Physics 798G Spring 2007

Lecture 7
Gauss's Law Test of the $1/r^2$ Law
Using an SGG

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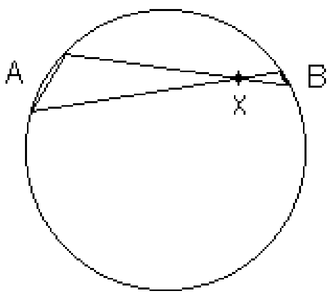
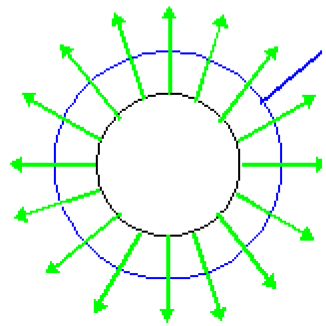
February 15, 2007

Newtonian null sources

- Uniform spherical shell: The gravity field vanishes inside the shell.



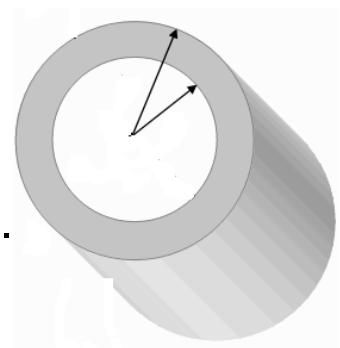
Newton



Gauss's law

Problem: The spherical shell must be made in two halves with holes to support a detector inside.

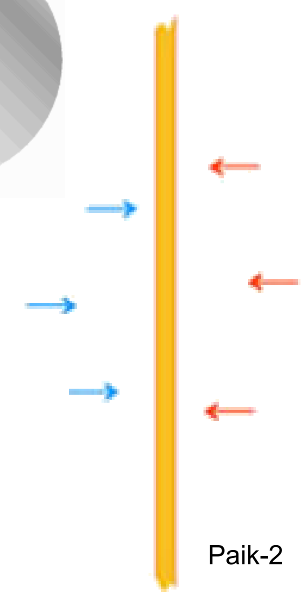
$\Rightarrow$ Source metrology error



- Infinitely long cylindrical shell: 2-D spherical shell.

  Problem: The cylindrical shell must be truncated.

  $\Rightarrow$ Newtonian error from the missing mass



- Infinite plane slab: 1-D spherical shell.

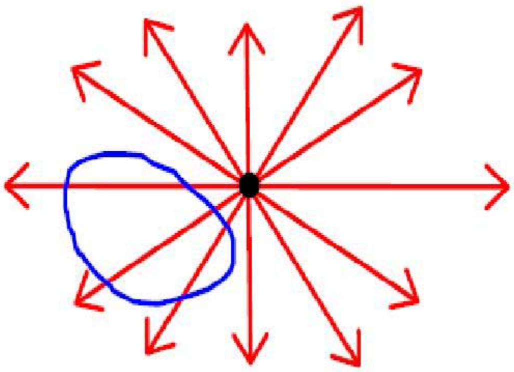
  The field is constant on either side of the plane.

  Problem: The plane slab must be truncated.

  $\Rightarrow$ Newtonian error from the missing mass

Newtonian null detector?

- Gauss's law: $$\Phi_{\text{total}} \equiv \oint_S \mathbf{g} \cdot \mathbf{n}\, da = -4\pi G m$$ $$\Rightarrow \nabla \cdot \mathbf{g} = -4\pi G \rho$$



Total flux of field lines $\propto$ Total mass enclosed

- Gravitational field (vector): $g_i = -\partial \phi / \partial x_i$

$\Rightarrow$ Cannot be distinguished from platform acceleration.

- Gravity gradient (tensor): $\Gamma_{ij} \equiv -\partial^2 \phi / \partial x_i \partial x_j$

$\Rightarrow$ In vacuum, $\sum_i \Gamma_{ii} = -\nabla^2 \phi = -4\pi G \rho = 0$ (traceless)

- Gravity gradiometer: $g_i(x_{j,2}) - g_i(x_{j,1}) \approx \frac{\partial g_i}{\partial x_j} \delta x_j = -\frac{\partial^2 \phi}{\partial x_i \partial x_j} \delta x_j$

The output sum of a 3-axis diagonal-component gravity gradiometer must remain constant as a source mass is moved outside.

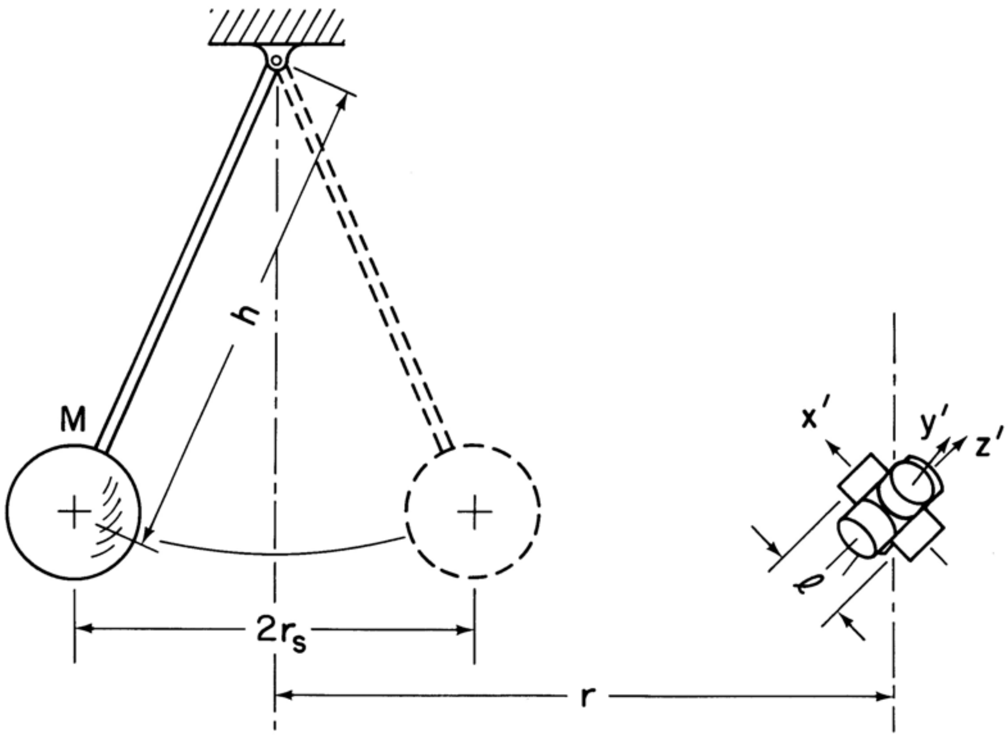
$\Rightarrow$ "Source-independent" null test! Paik, *PRD* 19, 2320 (1979)

Gauss's law test of the $1/r^2$ law

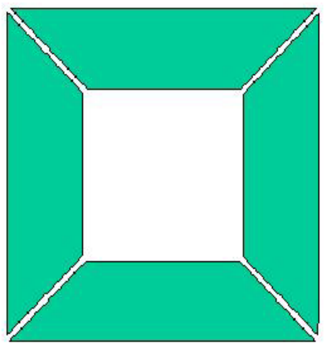
- Source mass: 1.5-ton Pb pendulum (driven pnematically)
- Detector: 3-axis Superconducting Gravity Gradiometer (SGG)

  Finite baseline (δx ~ 20 cm) $\Rightarrow$ Only a near-null detector

  Finite baseline error: $O(\delta x/r)^2$ $\Rightarrow$ Negligible for geophysical scale exp.



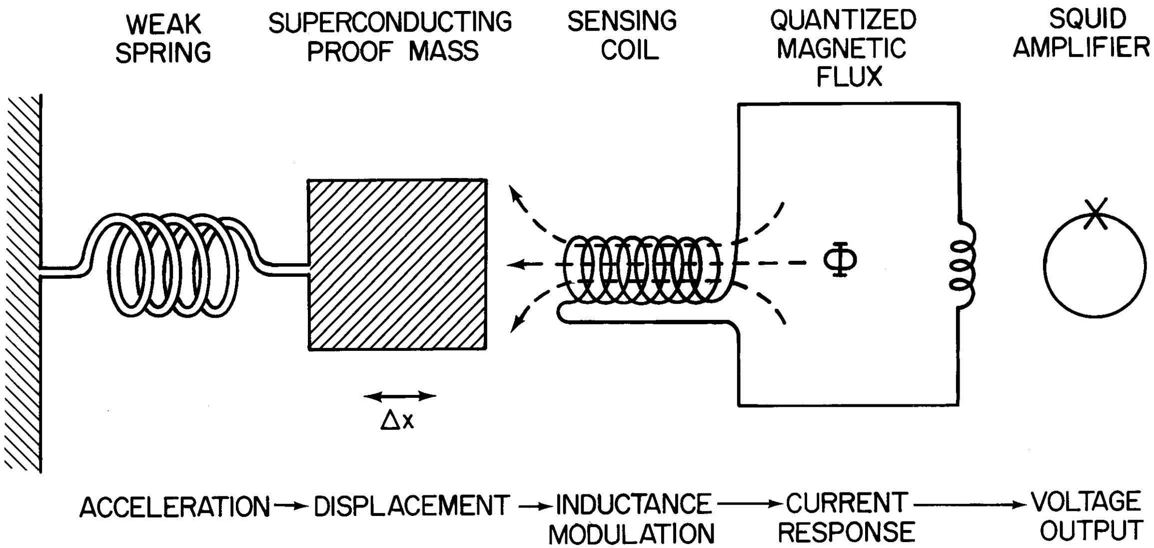
True null detector



First result:

Chan, Moody, & Paik,
PRL **49**, 1745 (1982).

Superconducting linear accelerometer



$$\omega_0^2 = \omega_{\mathrm{m}}^2 + \frac{2}{1+\gamma}\frac{B^2 A}{\mu_0 m d},$$

$$\gamma \equiv \frac{L_0}{L_p}$$

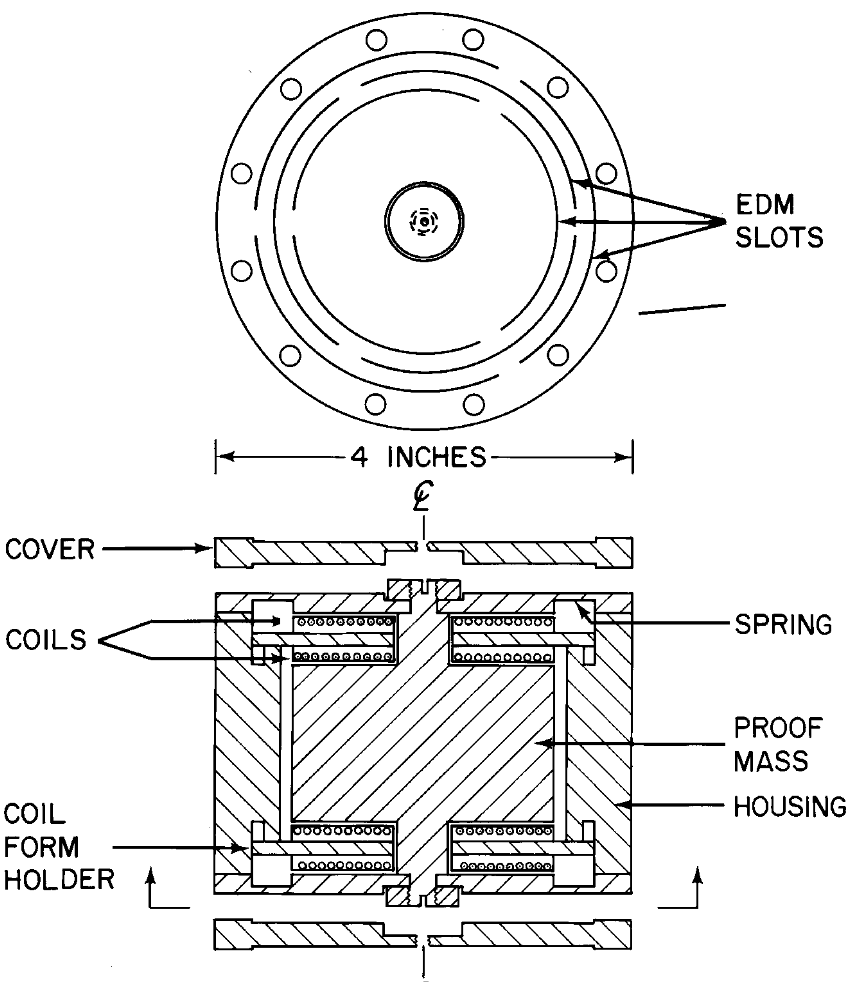
- Large energy coupling: $\beta = 1 - \left(\frac{\omega_{\mathrm{m}}}{\omega_0}\right)^2 = \frac{2}{1+\gamma}\frac{B^2 A}{\mu_0 m \omega_0^2 d} \approx \frac{1}{2}$: optimal

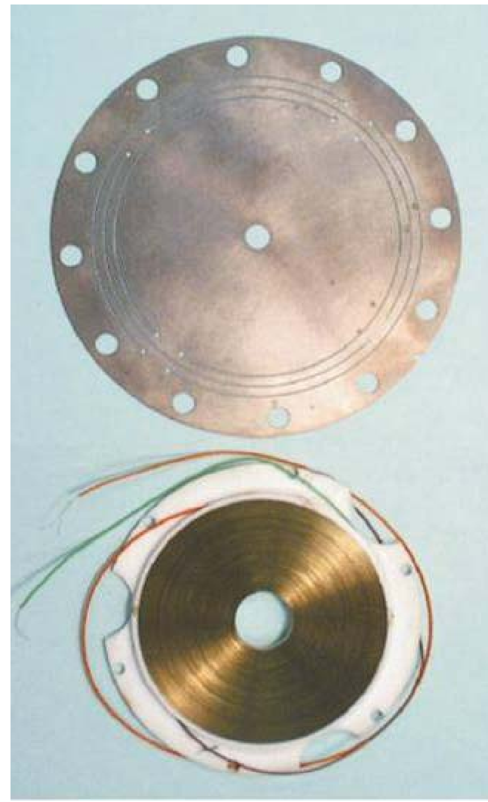
- Low noise: $S_a(f) = \frac{4}{m}\left[k_{\mathrm{B}} T \frac{\omega_0}{Q} + \frac{\omega_0^2}{2\beta\eta} E_A(f)\right]$

$m = 1.2$ kg, $\ell = 0.19$ m, $\omega_0/2\pi \approx 10$ Hz, $Q \approx 10^6$, $T = 4.2$ K, $\beta \approx \eta \approx 0.5$, $E_A(f)$
$= (1 + 0.1\ \mathrm{Hz}/f)\ 5 \times 10^{-31}$ J Hz^{-1} $\Rightarrow$ ${}_a^{1/2}(f) \approx 2\times10^{-13}\ \mathrm{m\,s^{-2}\,Hz^{-1/2}},\ f > 0.1\,\mathrm{Hz}$

Design of component accelerometers

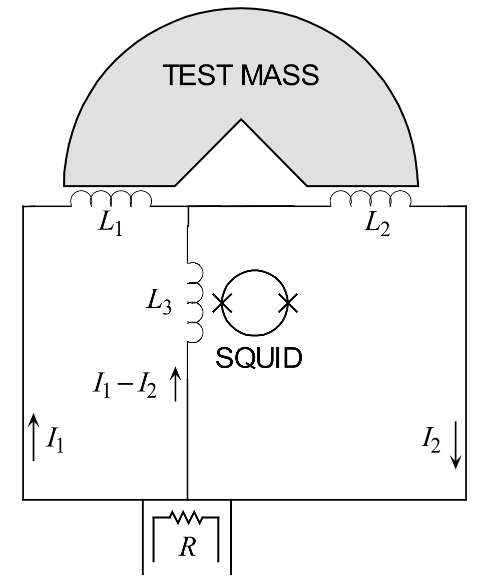
Linear accelerometer (a_i)

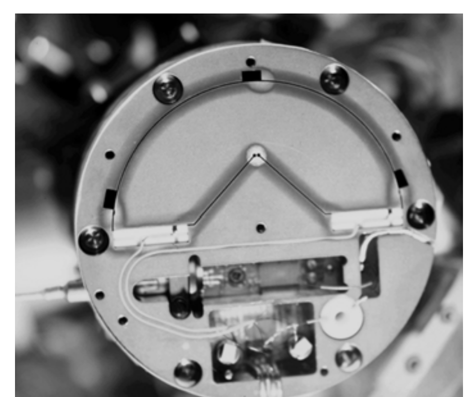




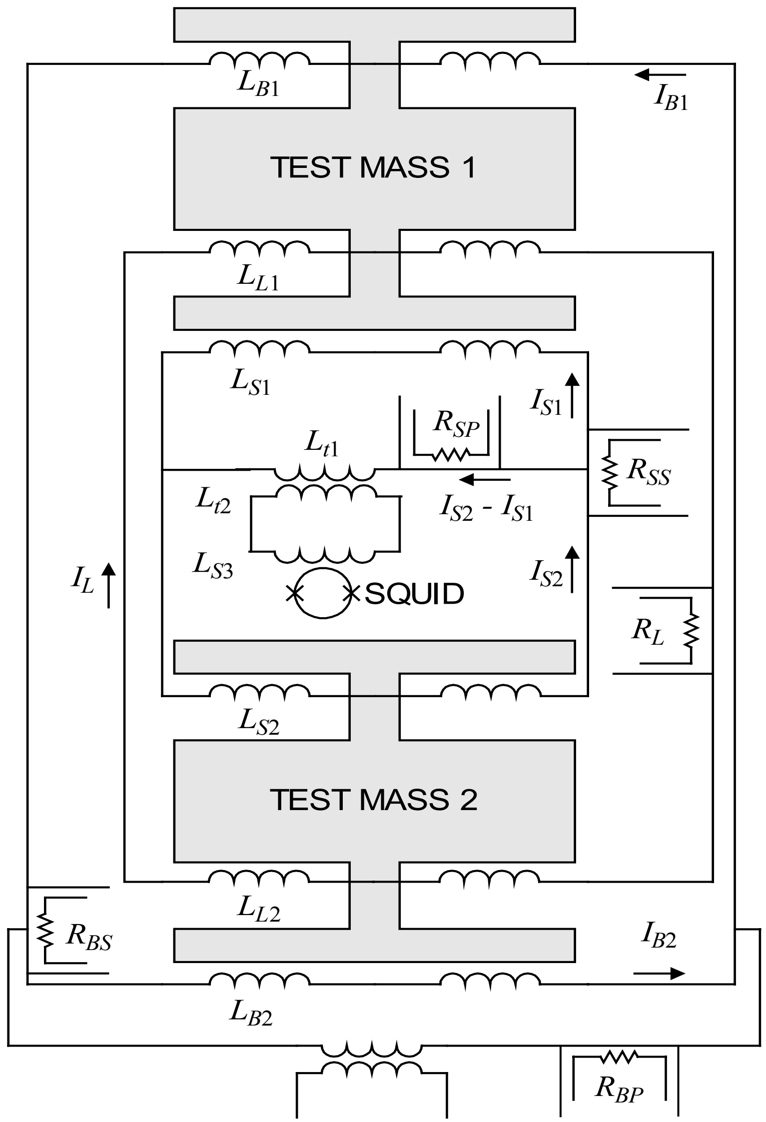
Cantilever spring and niobium pancake coil

Angular accelerometer (α_i)





Superconducting Gravity Gradiometer



- Test masses are coupled by persistent currents in a differencing circuit to a dc SQUID. $\Rightarrow \Gamma_{ii}$
- The ratio I_{S2}/I_{S1} is adjusted to balance out the common mode.
- Test masses are coupled by persistent currents in a summing circuit to another SQUID. $\Rightarrow a_i$
- Angular acceleration is separately measured. $\Rightarrow \alpha_l$
- Intrinsic gradient noise:

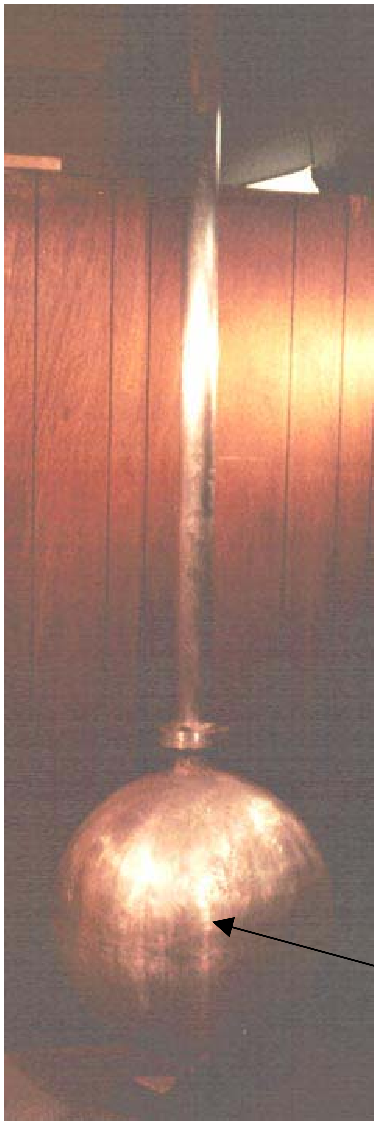
$$S_\Gamma(f) = \frac{8}{m\ell^2}\left[k_B T\frac{\omega_0}{Q} + \frac{\omega_0^2}{2\beta\eta}E_A(f)\right]$$

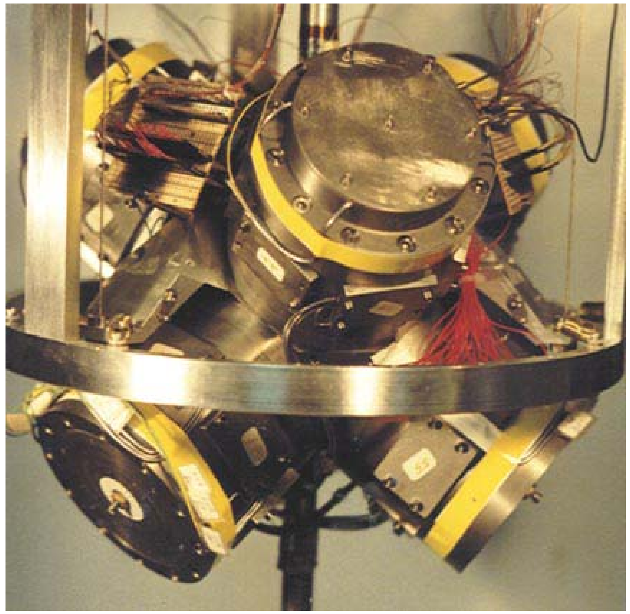
$$S_\Gamma^{1/2}(f) \approx 2\times10^{-3}\,\mathrm{E\,Hz^{-1/2}},\, 1\,\mathrm{E} \equiv 10^{-9}\,\mathrm{s^{-2}}$$

$$(\Gamma_{zz,\mathrm{Earth}} \approx 3\times10^3\ \mathrm{E})$$

$\Rightarrow$ Highly sensitive and stable.

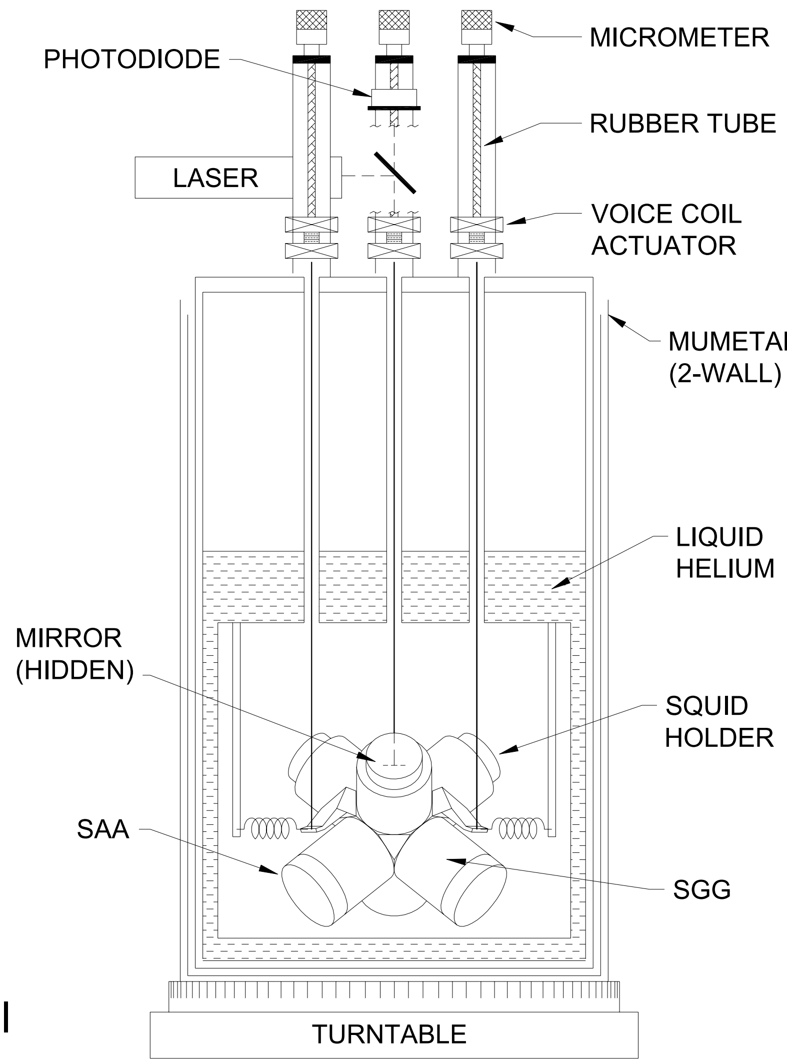
Gravity source, detector, and cryostat





3-axis SGG in umbrella orientation

Moody *et al, RSI* **73**, 3957 (2002)



1.5-ton Pb pendulum

Angular position read by optical shaft encoder

Major errors in SGG

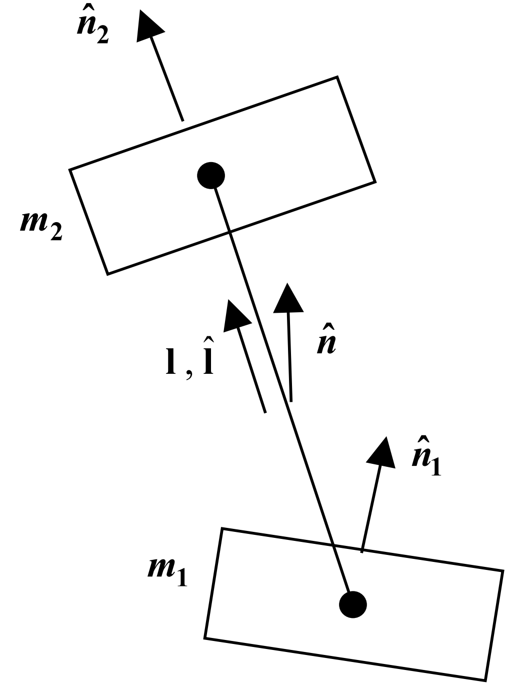
- Linear acceleration sensitivity due to axis misalignment:

$$\delta\Gamma_a = \frac{1}{\ell}\delta\hat{n}\cdot\vec{a},\ \ |\delta\hat{n}| \approx 10^{-4}$$

- Angular acceleration sensitivity due to misconcentricity:

$$\delta\Gamma_\alpha = \left(\delta\hat{\ell}\times\hat{n}\right)\cdot\vec{\alpha},\ \ \left|\delta\hat{\ell}\right| \approx 10^{-4}.$$

Residual balance: sensitivity to a_i and α_i is reduced to $<10^{-7}$ by error compensation.



- Orientation sensitivity:

$$\mathbf{\Gamma}' = \mathbf{R}(\delta\theta)\mathbf{\Gamma}\mathbf{R}^{-1}(\delta\theta),\quad \delta\Gamma_{ii} \approx O(\Gamma\,\delta\theta)$$

- Centrifugal acceleration sensitivity:

$$\Gamma'_{ij} = \Gamma_{ij} - \left(\Omega_i\Omega_j - \Omega^2\delta_{ij}\right),\ \delta\Gamma_{ii,\Omega} = -\Omega_j\delta\Omega_j - \Omega_k\delta\Omega_k + O(\delta\Omega^2)$$

⇒ Critical error due to nonlinearity

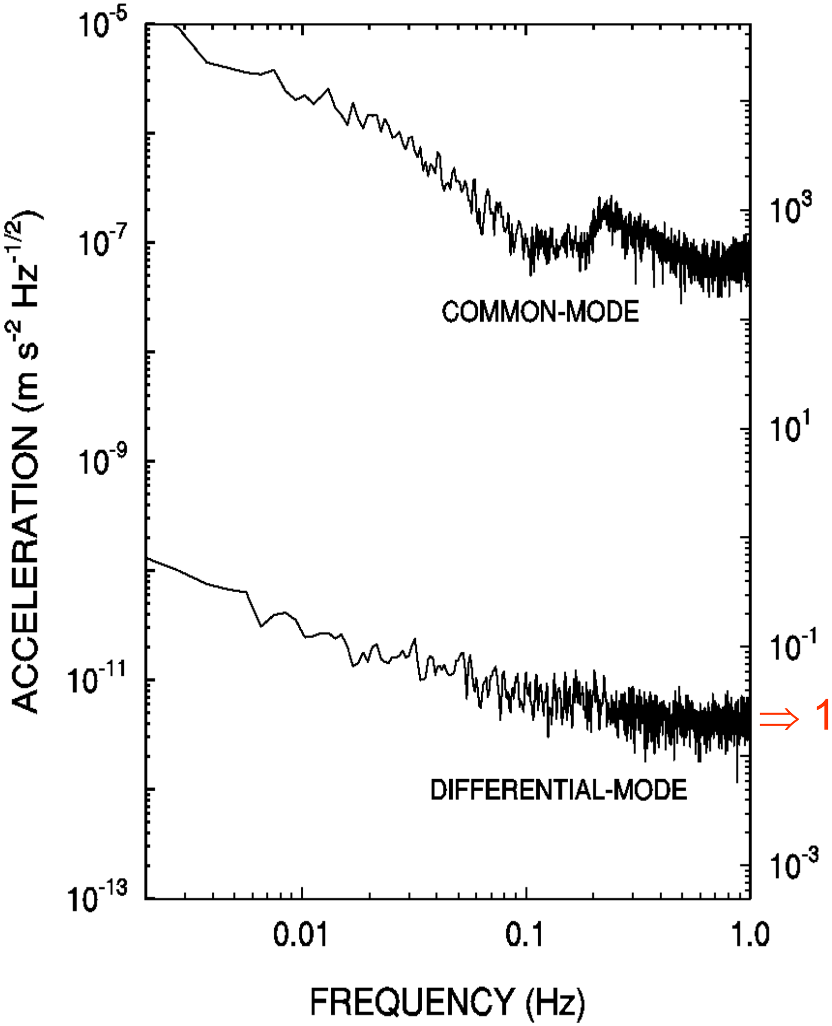
- Temperature sensitivity due to variation of penetration depth with temperature:

$$\lambda(T) = \lambda(0)/[1-(T/T_c)^4]^{-1/2}$$

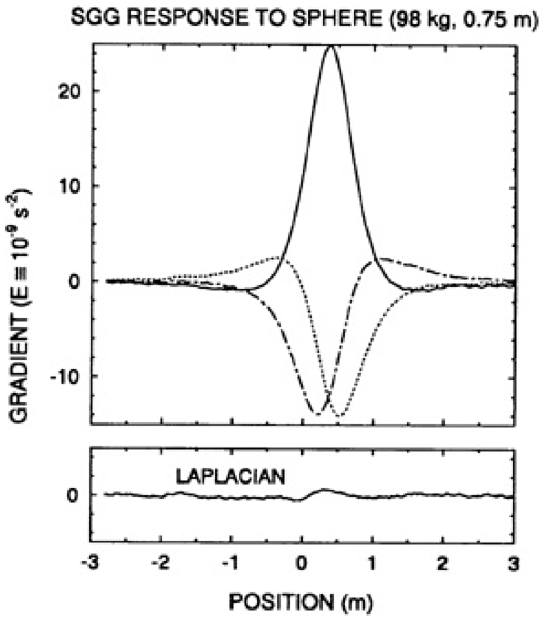
⇒ Low-pass filtered by soft mounting

Performance of SGG

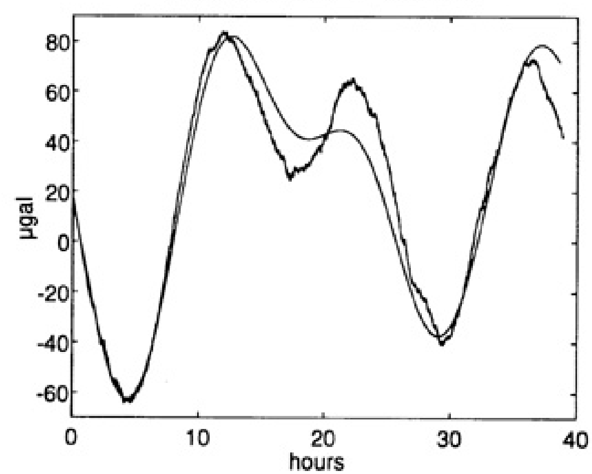
Gravity gradient noise PSD



$\Rightarrow 10^{-15}$ m Hz$^{-1/2}$



Gravimeter Mode



Additional errors in Gauss's law test

- 3-axis SGG measures $\sigma_{1'}\delta g_{1'} + \sigma_{2'}\delta g_{2'} + \sigma_{3'}\delta g_{3'}$

$$\approx \delta(\text{scale factor}) + \delta(\text{orthogonality}) + \delta(\text{finite baseline})$$

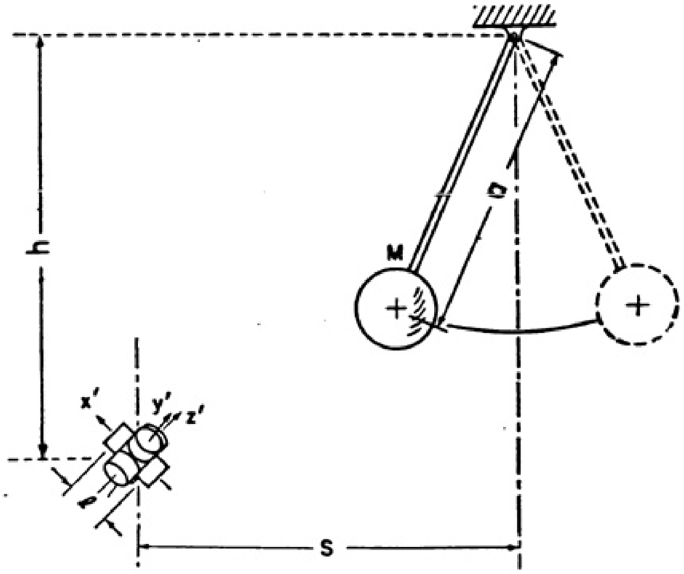
- Scale factor error can be removed by rotating 1-axis SGG about vertical.
- Upon rotation and summation (Parke 1990),

$$\sum_{j'} \Gamma_{j'j'} = 3\sqrt{3}\left(\hat{z}\cdot\overleftrightarrow{\Gamma}\cdot\hat{z}\right)\left(\delta\hat{x}\cdot\hat{z}\right) + 3\left(\vec{\alpha}\cdot\hat{z}\right)\left[\left(\delta\hat{\ell}\times\hat{x}\right)\cdot\hat{z}\right] - \frac{3}{\ell}\left(\ddot{\vec{r}}+\vec{g}\right)\cdot\hat{z}\left(\delta\hat{n}\cdot\hat{z}+\delta h^{c}/\sqrt{3}\right)$$

Non-orthogonality — Angular acceleration — Linear acceleration

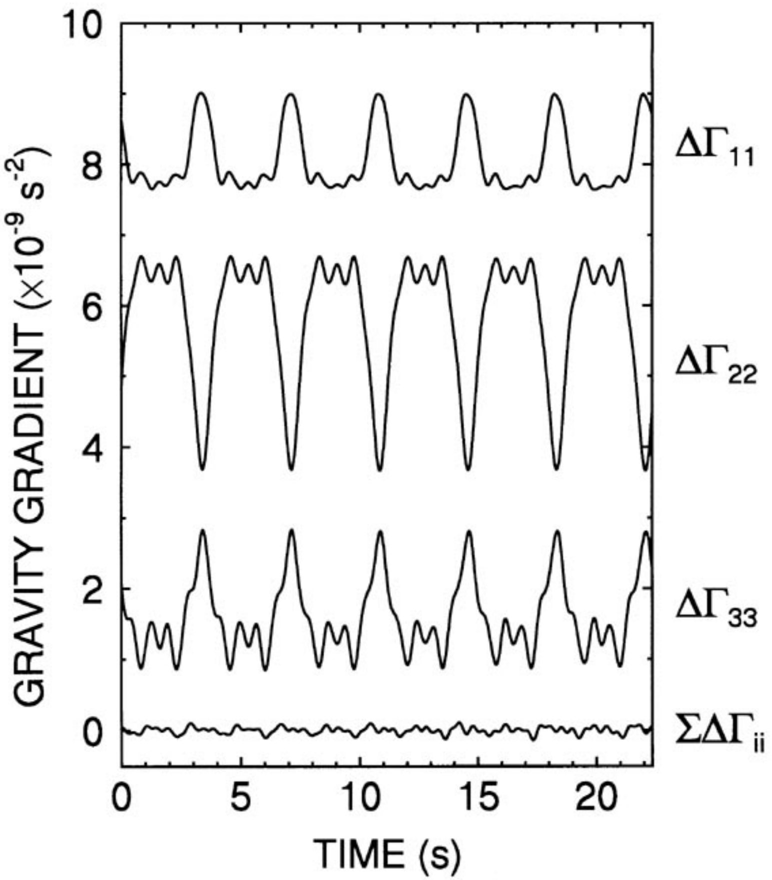
- Orthogonality error can be nulled by choosing source-detector orientation such that $\langle\Gamma_{zz}\rangle \approx 0$.
- Source-driven accelerations are averaged out except for α_z .

  $\Rightarrow$ Had to detect α_z with fiber optics gyro and remove the error.
- Finite baseline error is computed and removed.



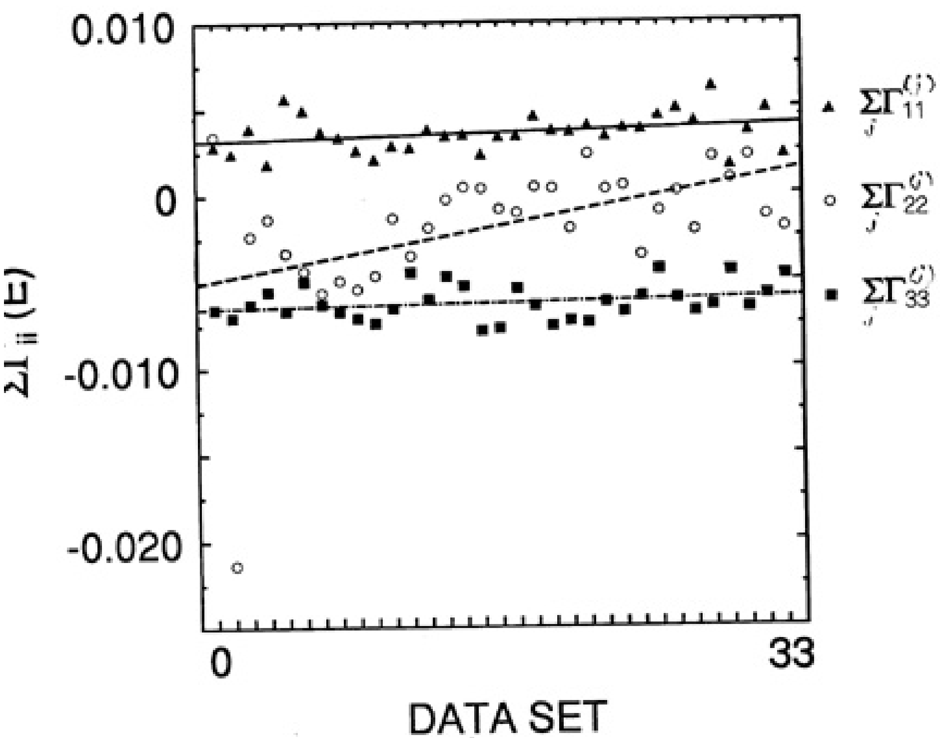
Experimental data

- Inverse-square law data



Source-driven acceleration errors are mostly in even harmonics and filtered out.

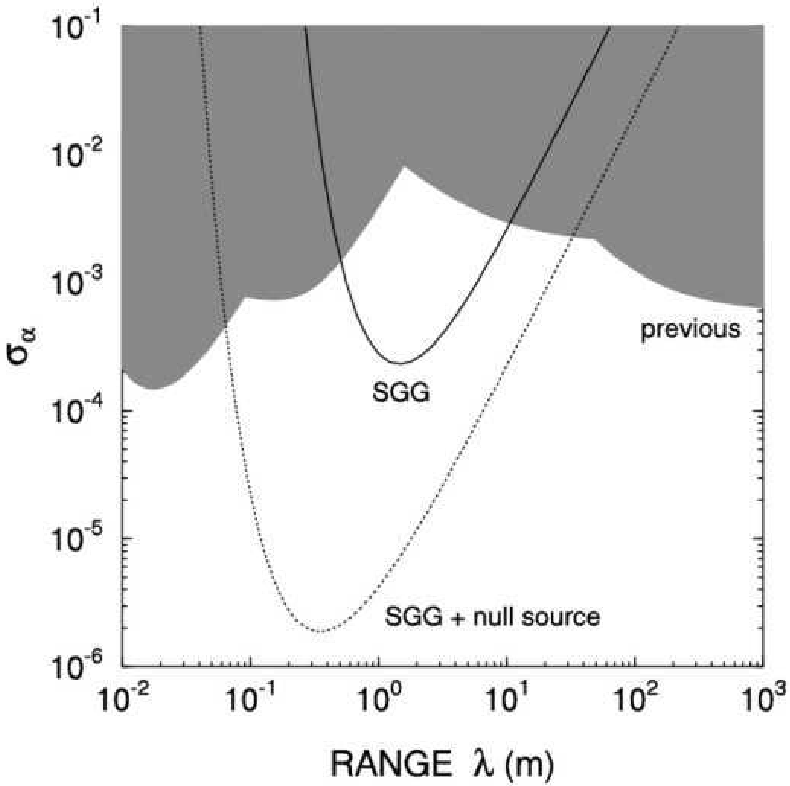
- Data was taken over 33 nights, with SGG rotated automatically by 120° at a time.



The data for Gradiometer 2 showed a sign of a current leakage and was discarded.

$1/r^2$ law result

- Resolution of the Gauss's law test



$$\sum \Gamma_{ii} = (0.58 \pm 3.10) \times 10^{-4}\ (2\sigma)$$

$$\alpha = (0.9 \pm 4.6) \times 10^{-4} \text{ at } \lambda = 1.5\,\text{m}$$

Moody & Paik, *PRL* **70**, 1195 (1993)

- Double null experiment

Source: Cylindrical shell

Detector: 3-axis SGG