupling coefficients seem to be linear functions of energy and seem to get closer together at increasing energy. Figure 2 shows the extrapolation of the current measurements to higher energies.

The coupling strengths do approach one another, but they do not meet at the same point. This is unattractive to theoretical physicists. Grand Unified Theories propose that there is an overarching

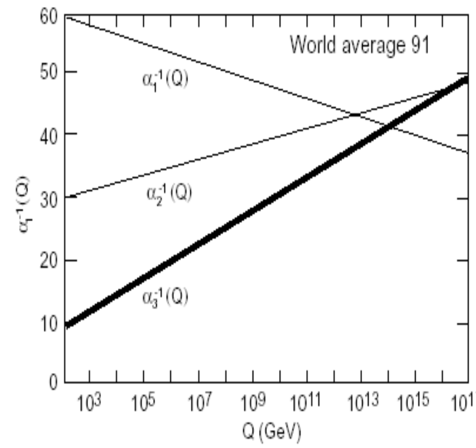

Figure 2: Dependence of force coupling strengths on energy. U(1):α_1, SU(2):α_2 (these together form the electro-weak force), and SU(3):α_3 (strong force)

symmetry that represents all of nature and may be broken at low energies but ought to unify all the forces at high enough energies. The manner in which the coupling strength depends on energy changes according to what model is used for the physics at any given energy. Hence, if this linear trend is modified by a new theory at higher energies the coupling coefficients may actually unite at one point.

1.4 How does supersymmetry help?

The two dilemmas outlined above can be solved rather elegantly with the introduction of an extension to the Standard Model. Supersymmetry proposes additional terms in the Lagrangian that would add extra particles and interactions to the theory. Namely, there would be a fermionic partner to the scalar boson Higgs particle. This helps solve the problem of quadratic divergences as the fermionic and bosonic portions of the integral are added with an opposite sign. Hence the fermionic quadratic divergences would cancel out the scalar bosonic quadratic divergences, leaving only the logarithmic divergences for both. This re-stabilizes the mass of the Higgs. Similarly, by adding in new particles and interactions the coupling coefficients have a different dependence on the energy scale. Figure 3 shows how a fine tuned supersymmetric model would predict the coupling coefficients to meet at a single point, and that the forces would be contained under one symmetry at higher energies.

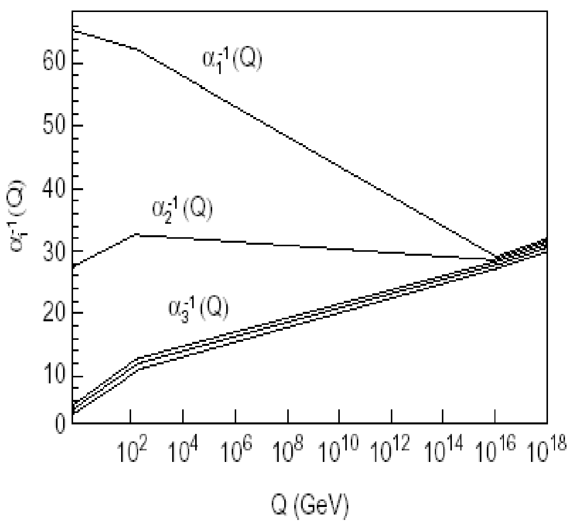

Figure 3: Dependence of force coupling strengths on energy with supersymmetry

There are many different variations of supersymmetric models, each tuning the model in such a way that the forces unify slightly differently. However, the opportunity to choose parameters in such a way that the coupling coefficients both agree with current experimental evidence at our energy scale and converge at higher energy scales is one reason supersymmetric models have become so widely supported.

2 The Basics of supersymmetry

2.1 The Mathematics

Typical symmetries involve changing either an intrinsic property of the field such as isospin or charge, or an external property of the field such as momentum. Supersymmetry is unique because it proposes a transformation that changes the *nature* of the field, transforming a fermionic field into a combination of fermionic and bosonic fields and vice-versa.

The most basic example of a supersymmetric model deals with only one dimension and three simple fields. This is a model first proposed in 1973 by Wess and Zumino. The Lagrangian for this model is shown in Eq. (3).

$$\begin{aligned} L = & \frac{1}{2}\left(\partial_\mu A\right)^2 + \frac{1}{2}\left(\partial_\mu B\right)^2 + \frac{i}{2} m\overline{\psi}\partial\psi \\ & - \frac{1}{2} m\overline{\psi}\psi - \frac{1}{2} m^2 A^2 - \frac{1}{2} m^2 B^2 \\ & + mgA\left(A^2 + B^2\right) - \frac{1}{2} g^2 \left(A^2 + B^2\right)^2 \\ & - ig\overline{\psi}A\psi + if\overline{\psi}\gamma_5 B\psi \end{aligned} \tag{3}$$

The fields A and B are real scalar fields such as the Higgs field, and the ψ field is a fermion field such as the electron. [1] The first line contains kinetic energy terms, the second line the mass terms and the last two lines contain the interaction terms between the various fields. As described earlier, symmetries are associated with transformations of the fields. Typically these transformations are applied as an infinitesimal addition to the field itself, for example the scalar field A will transform into a new field $A' = A + \delta A$. Here is where the"super" part of supersymmetry takes effect. In supersymmetric models the transformations of the fields transform the *nature* of the fields, mixing together the A, B and ψ fields. The Wess-Zumino transformations are shown in Eqs. (4) through (6).

$$\delta A = i\overline{\alpha}\gamma_5\psi \tag{4}$$

$$\delta B = -\overline{\alpha}\psi \tag{5}$$

$$\begin{aligned} \delta\psi = & F\alpha - iG\gamma_5\alpha + \\ & \left(\partial\gamma_5 A\right) + i\left(\partial B\right)\alpha \end{aligned} \tag{6}$$

Notice the scalar fields A and B obtain a fermion part, ψ and the A and B fields mix into the transformation of the ψ fields. If these transformed fields are plugged into the Lagrangian in Eq.(3), most of the changes cancel out and it remains unchanged up to a full derivative. This means that the physics that is observed is the same if the fields are not mixed (before the transformation) or mixed (after the transformation). The proposition on the impact on the Standard Model is that we are observing the physics of *some* of the particles, but not *all* the particles that exist. In particular there would be no change in the overall physics of the Standard Model if, indeed, there were bosonic partners to all the observed fermions and fermionic partners to all the observed bosons.

2.2 The New Particles

These new supersymmetric partners would have all the same quantum numbers as their regular matter partners except for two: the intrinsic spin would clearly differ by 1/2, and a new quantum number to reflect the supersymmetric origin is invented, R. One can calculate the R value for any particle with the formula $R = (-1)^{3(B-L)+2S}$. This gives +1 for all currently observed particles and -1 for all supersymmetric partners. In most models R is a conserved quantity so one supersymmetric particle can not decay to just one regular particle.

To distinguish the particles a new naming scheme is adopted. This is outlined in Fig. 4. All fermions have an "s" added to the beginning of the name. For example the partner for the electron is a selectron. All bosons have an "ino" added to the end of the name. The $W\pm$ particle becomes a Wino. The symbols for both the fermions and bosons gain a ~ above the symbol.

As stated before, in a perfect SUSY model the partners would have the same quantum numbers, including mass. This can not be physically accurate as the super partners would have already been detected. The mass symmetry must be broken in some way in order to make the partners slightly more massive on an overall energy scale. The masses must be close enough so that the contributions to the one loop diagrams mostly cancel, but not close enough that we would have seen them already. This places an upper limit on the difference of the square of the masses at 1 TeV.

$$\left| m_{higgs}^2 - m_{higgsino}^2 \right| < 1TeV \tag{7}$$

This provides a natural limit on the searches for experimental evidence of a SUSY model. It is not possible simply to push the theoretical masses beyond the range of experiment if no new particles are found at energies close to 1 TeV above the masses of observed particles.

2.3 Common Types of SUSY

The two most commonly discussed types of SUSY models are the Minimally supersymmetric Standard Model (MSSM) and Minimal Supergravity (mSugra). MSSM is minimal in that it introduces the fewest new particles necessary to unify the components of the Standard Model. A new particle is necessary for every degree of freedom that the observed particles have. In order to specify masses and coupling strengths and mixing angles there are over 120 new parameters that must be specified in the MSSM. Nearly all of these can be specified by hand and fine tuned to agree with any future experimental results. Unfortunately, this makes it extremely hard to make precise predictions with this model. mSugra reduces this number of new parameters down to 5 by assuming that similar particles have equal masses. It groups the parameters together and dictates that all parameters in a group have nearly the same value. It was originally named Minimal Supergravity because it included the coupling of the 3 Standard Model forces to gravity at high energies. There are a number of theories that achieve this now and so mSugra is not unique in this facet.

3 Relevance to dark matter

3.1 How does SUSY provide a solution to dark matter?

The quandary of the cold dark matter that is detected by cosmological experiments is one of the prevailing mysteries in physics today. This matter must have no electrical or color charge. If it were electrically charged, it would have bound with charged particles in normal matter and produced isotopes much heavier than seen. If it had color charge, it could interact via the strong force with the partons in hydrogen nuclei passing through it, resulting in an observable signal. Instead, there have been no known interactions with dark matter. Hence dark matter candidates are presumed to interact mostly through the weak force and perhaps somewhat gravitationally.

The lightest supersymmetric Particle (LSP) is a prime candidate for such matter. Assuming the R quantum number is conserved the LSP would be stable. It could not decay into another supersymmetric particle because it is the lightest, and it could not decay into a normal particle because that would violate R-parity. If the LSP has a mass around 1 TeV, then it would most probably not be electrically or color charged for the same reason that dark matter is assumed to have no charge. There are strict lim-

Particle	*Name*	*Feels These Forces*[a]	*Mediates These Forces*[b]	*Superpartner*
e, μ, τ	charged leptons (electron, muon, tau)	EM, W	—	sleptons $\widetilde{e}, \widetilde{\mu}, \widetilde{\tau}$ (selectron, smuon, stau)
ν_e, ν_μ, ν_τ	neutrinos	W	—	sneutrinos $\widetilde{\nu}_e, \widetilde{\nu}_\mu, \widetilde{\nu}_\tau$
u, c, t	up, charm, top quarks	EM, W, S	—	squarks $\widetilde{u}, \widetilde{c}, \widetilde{t}$
d, s, b	down, strange, bottom quarks	EM, W, S	—	squarks $\widetilde{d}, \widetilde{s}, \widetilde{b}$
γ	photon	[c]	EM	photino[d] $\widetilde{\gamma}$
$W^\pm$	weak boson	EM, W	W	Wino[d] $\widetilde{W}^\pm$
Z	weak boson	W	W	Zino[d] $\widetilde{Z}$
g	gluon	S	S	gluino $\widetilde{g}$
G	graviton	GR	GR	gravitino $\widetilde{G}$
h	Higgs boson[e]	W	generates mass	higgsino[e] $\widetilde{h}$

Figure 4: Naming scheme for supersymmetric partner particles [5]

its placed on the abundance of charged particles at relatively low energies by the abundances of nuclear isotopes. Additionally, since the coupling strength tends to be proportional to the inverse of the mass particles that would interact via the strong force tend to be heavier than those that feel the weak force. The LSP, therefor, would be a weakly interacting massive particle (WIMP) that is as pervasive (possibly more so) as regular matter but barely detectable.

3.2 What are the candidates?

There are not a large number of supersymmetric partners in the MSSM that have neither electrical or color charge. The mass of the neutrino is so small that the sneutrino seems a prime candidate. This has been ruled out by experimental evidence, though, through the Large Electron Positron Collider at CERN during the 1980's and '90's. The graviton is massless and so the gravitino seems a likely choice for the LSP. So little is known about the relevance of gravity to the Standard Model, though, that this would be nearly impossible to rule out experimentally.

The most favored choices are the neutralinos. These are linear combinations of the neutral components of the Higgsino and the Wino and Bino (partners of the gauge bosons in electro-weak force). Eq.(8) shows a general form for a neutralino:

$$\chi = \alpha\widetilde{B} + \beta\widetilde{W^3} + \gamma\widetilde{H_1} + \delta\widetilde{H_2} \tag{8}$$

The B and W are the two neutral fields introduced in the electroweak symmetry breaking formalism and compose the Z boson in the Standard Model. The superpartners to these fields along with some of the components of the superpartners to the Higgs field (H_1, H_2) add together in varying degrees and make up a bosonic supersymmetric field. The coefficients, $\alpha, \beta, \gamma, \delta$, depend on the parameters of the model. In some limiting cases this neutralino could be a pure state of the photino or the Bino. Figure 3.2 shows a region of the SUSY parameter space along with some dashed line mass contours of the neutralino.

M_2 and μ are parameters in the MSSM. The regions that are cross hatched have been experimentally excluded because they would lead to a charged superpartner with a mass that would have been seen at the Large Electron Positron Collider but was not. The light shaded region has also been excluded by further LEP experiments. Clearly the pure photino and $\overline{S}^0$ have been ruled out, but the Bino and Higgsino with masses greater than 100 GeV are still plausible candidates.

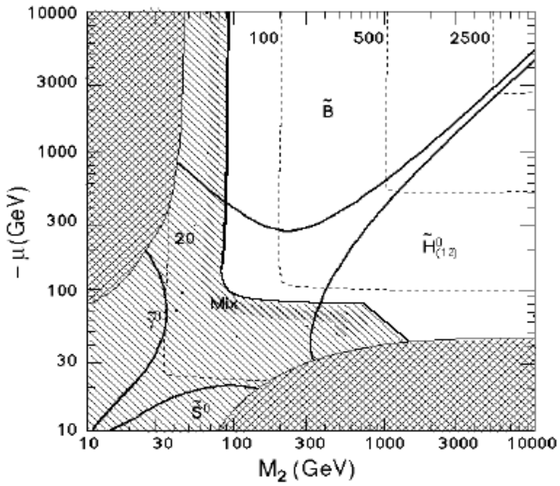

Figure 5: A section of the parameter space for MSSM with possible mass contours for the LSP. The shaded regions have been experimentally excluded. [2]

4 Conclusion

The category of supersymmetric models provide solutions to several of the most distressing problems with the Standard Model. It is a very natural extension of the model that has already been well tested experimentally. There has not yet been any experimental evidence for supersymmetry, though. If evidence for it is not seen at the new Large Hadron Collider the skeptics will gain ground and the number of supporters may dwindle. The paradoxes inherent in the Standard Model are not to be taken lightly, and if supersymmetry is not the answer the entire model might be drawn into question. Just as the Aristotilean model of the geocentric universe was massaged into a greater number of circles, so the Standard Model may be massaged into a more complex symmetric model. The end result, though, may be an entirely new approach. If no saving graces are found at energies soon to be probed there may be renaissance in the world of elementary particles, a return to the fundamental understanding of what we see and how it can be explained. In either case some sign of new physics is sure to be seen when the most powerful collider starts in the fall of 2007. More information is soon to come, and hopefully many new insights along with it.

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