Take-home messages from 1st QFT topic (renormalizability of QED)

- renormalizable theory (in general): coupling constants of non-negative mass dimension ...

$\Rightarrow$ only a finite number of amplitudes (with "small" number of external lines) are divergent: "absorb" infinities in finite number of input/free parameters (to be fixed by measurements) ...

$\Rightarrow$ other ("infinite" number!) of amplitudes (large number of external lines) are finite, predicted in terms of above parameters ...

e.g. Yukawa theory ($\mathcal{L} \ni h\bar{\Psi}\psi\phi + \lambda\phi^4 + m\bar{\Psi}\psi + m_\phi^2\phi^2 + \mu\phi^3$)

and gauge theory

- gauge theory more minimal/predictive (due to gauge invariance: only 1 coupling and mass term for fermion)

- So, take gauge theory as prototype for all forces (ultimately data decides!)
- Specifically for QED, we find

(1). Photon massless even at loop-level: sort of expected from gauge invariance (related to no tree-level mass), if regulator preserves gauge invariance... ... in detail, use WT identity in $\Pi_{\mu\nu}$, DIMREG explicitly ...

(2). $Q_B = \underbrace{\frac{Z_1}{Z_2}}_{=1 \text{ due to WT identity}} Q_{(\text{observed})}$: not renormalized,

cf. $e_B = e/\sqrt{Z_3}$ & $m_B = m - \left(1 - \frac{1}{Z_2}\right)\delta m$

- More accurately, $\frac{(Q_B)_i}{(Q_B)_j} = \frac{Q_i}{Q_j}$: due to gauge invariance/WT identity, but "non-trivial" (cf. photon massless): Q_B's different for electron vs. quark (so are $Z_{1,2}$: no principle relating e^- to q), so why should there be any "magic" at loop-level ?! (Again, no such result for masses : $m_i / m_j \neq (m_B)_i / (m_B)_j$)

[Also, prediction here if Q_B's for e^- vs. q can be fixed by, e.g., GUT...]

(3). (Gauge) coupling runs : IR-free for QED (in general, with only matter fields in vacuum polarization, $\Pi_{\mu\nu}$)

... onto other 2 forces, where we would like to use QED model (or gauge theory), given its above successes...
... but face "hurdles" initially :

Weak (nuclear) force : gauge bosons massive...
so, just add bare mass term ? But that makes theory non-renormalizable (spoils motivation / QED attraction) $\Rightarrow$ use Higgs mechanism (spontaneous symmetry

breaking) instead (which is a robust, interesting QFT phenomenon "by itself", i.e., irrespective of its above phenomenological "application")

whereas, strong (nuclear) force presents a different "challenge": it binds constituents / quarks into proton..., but partons (empirically) weakly-coupled at energies $\gg$ GeV $\Rightarrow$ non-abelian gauge theory (loops of self-coupled gauge bosons) gives running of opposite sign to QED (again, plausible/fascinating theory in its own right)

... so, lesson is that gauge theories are "rich": can describe massive gauge bosons, as well as "confinement"... no need to abandon gauge invariance, hence renormalizability (predictivity)!