Spontaneous breaking of continuous [U(1) or abelian] symmetry : gives massless scalar field [Nambu-Goldstone boson (NGB), with derivative-only interactions]

— Complex scalar field (vs. real for discrete symmetry), $\Phi(x)$ [decompose into real and imaginary (linear representation) or modulus and phase (radial representation)] with potential only dependent on $\Phi^\dagger \Phi$

$$\mathcal{L} = (\partial_\mu \Phi^\dagger)(\partial^\mu \Phi) - \underbrace{\left[\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2\right]}_{V(\Phi)\text{: function of } \Phi^\dagger \Phi}$$

symmetric under $\Phi \to e^{i\alpha} \Phi$ (α is constant, i.e., space-time independent)

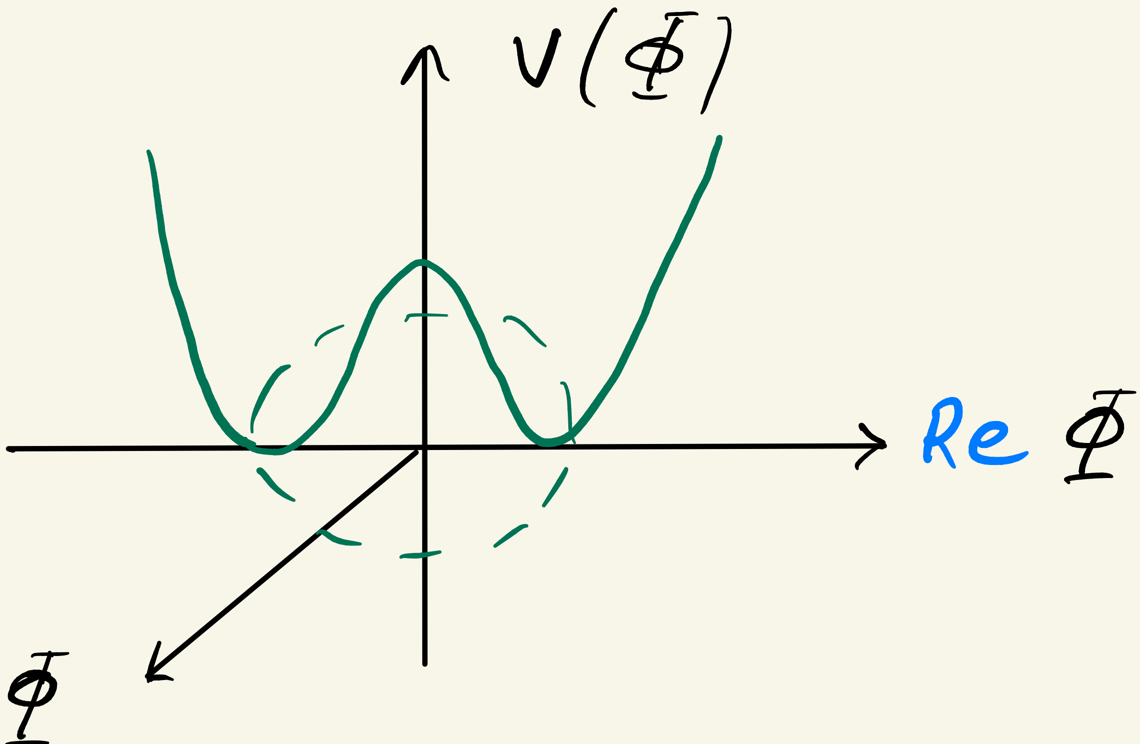
As for real scalar (discrete symmetry), start at classical level, choosing $\mu^2 < 0$:

$$(\Phi^\dagger \Phi)_{\text{classical}} = v^2/2 = -\mu^2/\lambda, \text{ i.e.,}$$

$$\Phi_{\text{classical}} = v/\sqrt{2}\, e^{i\theta}, \text{ where } \theta \text{ is arbitrary}$$

— modulus of Φ fixed, but not phase $\Rightarrow$

upon quantization, "expect" one massless particle ("corresponding to" fluctuations along trough: no analog for real scalar) while other is massive (fluctuations along radial direction, kind of like real scalar)



- choice of θ (phase of VEV) breaks symmetry
- continuous degeneracy of vacuua (parametrized by θ: all values have same energy)

Onto quantum theory: again

classical value of Φ interpreted as VEV: $\langle 0|\Phi|0\rangle = v/\sqrt{2}\, e^{i\theta}$

- Expand Φ around (non-zero) VEV / shift field):

$\Phi(x)$ [linear representation, more familiar]

$$= \frac{1}{\sqrt{2}}\left\{\left[v + \eta_\ell(x)\right] e^{i\theta} + i\,\zeta_\ell(x) e^{i\theta}\right\}$$

linear

with $\langle 0|\eta_\ell(x)|0\rangle = 0 = \langle 0|\zeta_\ell(x)|0\rangle$

$\eta_\ell(x)$, $\zeta_\ell(x)$ are real fields, orthogonal to each other $\Rightarrow$ expand $\eta_\ell(x)$, $\zeta_\ell(x)$ in terms of different a, $a^\dagger$'s

- Rewrite above $\mathcal{L}$ in terms of η_ℓ, ζ_ℓ ($e^{i\theta}$ disappears as expected by $\Phi \longrightarrow e^{i\alpha}\Phi$

being symmetry):

$$\mathcal{L}_{SSB} = \frac{1}{2}(\partial^\mu \eta_e)(\partial_\mu \eta_e) + \frac{1}{2}(\partial^\mu \xi_e)(\partial_\mu \xi_e)$$

$$\left. - \lambda^2 v^2 \eta_e^2 \; (\text{no } \xi_e^2) \right\} \text{mass term}$$

$$\left. -\lambda v \eta_e (\eta_e^2 + \xi_e^2) - \frac{\lambda}{4}(\eta_e^2 + \xi_e^2)^2 \right\} \text{interactions (seem non-derivative)}$$

Note characteristics of $\mathcal{L}_{SSB}$:

(i). η_e (particles) massive (must be fluctuations in radial direction: more clear using radial representation)

(ii). ξ_e massless (fluctuations along trough)

(iii). processes with odd number of η_e's allowed (symmetry broken) (iv). relations among couplings [cubic (2 types) & quartic (3 types) in terms of λ, v (two parameters)]: "remnant" of symmetry, which will be used later to show χ_ℓ (but not η_ℓ) has effectively derivative interactions

(v). manifestly renormalizable (coupling constants of mass dimension ≥ 0)

Onto radial representation for Φ, which will make features (ii) & (iv), i.e., χ_ℓ massless and derivatively-coupled, "obvious"...

... but (v). renormalizability "obscure"