Physics 798G Spring 2007

Lecture 5
Classical and Modern Tests of General Relativity

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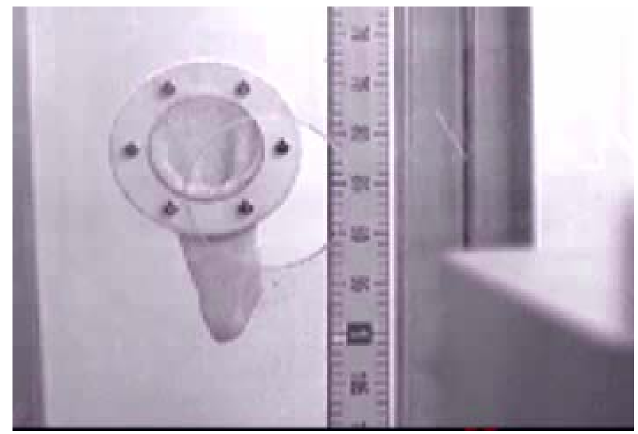
From Newton to Einstein 1

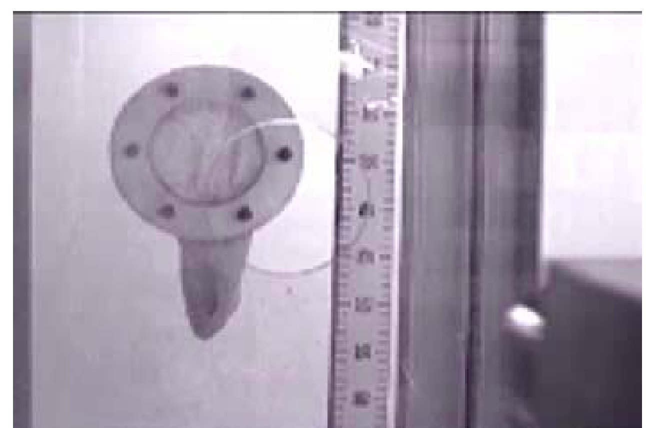
- Galileo's Universality of Free Fall: All bodies fall with the same acceleration regardless of their mass or internal composition.

$\Rightarrow$ Newton's Weak Equivalence Principle: m_P (passive grav) = m_I

Stationary



Free Fall

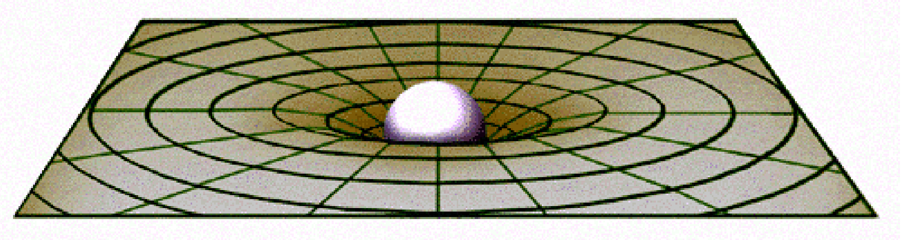


From Newton to Einstein 2

- Einstein Equivalence Principle: In a freely falling frame, all the laws of physics behave as if gravity is absent.

  (EEP $\Rightarrow$ WEP + LLI + LPI)

  $\Rightarrow$ Metric theories of gravity (curvature of spacetime)



- Strong Equivalence Principle: WEP is valid for self-gravitating bodies as well as test bodies. (GR may uniquely embody SEP.)

Special Relativity $\Rightarrow$ General Relativity

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu,$$

$$\eta_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{bmatrix}$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu,$$

$$g_{\mu\nu} = \begin{bmatrix} -\left(1-\frac{2GM}{rc^2}\right) & 0 & 0 & 0 \\ 0 & \left(1-\frac{2GM}{rc^2}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{bmatrix}$$

General Relativity 1

- Law of gravity: Newton: $\nabla^2\phi = -4\pi G\rho$ $\Rightarrow$ Einstein: $G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$

$$G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu}(g^{\alpha\beta} R_{\alpha\beta}) : \text{ Einstein tensor (curvature)}$$

$$R_{\mu\nu} = g^{\alpha\beta} R_{\beta\mu\alpha\nu}, \quad R_{\alpha\beta\gamma\delta} = \tfrac{1}{2}\left(g_{\alpha\delta,\beta\gamma} - g_{\beta\delta,\alpha\gamma} + g_{\beta\gamma,\alpha\delta} - g_{\alpha\gamma,\beta\delta}\right)$$

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu} \ : \text{ stress-energy tensor (perfect fluid)}$$

- "Field" equations: Maxwell: $\Rightarrow$ Einstein: ($\phi \approx 0$, $v \ll c = 1$)

$$\nabla \cdot \mathbf{E} = 4\pi\rho, \quad \nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0,$$
$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{B} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \frac{4\pi}{c}\mathbf{J}.$$

$$\nabla \cdot \mathbf{E}_g \approx 4\pi\rho, \quad \nabla \times \mathbf{E}_g + \frac{\partial \mathbf{B}_g}{\partial t} \approx 0,$$
$$\nabla \cdot \mathbf{B}_g \approx 0, \quad \nabla \times \mathbf{B}_g - \frac{\partial \mathbf{E}_g}{\partial t} \approx -16\pi\rho\mathbf{v}.$$

- Equation of motion: Newton: $\frac{d^2x}{dt^2} = \frac{F}{m}$ $\Rightarrow$ Einstein:

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0$$

geodesic equation

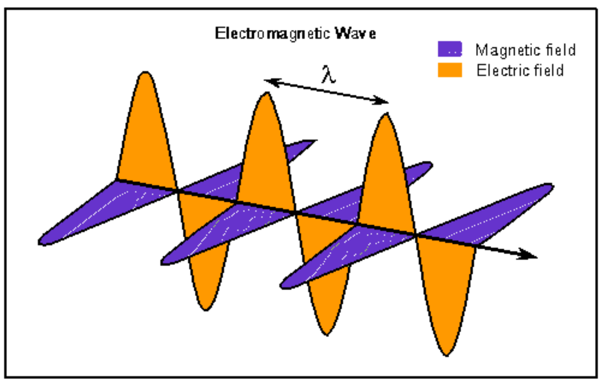
$$\Gamma_{\alpha\beta\gamma} = \tfrac{1}{2}\left(g_{\beta\alpha,\gamma} - g_{\gamma\beta,\alpha} + g_{\alpha\gamma,\beta}\right) : \text{ metric connections}$$

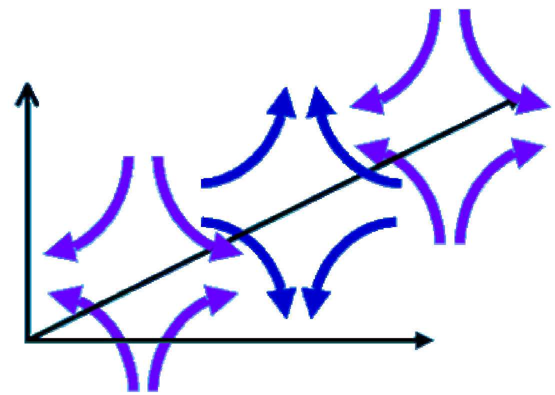
General Relativity 2

- Weak field, low velocity limit: $G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad \Rightarrow \quad \nabla^2 \phi = -4\pi G \rho$
- Wave equation: EM wave: Gravitational wave:

$$\left(\nabla^2 - \frac{1}{c^2} \frac{1}{\partial t^2} \right)(\mathbf{E}, \mathbf{B}) = 0,$$

$$\left(\nabla^2 - \frac{1}{c^2} \frac{1}{\partial t^2} \right) h_{\mu\nu} = 0, \quad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$





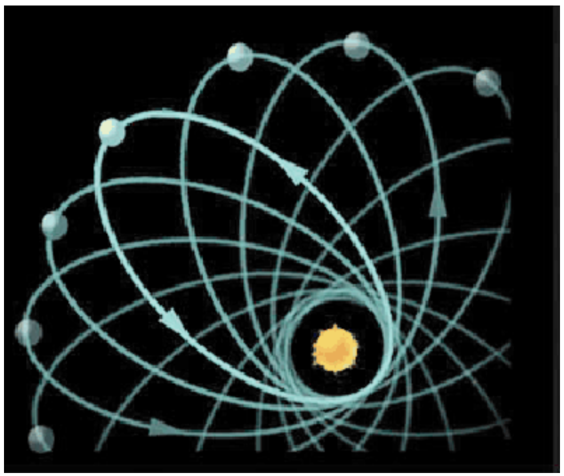
- Expanding universe ⇒ Big Bang cosmology
- Generalized field equation: $G_{\mu\nu} - \lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$

λ: cosmological constant (~10^{-29}g/cm^3 : "dark energy"?)

Classical Tests of General Relativity 1

1. Perihelion shift of Mercury

The deviation of Mercury orbit around the sun from its Kepler orbit due to a correction to the $1/r^2$ law.



Einstein:

$$\frac{d\phi}{dt} = \frac{6\pi G}{c^2} \frac{M}{a(1-e^2)\tau}$$

Cause	Perihelion shift (arcsec/century)
Perturbation from other planets	532
Additional shift	43.1 ± 0.5
Another planet (Vulcan)?	undetected
Oblateness of the sun?	< 0.1
General Relativity	43.0

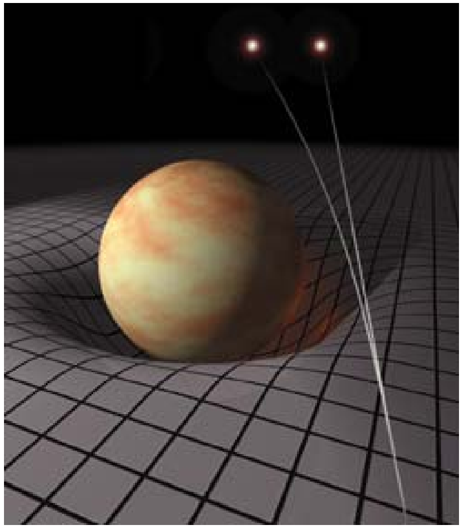
Classical Tests of General Relativity 2

2. Deflection of light

When light passes near the sun, its path is slightly bent.

Einstein: $\Delta\phi = \frac{4G}{c^2}\frac{M_S}{b}$, twice the Newtonian effect

$\Rightarrow$ Gravitational lensing



Prediction/Experiment	Deflection (arcsec)
General Relativity	1.750
Eddington (1919)	1.98 ± 0.16 (Sobral) 1.16 ± 0.40 (Principe)
Fomalont and Sramek (1976)	1.760 ± 0.016

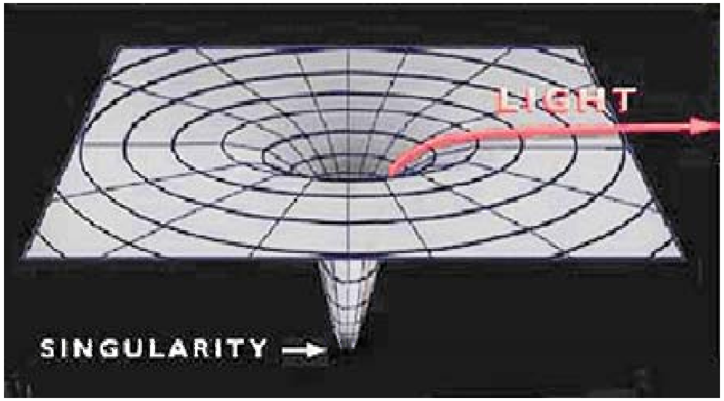
Classical Tests of General Relativity 3

3. Gravitational red shift

Clocks in a gravitational potential well observed from a stationary clock at a distant point appear to tick slower.

Einstein: $$\frac{\Delta \nu}{\nu} = \frac{\Delta U}{c^2} = \frac{gh}{c^2}$$

(Einstein Equivalence Principle)

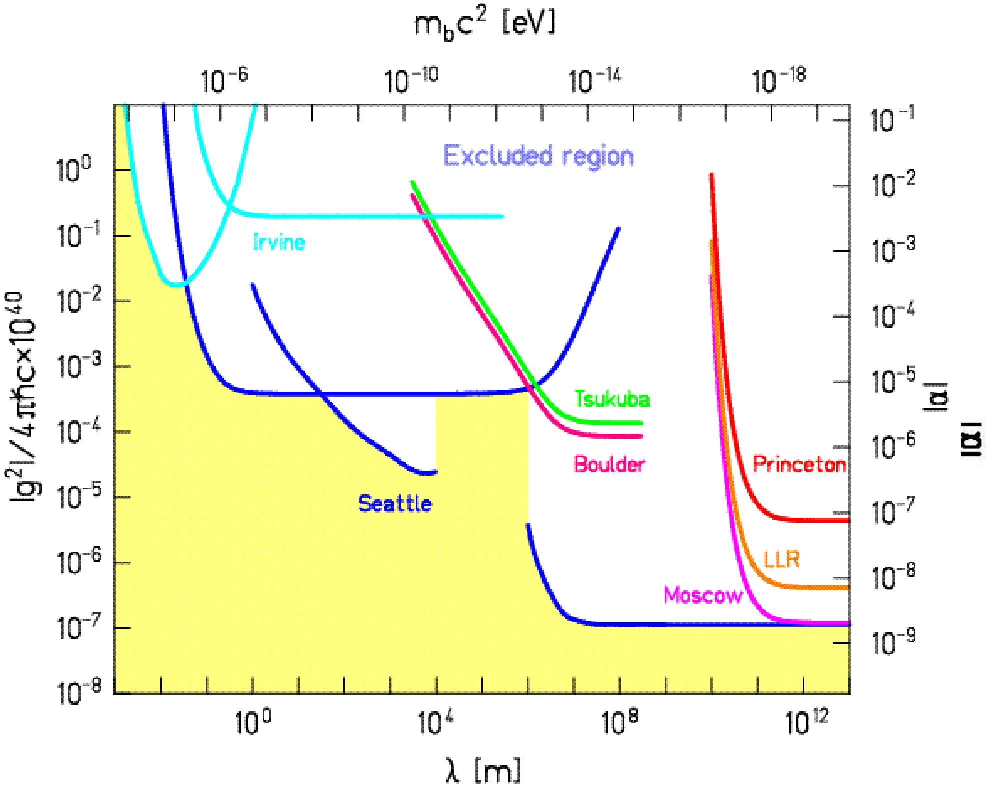


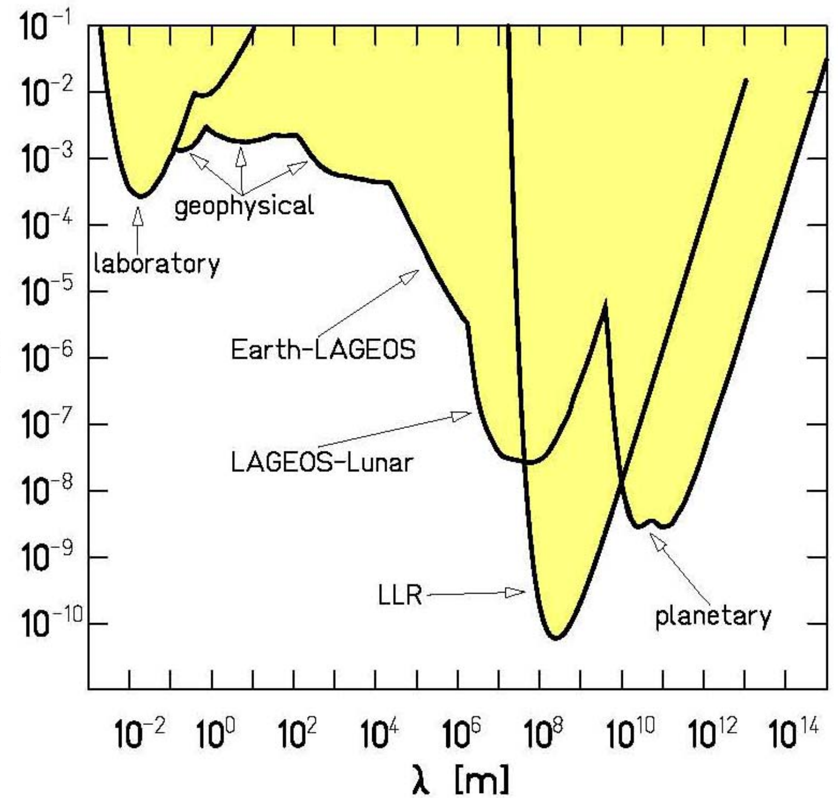
Experiment	Red shift	Agreement with GR
Pound-Rebka (1960) Pound-Snider (1964)	5×10^{-15} for h = 22.6 m	10^{-1} 10^{-2}
Vessot (1980) “Gravity Probe A”	4×10^{-10} for h = 10,000 km	7×10^{-5}

Modern Tests of General Relativity 1

1. Weak Equivalence Principle and the Inverse-Square Law

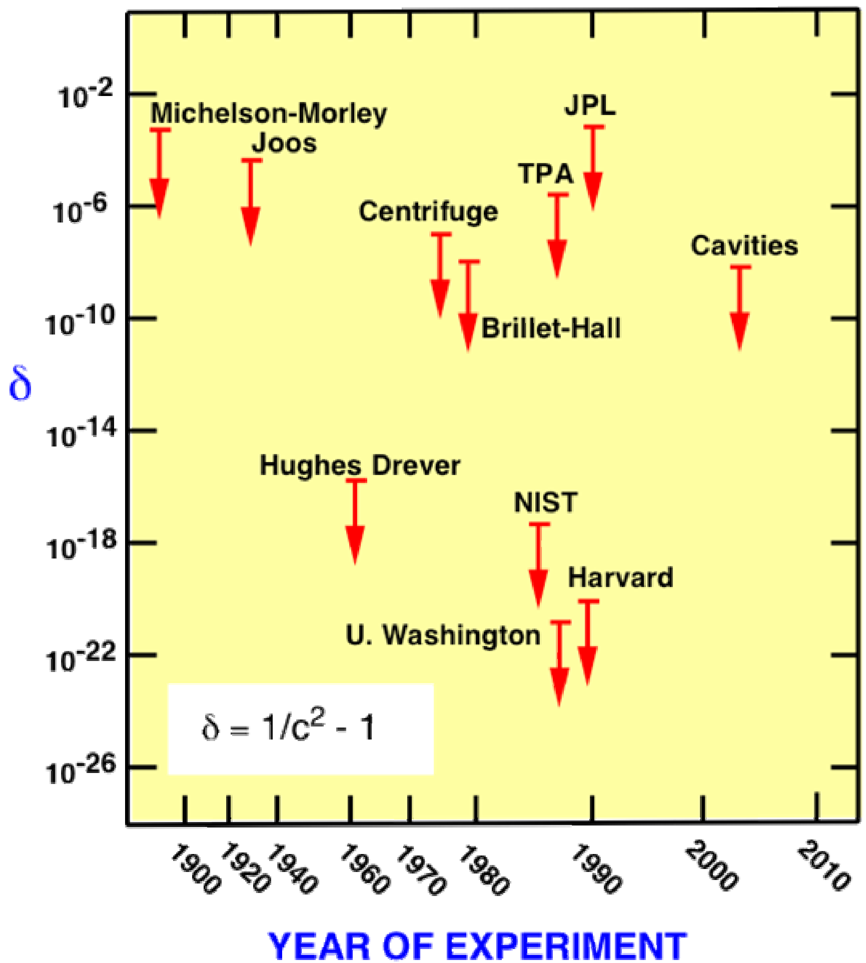
For a generalized potential: $$V(r) = -G\frac{m_1 m_2}{r}\left(1+\alpha\, e^{-r/\lambda}\right),\ \lambda = \frac{\hbar}{m_b c}$$

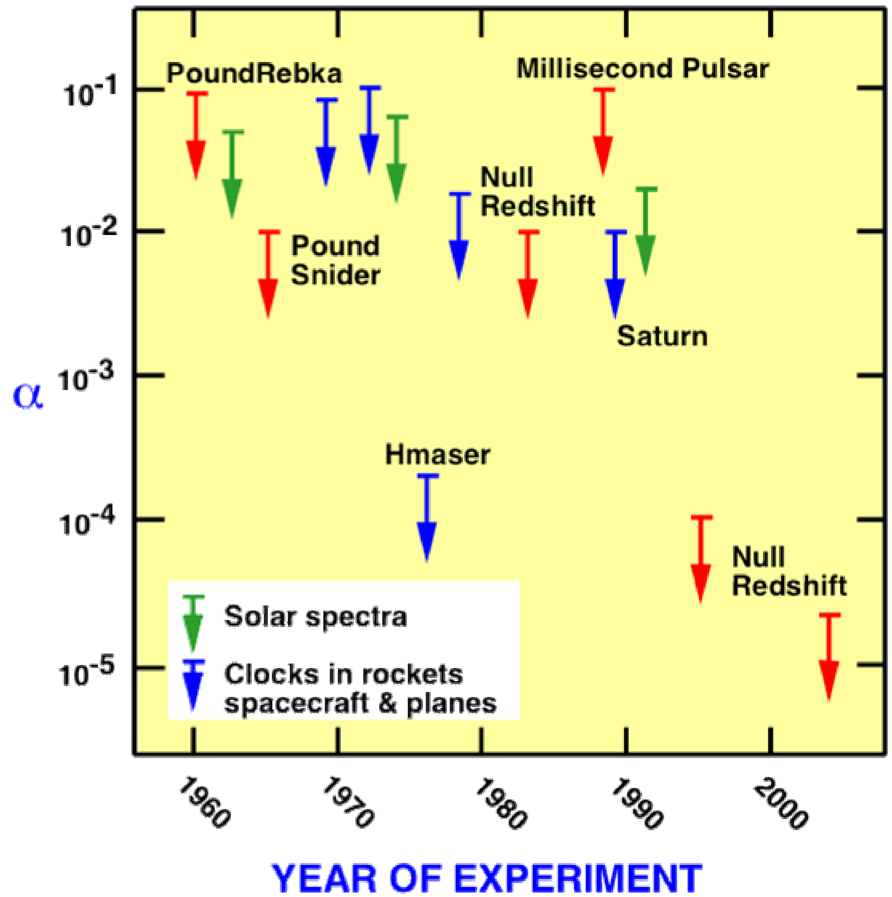




Modern Tests of General Relativity 2

2. Local Lorentz Invariance and Local Position Invariance



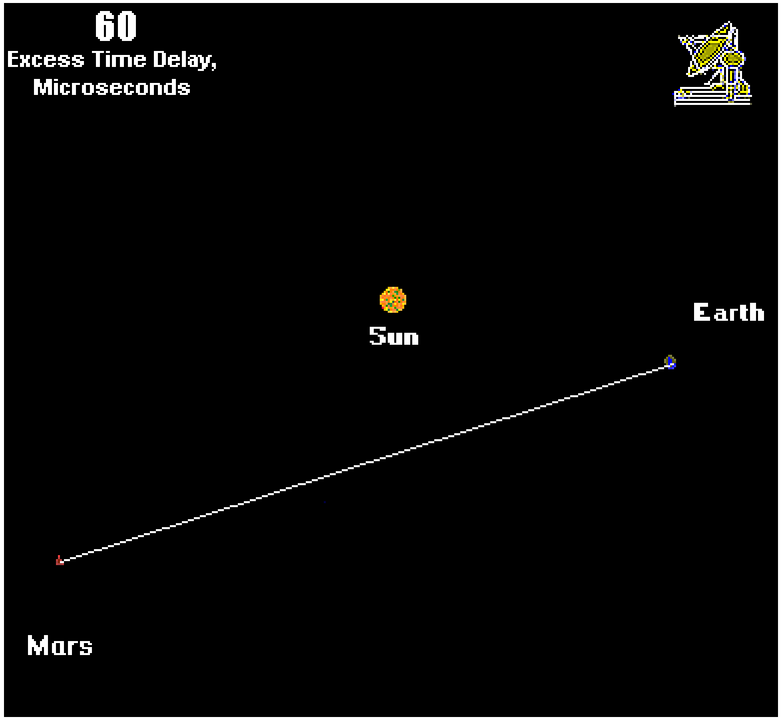


$$\Delta\nu/\nu = (1+\alpha)\Delta U/c^2$$

Modern Tests of General Relativity 3

3. Gravitational time delay (Shapiro effect)

As the radar and radio beams pass close to the Sun, a delay in the transit time is measured. This delay is caused by the gravitational force of the Sun.



Gravitational time delay:

$$\Delta t = \frac{4GM}{c^3}\left[\ln\left(\frac{4r_E r_M}{b^2}\right)+1\right]$$

Experiment	Agreement with GR
Shapiro (1968) Radar to Mars	10^{-1}
Shapiro *et al*. (1970's) Radio from Viking	10^{-3}

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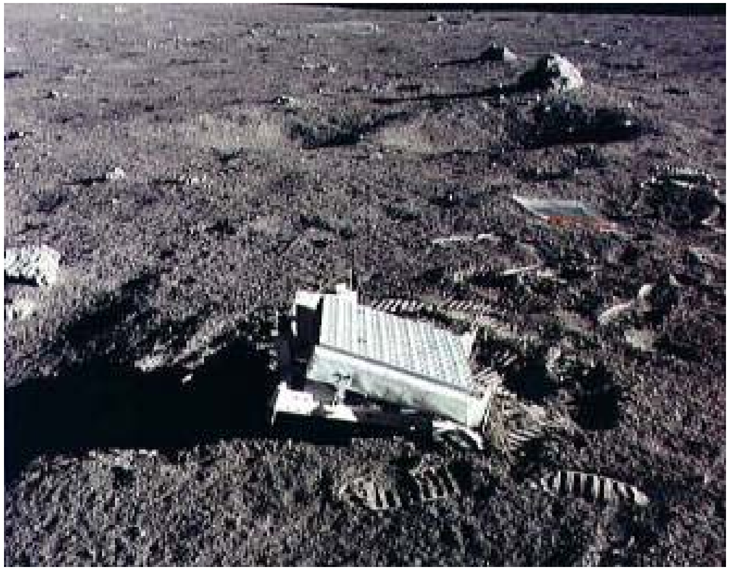
4. Strong Equivalence Principle

Free fall is independent of gravitational self energy GM^2/r.

$$\frac{m_g}{m_i} = 1 + \eta E_{self} + \eta' E_{self}^2, \quad E_{self} = -\frac{GM}{rc^2} \quad \Rightarrow \quad \text{Uniquely GR}$$

(The Nordtvedt effect)

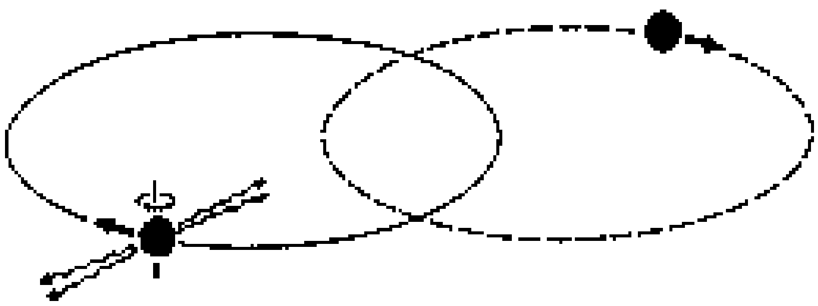


Experiment	Limit on $|\eta|$
Lunar laser ranging (1972-) weak field	$< 1.6 \times 10^{-3}$
Binary pulsar (2004) strong field	$< 10^{-3}$

Modern Tests of General Relativity 5

5. Binary pulsar timing

GR predicts that such a system will radiate energy in gravitational waves, causing the stars to slowly spiral towards each other.



Decay of the orbit period:

$$\frac{1}{\tau}\frac{d\tau}{dt} = -\eta(e)\frac{12}{5}\frac{G^3M^3}{c^5a^4}$$

Prediction/Experiment	$d\tau/dt$
General Relativity	$-2.403(2) \times 10^{-12}$
Taylor *et al.* (1975-)	$-2.40(9) \times 10^{-12}$