Overview of Superconducting Qubits

Many of these slides are due to
Prof. Dale van Harlingen, UIUC,
and were used in his Physics 498 Superconducting Quantum Devices class

These slides are intended for use by students in Phys798C Superconductivity
at the University of Maryland, Fall 2025

Classic vs. Quantum Logic

CLASSICAL LOGIC: two distinct states "bit"

$$0 \quad or \quad 1$$

can do all operations from NOT and exclusive-OR gates

Single-bit operation — Two-bit operation

perform series of operations on bits to get final answer

QUANTUM LOGIC: superposition of two quantum levels "qubit" = quantum bit

$$\Psi = a|0\rangle + b|1\rangle$$

can do all operations from single-qubit and controlled-NOT functions

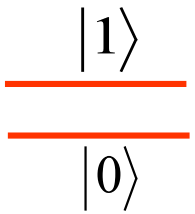
unitary transformations (e.g. rotations) — Two-qubit operation

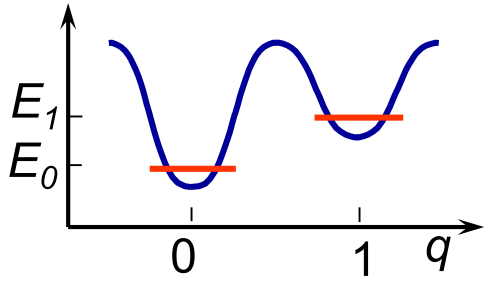
"Entangle" qubits and allow quantum evolution to "project out" answer

Key to quantum computation = entanglement

qubit = quantum two-level system $|0\rangle$ *and* $|1\rangle$

Superposition: $\Psi = a|0\rangle + b|1\rangle$





Entanglement: interference of two qubits $\Psi = A|00\rangle + B|01\rangle + C|10\rangle + D|11\rangle$

Performing logic operations with entangled states allows the quantum evolution to sample multiple states …
effectively massive parallel computation

Quantum mechanically, a register of N entangled qubits can store 2^N states in superposition:

e.g. A 300-qubit register can simultaneously store $2^{300} \sim 10^{90}$ numbers

2037035976334486086268445688409378161051468393665936250636140449354381299763336706183397376

This is more than the total number of particles in the Universe!

(Google SC quantum computer has 57 qubits = only 144,115,188,075,855,872 states)

Some problems benefit from this exponential scaling, enabling solutions of otherwise insoluble problems.

Macroscopic Quantum Phenomena

Superconductors are intrinsically coherent quantum systems --- we have seen this in Cooper pairing, Josephphson tunneling, and the RSJ model in which we characterized the junction phase as a "particle" with inertia and damping moving in a "washboard" potential that incorporates the periodicity of the current-phase relation

However, there is another layer at which SC systems are "quantum" in which the "phase particle" itself, which represents the whole system, can behave quantum mechanically.

Three phenomena:

1. Macroscopic Quantum Tunneling (MQT) --- tunneling through the washboard potential

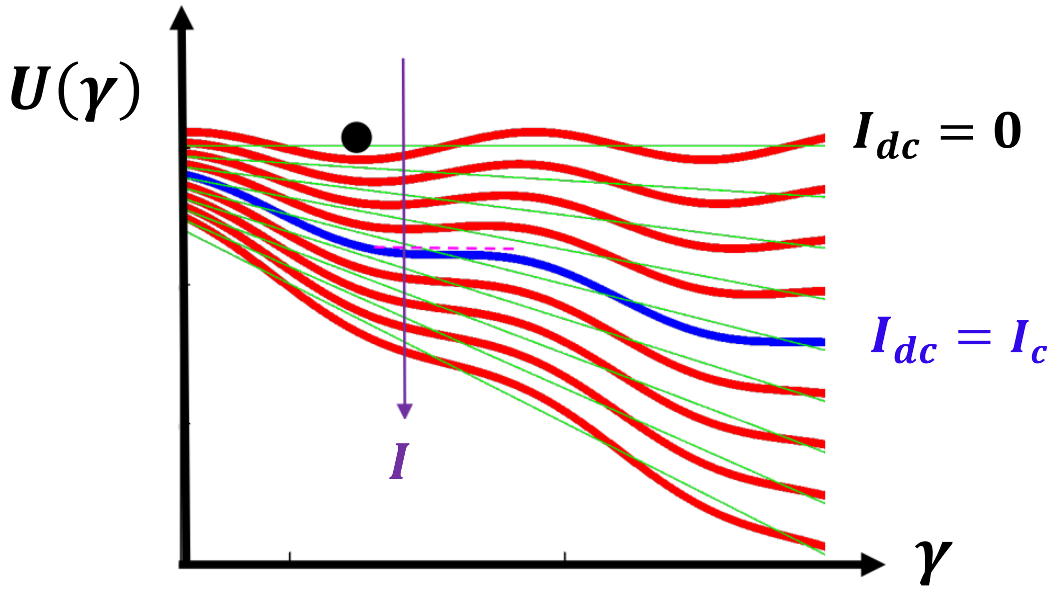
2. Quantized energy levels --- quantum mechanical behavior of the phase particle

3. Macroscopic Quantum Coherence (MQC) --- superposition of discrete quantum states

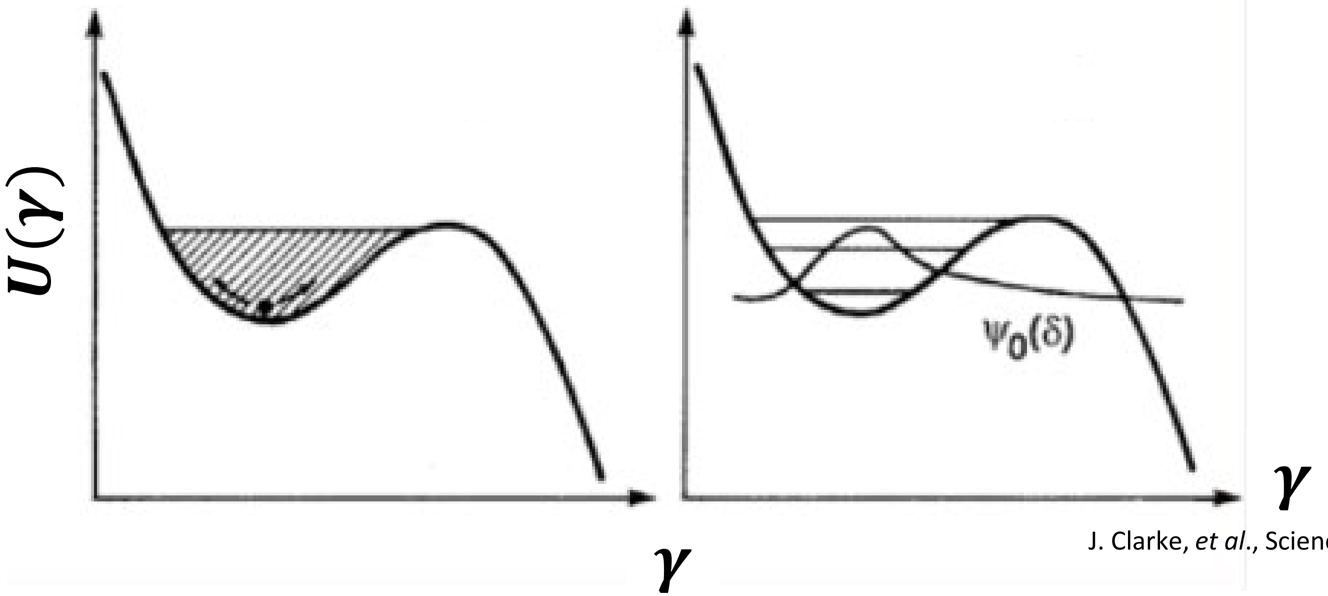
This was a major focus on superconductor device research in the 1980's

When is a Current-Biased Josephson Junction in the Quantum Limit?

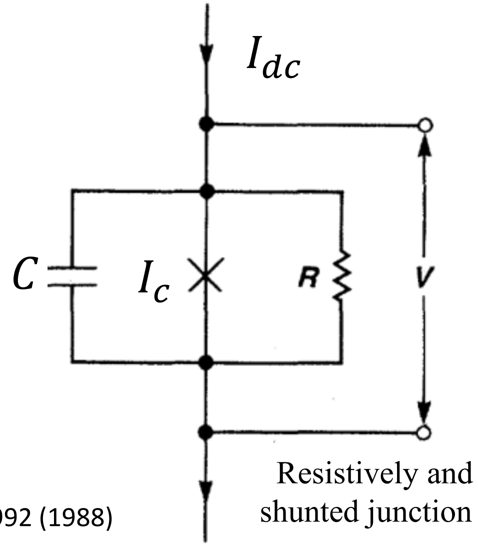
A current-biased Josephson junction has a potential given by $U(\gamma) = -\frac{\Phi_0}{2\pi}\,(I_{dc}\,\gamma + I_c \cos\gamma)$,
where γ is the gauge-invariant phase difference and Φ_0 is the flux quantum.
This creates the tilted washboard potential as a function of gauge-invariant phase.



When is the Problem Quantum as opposed to Classical?



J. Clarke, *et al.*, Science 239, 992 (1988)



Resistively and capacitively-shunted junction (RCSJ) model

DC Josephson equation
$\boldsymbol{I = I_c \sin\gamma}$

The gauge-invariant phase point oscillates in an (approximately) harmonic oscillator potential characterized by a curvature given by the dc-current-dependent Josephson plasma frequency $\omega_p(I_{dc})$:

$$E_n \approx \hbar\omega_p\left(n+\frac{1}{2}\right) - \text{small anharmonic corrections.} \qquad \text{with} \qquad \omega_p = \sqrt{\frac{2eI_c}{\hbar C}}\left(1-\left(\frac{I_b}{I_c}\right)^2\right)^{1/4}$$

$n = 0,1,2,3,\ldots$

To see quantum transitions between the states the thermal energy must be small compared to the energy level spacing

$$\boldsymbol{k_B T \ll \hbar\omega_p}$$

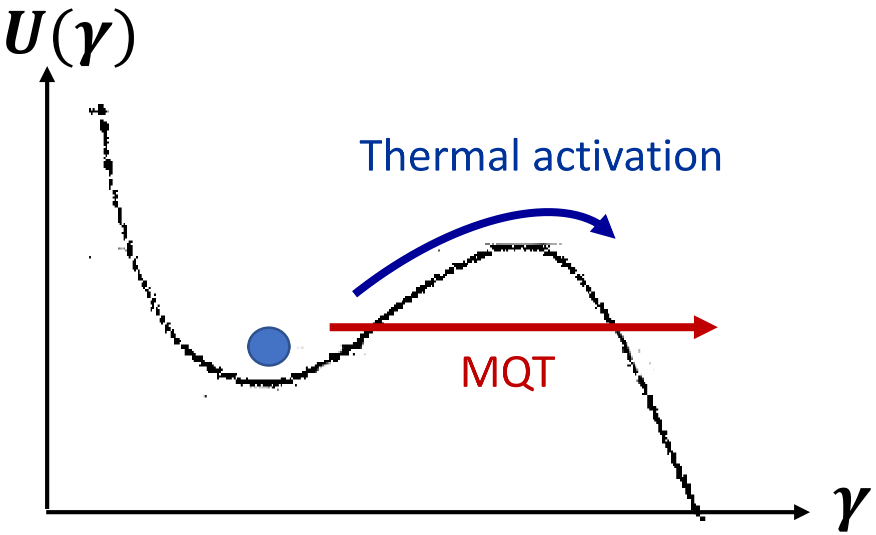
To minimize the effects of loss we also require: $\boldsymbol{Q = \omega_p RC \gg 1}$, in the language of the RCSJ model, shown above

This approach leads to a phase qubit

Macroscopic Quantum Tunneling (MQT) --- can a macroscopic system tunnel?

First test of QM in a macroscopic system

Consider onset of resistance --- Can the phase particle tunneling from well, like a quantum object?

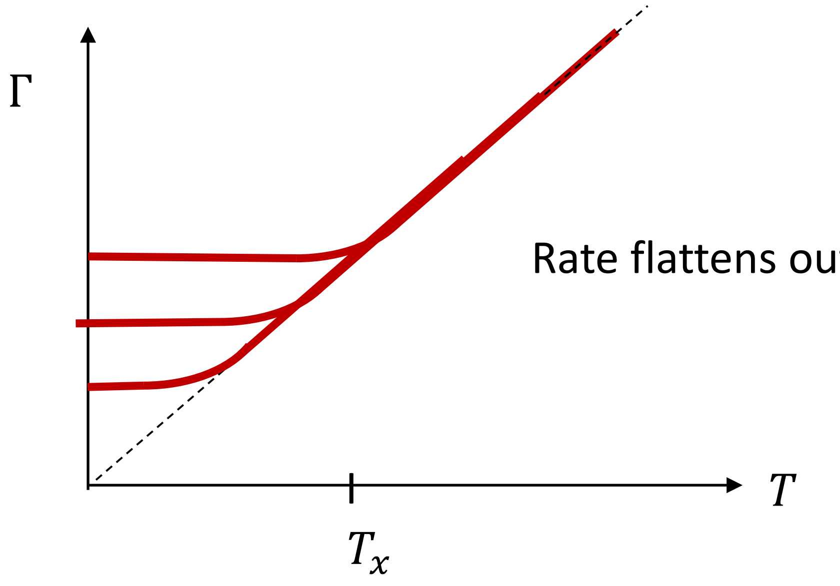


Electrons tunnel. Alpha particles tunnel? What about macroscopic objects?

Can your car tunnel out of your garage?

Thermal $\quad \Gamma_T = A_T \left(\frac{\omega_p}{2\pi}\right) e^{-\frac{\Delta U}{k_B T}}$ where $Q = \omega_P RC$

Quantum $\quad \Gamma_Q = A_Q \left(\frac{\omega_p}{2\pi}\right) e^{-7.2 \frac{\Delta U}{\hbar \omega_p}\left(1+\frac{0.87}{Q}\right)}$



Rate flattens out at low T

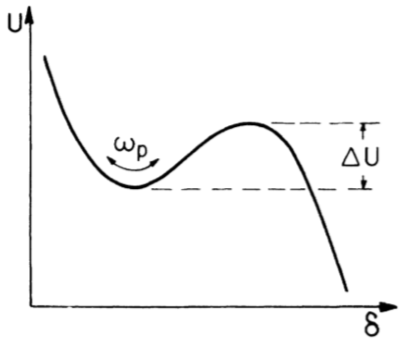
Absence of damping (Q large), MQT dominates for $\hbar\omega_p = 7.2 k_B T$

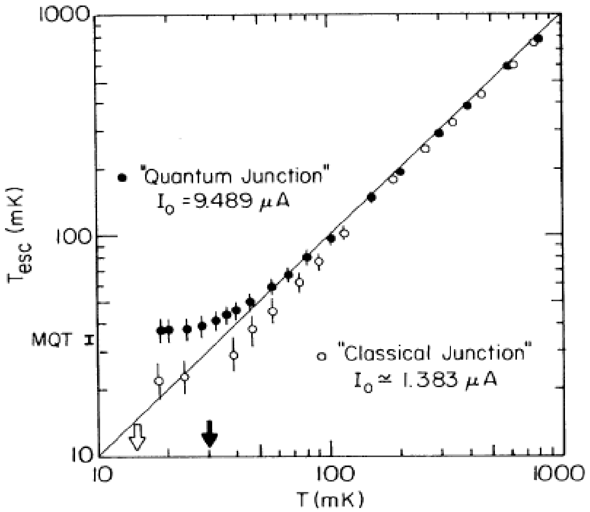
VOLUME 55, NUMBER 18 PHYSICAL REVIEW LETTERS 28 OCTOBER 1985

Measurements of Macroscopic Quantum Tunneling out of the Zero-Voltage State of a Current-Biased Josephson Junction

Michel H. Devoret,[a] John M. Martinis, and John Clarke
Department of Physics, University of California, Berkeley, California 94720, and Materials and Molecular Research Division, Lawrence Berkeley Laboratory, Berkeley, California 94720
(Received 26 July 1985)

The escape rate of an underdamped ($Q \approx 30$), current-biased Josephson junction from the zero-voltage state has been measured. The relevant parameters of the junction were determined *in situ* in the thermal regime from the dependence of the escape rate on bias current and from resonant activation in the presence of microwaves. At low temperatures, the escape rate became independent of temperature with a value that, with no adjustable parameters, was in excellent agreement with the zero-temperature prediction for macroscopic quantum tunneling.



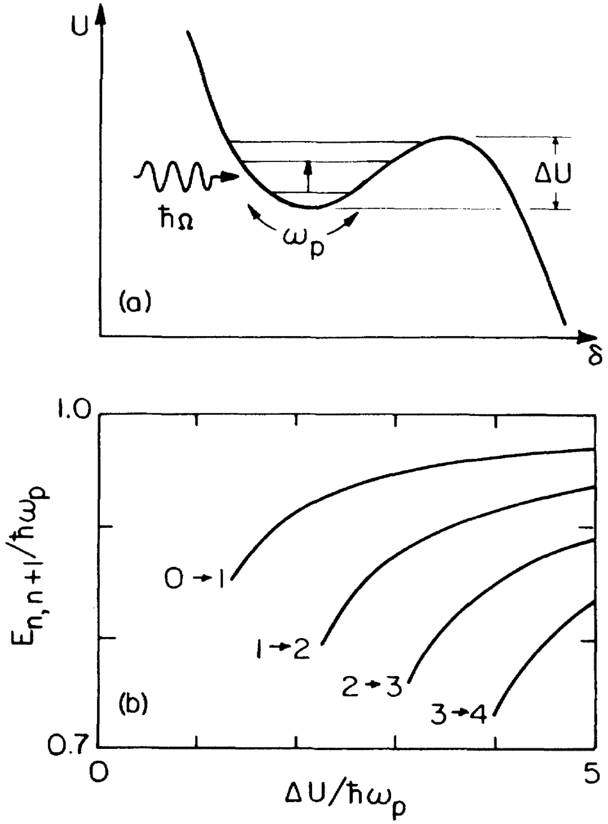


In this experiment, the curvature around the transition comes from partial thermal activation followed by MQT through the narrower barrier --- this is predicted to be quadratic

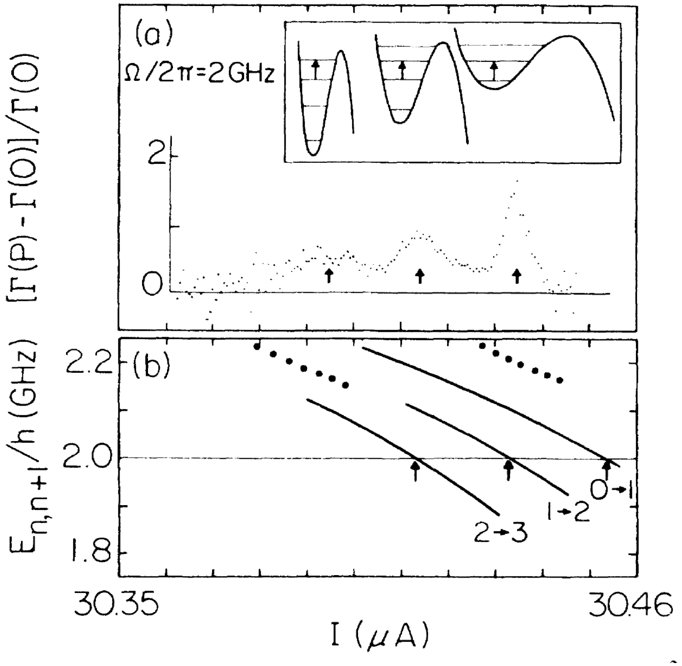
Quantized energy levels --- quantum particles in an anharmonic potential

Measured by “resonant activation” --- apply microwave irradiation and excite phase particle to excited levels to enhance tunneling out of the well

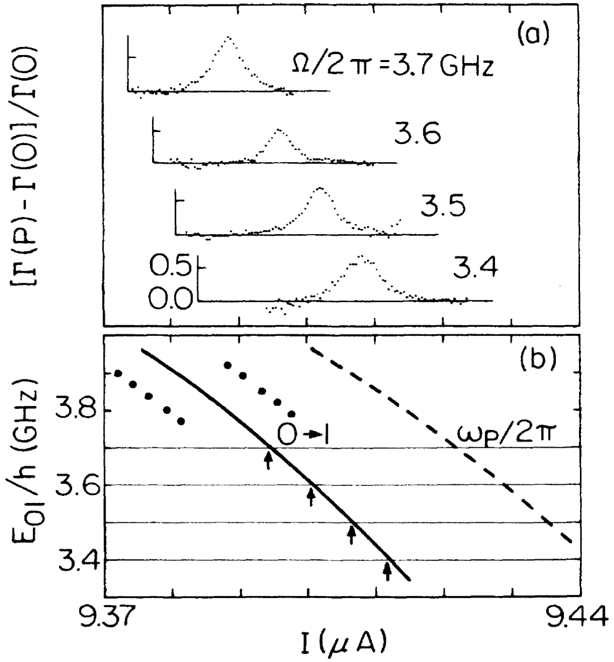
Structure in the transition distribution depends on the discrete energy levels occupied



For different transitions
at a fixed frequency



For the 0→1 transition
at different frequencies



When is an RF SQUID in the Quantum Limit?

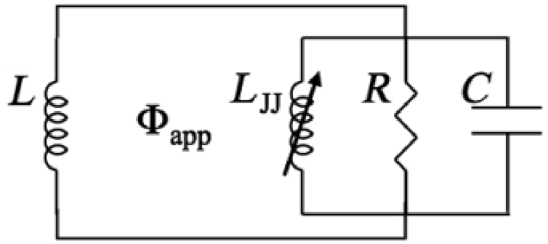
SQUID = Superconducting Quantum Interference Device

RF SQUID: a superconducting loop (inductance L) interrupted by a single Josephson junction

Magnetic flux Φ_{app} applied to the loop

Josephson junction contributes nonlinear and tunable inductance $L_{JJ} = \frac{\Phi_0}{2\pi\, I_c\ \cos\gamma}$

RCSJ model of JJ: shunt capacitance C and resistance R



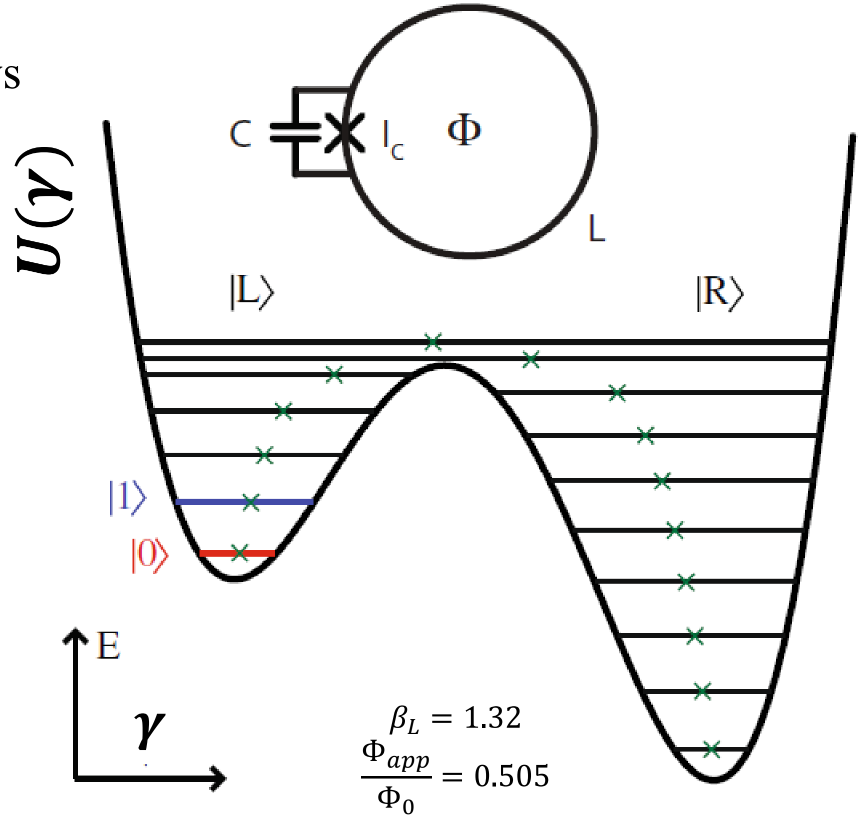
$$U(\gamma) = -\frac{1}{2L}\left(\frac{\Phi_0}{2\pi}\right)^2\left(\gamma - 2\pi\frac{\Phi_{app}}{\Phi_0}\right)^2 - E_J\cos\gamma \qquad \text{with} \qquad E_J = \frac{\Phi_0 I_c}{2\pi}$$

Un-driven RF SQUID dynamics given by: $\frac{1}{2}C\left(\frac{\Phi_0}{2\pi}\right)^2\dot{\gamma}^2 + U(\gamma) = 0$

When is the Problem Quantum as opposed to Classical?

Example of quantized energy levels in an RF SQUID near $\frac{\Phi_{app}}{\Phi_0} \approx \frac{1}{2}$

$\beta_L = \frac{2\pi L I_c}{\Phi_0} > 1$ shows two-well minima



D. A. Bennett, *et al.*, Quantum Info Process 8, 217 (2009)

1) Low temperature such that $\boldsymbol{k_B T} \ll \hbar\boldsymbol{\omega_p}$

$$\omega_p = \frac{1}{\sqrt{\left(\frac{1}{L} + \frac{1}{L_{JJ}(T, \Phi_{app})}\right)^{-1} C}}$$

is the plasma frequency of a local well

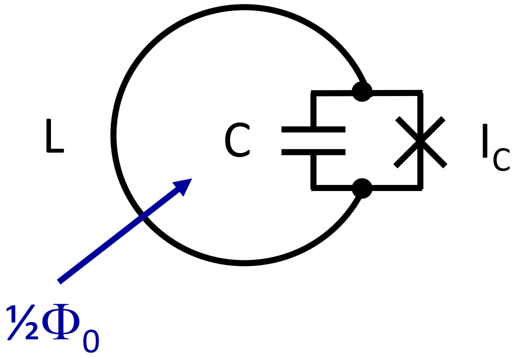
2) To minimize the effects of loss we also require: $\boldsymbol{Q = \omega_p RC \gg 1}$, in the language of the RCSJ model

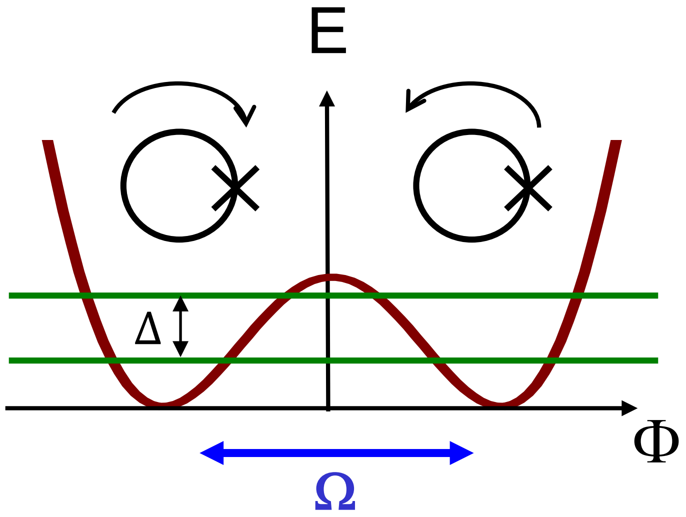
3) Intermediate $\beta_L \sim 1 - 3$ so that the potential supports 1 or 2 minima without too high a level density

4) Flux biasing near the half-quantum $\frac{\Phi_{app}}{\Phi_0} \approx \frac{1}{2}$ to create a symmetric double-well

This approach leads to a flux qubit

rf SQUID at $\Phi_x = \frac{\Phi_0}{2}$ → double-well potential characterized by the direction of circulating current





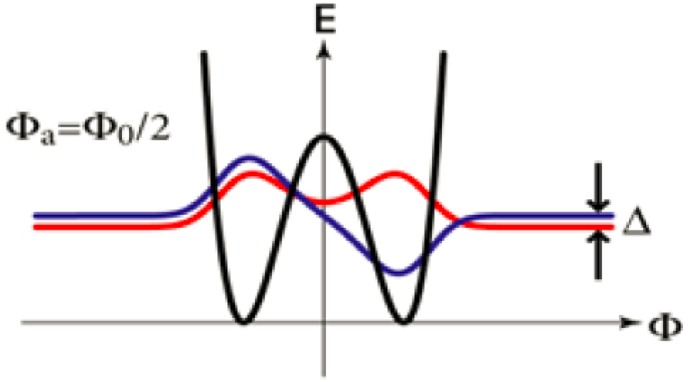
Tunnel coupling creates symmetric and anti-symmetric energy levels:

$$\psi_E = \frac{1}{\sqrt{2}}(\psi_R + \psi_L)$$

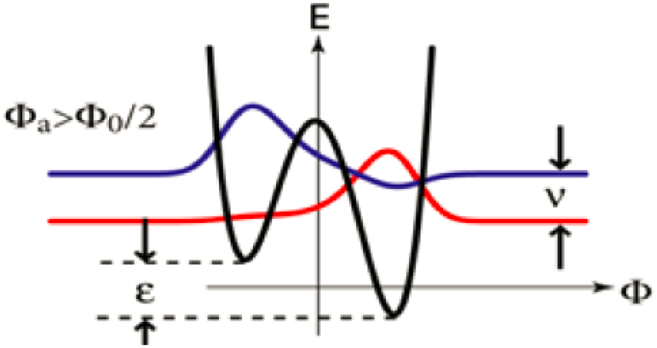
$$\psi_0 = \frac{1}{\sqrt{2}}(\psi_R - \psi_L)$$

Wavefunctions of the particle in the double-well potential:

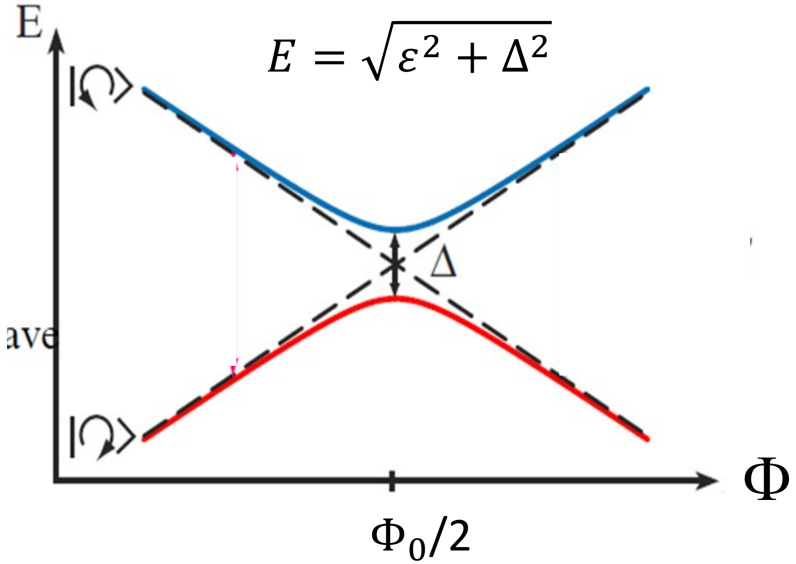
degeneracy point $\Phi = \Phi_0/2$



away from degeneracy point

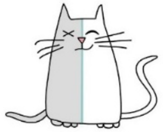


Energy level separation has avoided crossing:



Macroscopic Quantum Coherence (MQC) ---
can a macroscopic system be in a superposition of states?

A manifestation of the Schroedinger's cat paradox



"I have good news and bad news"

ANNALS OF PHYSICS **149**, 374–456 (1983)

Quantum Tunnelling in a Dissipative System

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In this paper we attempt to motivate, define, and resolve the question "What is the effect of dissipation on quantum tunnelling?" The question is of particular interest in the context of tunnelling of a macroscopic variable such as the trapped flux in a SQUID, where we show that it is crucial to resolve it in the context of tests of the validity of quantum mechanics at the macroscopic level, but it is also relevant to various microscopic tunnelling situations. We define the question as follows: Suppose we have a system, which has a metastable minimum and whose quasiclassical equation of motion in the region near the minimum is given by

$$M\ddot{q} + \eta\dot{q} + \partial V/\partial q = F_{\text{ext}}(t),$$

where the potential $V(q)$ and friction coefficient η are regarded as experimentally determined quantities (and the energy dissipated irreversibly per unit time is simply $\eta\dot{q}^2$). How does the tunnelling behaviour of such a system at $T = 0$ differ from that of one obeying a similar equation, with the same potential $V(q)$ and mass M, but with friction coefficient η equal to zero?

PHYSICAL REVIEW LETTERS

VOLUME 54 | 4 MARCH 1985 | NUMBER 9

Quantum Mechanics versus Macroscopic Realism: Is the Flux There when Nobody Looks?

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(Received 19 November 1984)

It is shown that, in the context of an idealized "macroscopic quantum coherence" experiment, the predictions of quantum mechanics are incompatible with the conjunction of two general assumptions which are designated "macroscopic realism" and "noninvasive measurability at the macroscopic level." The conditions under which quantum mechanics can be tested against these assumptions in a realistic experiment are discussed.

PACS numbers: 03.65.Bz, 05.30.−d, 74.50.+r, 85.25.+k

Despite sixty years of schooling in quantum mechanics, most[1] physicists have a very non-quantum-mechanical notion of reality at the macroscopic level, which implicitly makes two assumptions. (A1) Macroscopic realism: A macroscopic system with two or more macroscopically distinct[2,3] states available to it will at all times *be* in one or the other of these states. (A2) Noninvasive measurability at the macroscopic level: It is possible, in principle, to determine the state of the system with arbitrarily small perturbation on its subsequent dynamics. A direct extrapolation of quantum mechanics to the macroscopic level denies this. The aim of this Letter is (1) to point out that under certain conditions the experimental predictions of the conjunction of (A1) and (A2) are incompatible with those of quantum mechanics extrapolated to the macroscopic level, and (2) to investigate how far these conditions may be met in a realistic experiment.

To this end, let us consider the (as yet unobserved) phenomenon of "macroscopic quantum coherence" (MQC) in an rf SQUID.[4] We take the potential $V(q)$ for the trapped magnetic flux q to be reflection symmetric (see Fig. 1) with minima at $\pm q_0$ far enough apart that states in which q is close to $+q_0$ and $-q_0$ can be regarded as macroscopically distinct. For an isolated SQUID, quantum mechanics predicts that if the flux is initially in one well, it will oscillate back and forth with some frequency Δ_0. A more realistic quantum mechanical calculation[5] which includes the irremovable environmental effects shows that for low enough temperature and weak enough coupling to the environment, the oscillations are not entirely des-

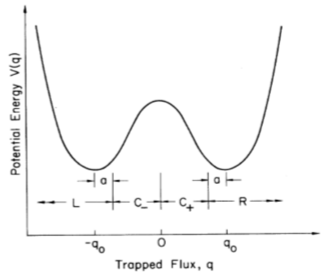


FIG. 1. The potential $V(q)$ for the trapped flux q. The various notations are explained in the text.

Why are superconductors good candidates for MQT and MQC?

Advantages

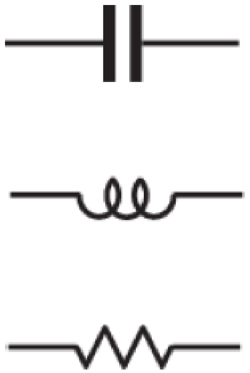
1. **Intrinsically low temperature quantum devices**
2. **Energy gap suppresses low energy excitations**
3. **Zero dc resistance**
4. **Josephson effect provides periodic potential**
5. **High characteristic frequencies**
6. **Convenient device coupling via flux transformers**
7. **Established fabrication techniques**

Disadvantages

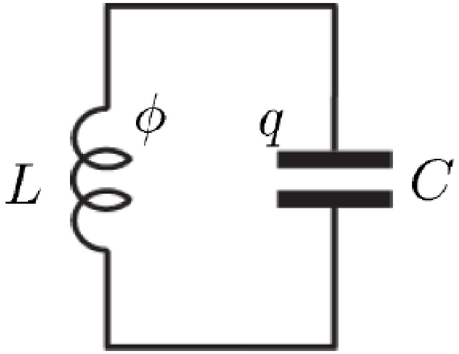
1. **Macroscopic devices --- strong coupling to environment**
2. **Strong sensitivity of Josephson tunneling to barrier details**

Constructing Linear Quantum Electronic Circuits

basic circuit elements:



harmonic LC oscillator:



$$\omega = \frac{1}{\sqrt{LC}} \sim 5\,\mathrm{GHz}$$

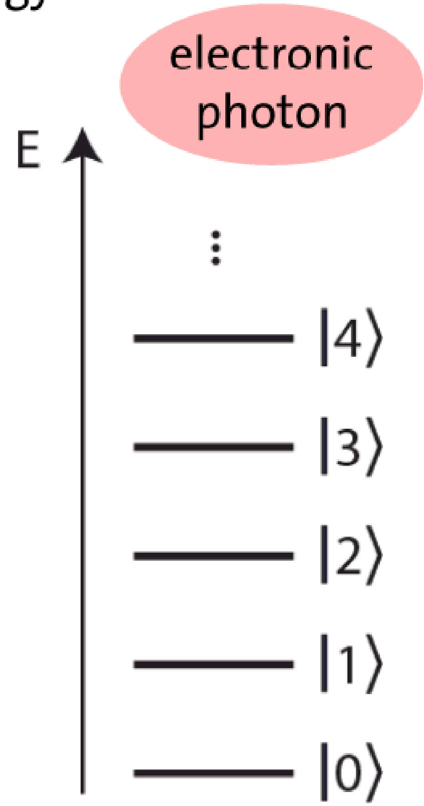
classical physics:

$$H = \frac{\phi^2}{2L} + \frac{q^2}{2C}$$

quantum mechanics:

$$\hat{H} = \frac{\hat{\phi}^2}{2L} + \frac{\hat{q}^2}{2C} = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2}) \qquad \left[\hat{\phi}, \hat{q}\right] = i\hbar$$

energy:



ETH
Eidgenössische Technische Hochschule Zürich
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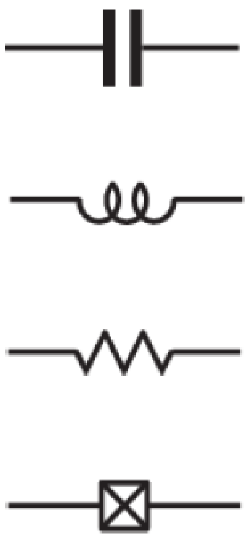
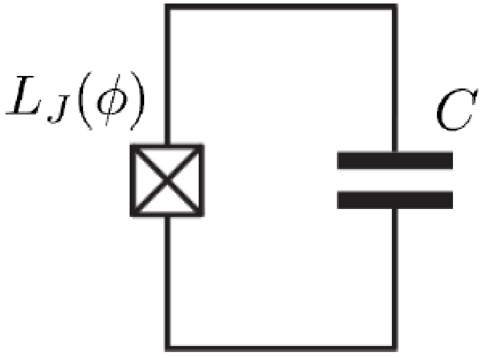
Constructing Non-Linear Quantum Electronic Circuits

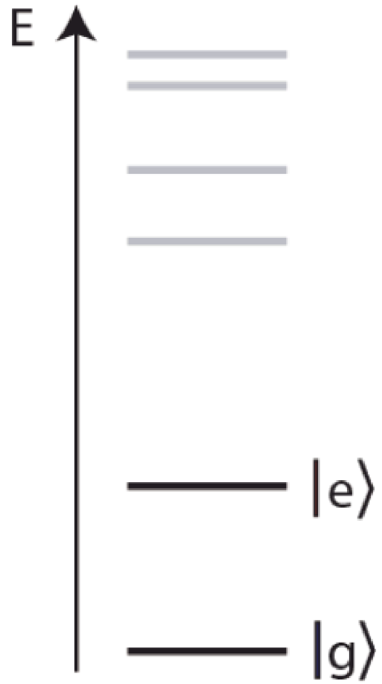
circuit elements:



anharmonic oscillator:



non-linear energy level spectrum:



Josephson junction:

a non-dissipative nonlinear element (inductor)

$$L_J(\phi) = \left(\frac{\partial I}{\partial \phi}\right)^{-1} = \frac{\phi_0}{2\pi I_c} \frac{1}{\cos(2\pi\phi/\phi_0)}$$

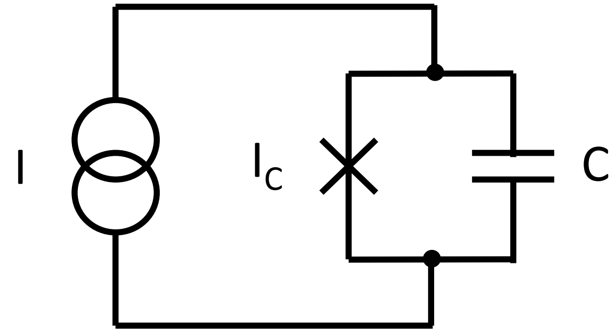
electronic artificial atom

Characteristic signatures of qubit behavior --- evidence for superposition of quantum states

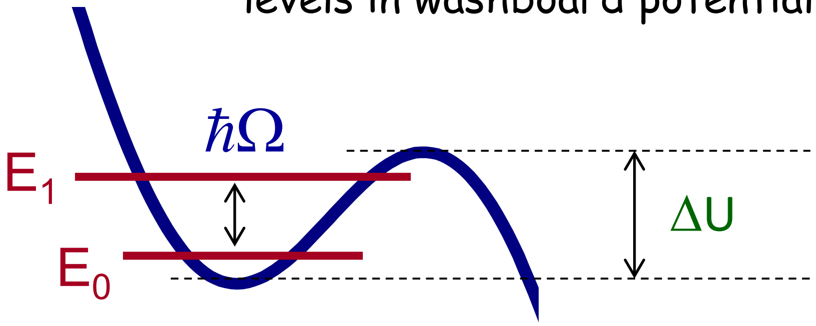
1. Avoided crossings --- evidence of energy splitting of quantum states
2. Energy relaxation --- finite lifetime in excited state (decay to ground state) --- way to measure T_1
3. Rabi oscillations --- resonances between ground and excited states
4. Ramsey fringes – dephasing resonances between ground and excited states --- way to measure T_2
5. Spin echo effects --- way to reduce dephasing

Phase (single Josephson junction) qubit

Current-biased single junction



Oscillations between energy levels in washboard potential well

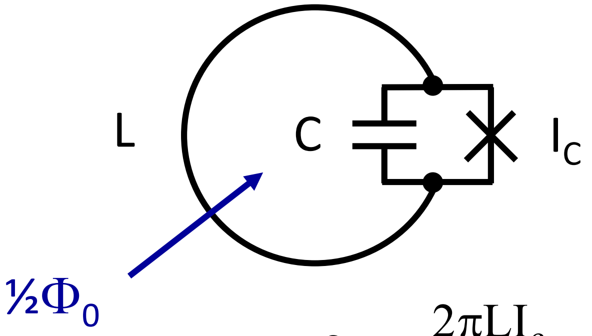


Barrier height: $$\Delta U = \frac{2\sqrt{2}\, I_c \Phi_0}{3\pi}\left(1-\frac{I}{I_c}\right)^{3/2}$$

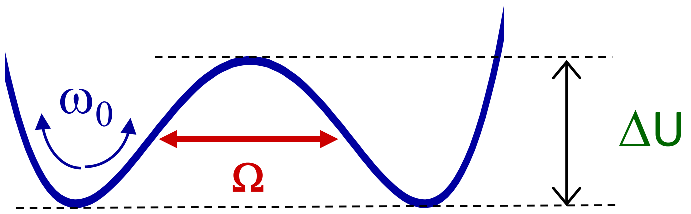
Plasma frequency: $$\omega_p = \left(\frac{2\sqrt{2}\,\pi I_c}{C\,\Phi_0}\right)^{1/2}\left(1-\frac{I}{I_c}\right)^{1/4}$$

Tunneling frequency: $$\Omega = \frac{E_1 - E_0}{\hbar} \approx \omega_p$$

rf SQUID qubit



$$\beta_L = \frac{2\pi L I_c}{\Phi_0} > 1$$



Barrier height: $$\Delta U = \frac{3E_J}{4\pi^2}(\beta_L - 1)^2$$

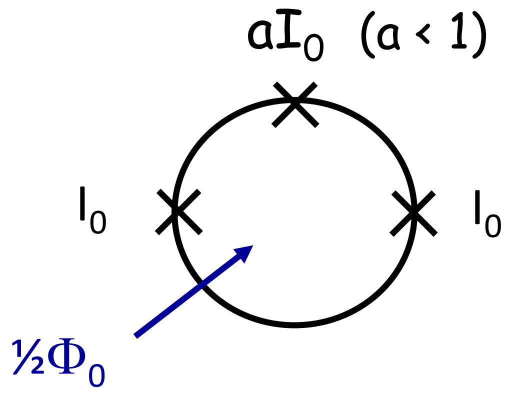
Plasma frequency: $$\omega_0 = 2\sqrt{\frac{\beta_L - 1}{LC}}$$

"Degree of classicality" : $$\lambda = \sqrt{\frac{8 I_c C \Phi_0{}^3}{\pi^3 \hbar^2}}$$

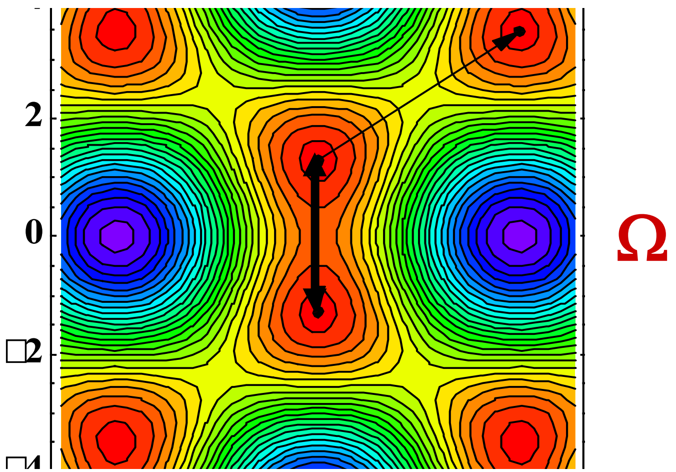
Tunneling frequency: $$\Omega = \omega_0 \exp\left[-\frac{(\beta_L - 1)^{3/2}\lambda}{\sqrt{2}}\right]$$

Three-junction qubit

T. P. Orlando *et al.*, PRL 60, 15398 (1999)



3-dimensional potential



Josephson energy: $E_J = I_0 \Phi_0 / 2\pi$

Charging energy: $E_C = e^2 / 2C$

Plasma frequency: $$\omega_m = \frac{1}{\hbar}\left[\frac{4(4a^2-1)}{a(1+2a)} E_J E_C\right]^{1/2}$$

Phase minimum : $$\phi_m^* = \cos^{-1}\left(\frac{1}{2a}\right)$$

Tunneling frequency: $$\Omega(E_J) = \omega_m \exp\left[-\frac{4a(1+2a)E_J}{E_C}\right]^{1/2}\left(\sin\phi_m^* - \frac{\phi_m^*}{2a}\right)$$

(intracell)