Introduction to $\boxed{\text{SSB / Higgs mechanism}}$

- QED is "nice" theory for EM force: couplings follow from principle of gauge invariance; renormalizable

$\Rightarrow$ why not "extend" QED suitably to describe other 2 forces of SM?

- However, naively, this seems difficult, given different nature of these forces vs. EM force (long range: $\sim e^2_{\text{eff}}(r)/r^2$, with effective coupling increasing – albeit slowly – with energy):

weak (nuclear) force is short range

$\Rightarrow$ force carriers (gauge bosons) are massive (also "non-abelian": couple 2 different fermions, e.g., electron to neutrino) ... while strong (nuclear) force is asymptotically free : stronger in IR (binding quarks/gluons into hadrons, but constituents of hadrons weakly coupled at energies $\gg$ GeV) $\Rightarrow$ opposite running to QED...

— Goal : obtain these new features without sacrificing gauge invariance (at least "to begin with") & renormalizability...

... segue into next 2 QFT topics :

Spontaneous symmetry breaking / Higgs mechanism: renormalizable way to give mass to gauge bosons (cf. explicit/bare mass term) for weak (nuclear) force... and

non-abelian gauge theory (gauge boson self-interactions) gives asymptotic freedom for strong force (and off-diagonal coupling for weak force)

— Begin with phenomenlogical description of weak (nuclear) force : Fermi theory / effective 4-fermion interaction:

Muon decay: $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ from

$$\mathcal{L}_{int.} \sim \frac{G_F}{\sqrt{2}} \left[\bar{\psi}_e \gamma^\mu (1-\gamma_5) \psi_{\nu_e}\right]\left[\bar{\psi}_{\nu_\mu} \gamma_\mu (1-\gamma_5) \psi_\mu\right]$$

creates e^- destroys μ^-

and neutron (radioactive) decay: $n \to p e^- \bar{\nu}_e$ from

$$\mathcal{L}_{int.} \sim (\text{same}) \frac{G_F}{\sqrt{2}} \left[\bar{\psi}_p \gamma_\mu (1-\gamma_5) \psi_n\right]\left[\bar{\psi}_e \gamma^\mu (1-\gamma_5) \psi_{\nu_e}\right]$$

destroys neutron

(realized later on: "replace" n, p with u, d quarks.)

– Note: from experimental data, deduce $\boxed{\gamma_\mu (1-\gamma_5)}$ (pseudo-vector or "V – A") structure ...

V → vector (γ_μ)

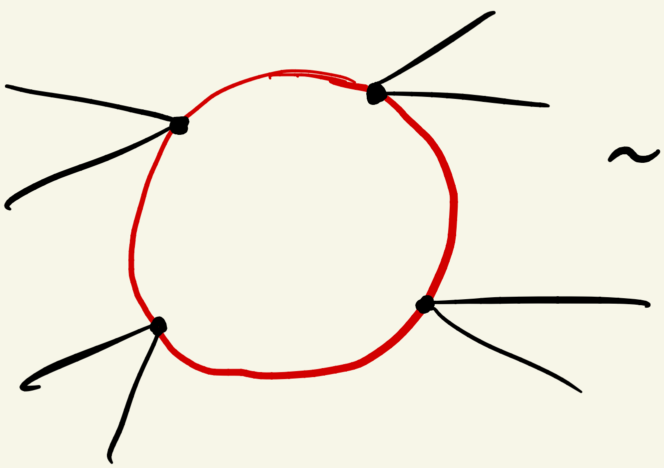
A → axial-vector ($\gamma_\mu \gamma_5$)

... vs. "purely" V (only γ_μ) in QED

$\Rightarrow$ only L chiralities of fermions (R chiralities of anti-fermions) interact (vs. both L, R chiralities of fermions coupling - equally - to photon)

- Fermi theory successful ... so why "tinker"?!

- "Problem" of Fermi theory: $[G_F] = -2 \Rightarrow$ (superficially & actually) non-renormalizable: cannot predict 8-fermion amplitude due to log-divergence at 1-loop



$$\sim G_F^4 \int \frac{d^4 k}{k^4} \sim \log \Lambda_{UV}$$

(Possible) Cure: "universality": same size & Lorentz structure of coupling between muon & radioactive decay and γ_μ form ["forget" $(1-\gamma_5)$ for now] (similar to QED) "suggests" underlying Fermi theory is (massive) intermediate vector (gauge) boson (IVB) theory (see below for why massive)...

...and QED shown to be renormalizable, so (hopefully) might be IVB theory then?

[Note: weak gauge boson (W) couples e^- to ν_e $\Rightarrow$ W is charged, cf. photon is itself (electrically) neutral, coupling only "diagonally" $\Rightarrow$ need non-abelian gauge theory, will return to this later.]

— How massive gauge boson exchange "reduces" to Fermi theory?

— first, add explicit mass term for gauge boson:

$$\mathcal{L}\,(\text{Proca}) = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \tfrac{1}{2} M_A^2 A^\mu A_\mu - A_\mu j^\mu$$

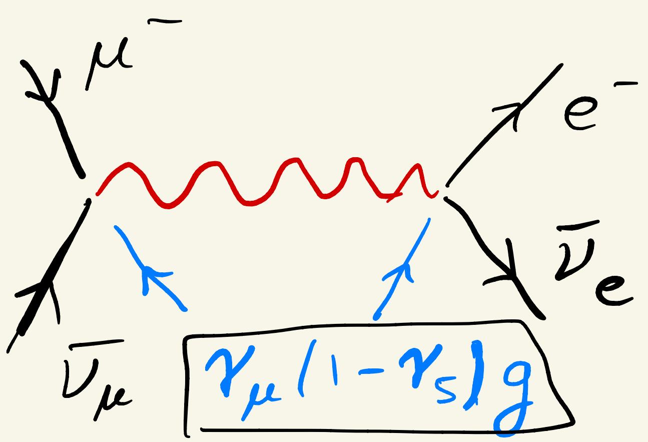
then, calculate gauge boson propagator in HW3.1

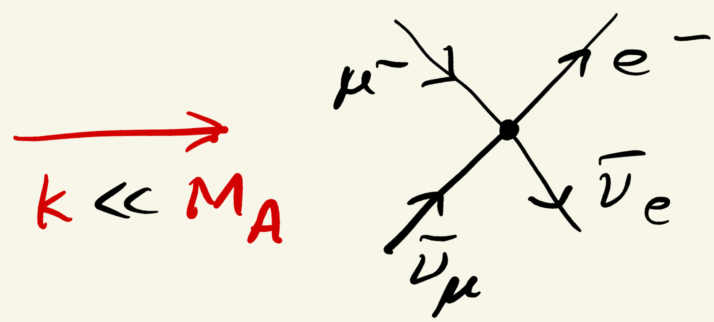
(no need to "fix" gauge, since M_A^2 not gauge-invariant, cf. massless gauge boson propagator "ambiguous" with only $F_{\mu\nu}F^{\mu\nu}$; see sec. 8·2 of LP):

$$D_{\mu\nu}(k) = \frac{\left(-g_{\mu\nu} + k_\mu k_\nu / M_A^2\right)}{k^2 - M_A^2}$$

(k is momentum in gauge boson line, related to external fermion momenta)

$\Rightarrow$ at low energies (IR limit), i.e., momenta of external particles (muon, neutron ...) $\ll M_A$ (so that k in gauge boson propagator $\ll M_A$ also), the propagator becomes $\sim 1/M_A^2 \Rightarrow$



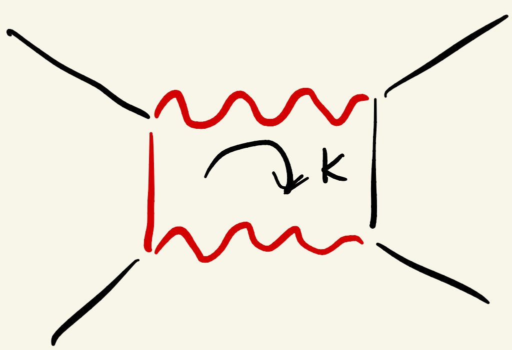


(contact interaction of Fermi theory)

so that we can identify G_F with $\sim g^2/M_A^2$

– Next, is this (massive) gauge boson theory renormalizable (like QED)?!

– By superficial/naive power-counting, it is not renormalizable: calculate D in HW 3.2: essentially, take UV limit of propagator (vs. IR to get Fermi theory above) $\sim 1/M_A^2$ (for $k \gg M_A$), cf. $\sim 1/k^2$ for massless gauge boson propagator $\Rightarrow$ D is "worse", e.g.,



$$\sim \int d^4k \left(\frac{1}{k}\right)^2 \left(\frac{1}{M_A^2}\right)^2 \sim \Lambda_{UV}^2 \text{ here}$$

$$\text{vs. } \sim \int d^4k \left(\frac{1}{k}\right)^2 \left(\frac{1}{k^2}\right)^2 \sim \text{finite for massless gauge boson}$$

...even if (gauge) coupling is dimensionless: again, due to behavior of propagator

— However (you knew that was coming!), above D is superficial : $1/M_A^2$ scaling ("responsible" for non-renormalizability) comes with $k_\mu k_\nu$ Lorentz structure.

Now, at vertex of gauge boson with fermions, this gives (schematically)

$\sim k_\mu j^\mu \longrightarrow \partial_\mu j^\mu$ (in position space)

$j \rightarrow$ fermion current

(Similarly for k_ν ...)

So, 2 cases to consider (note: we will return to this point in context of Higgs mechanism):

(a) A_μ couples to conserved current, e.g., $\mathcal{L}_{Dirac} = \bar{\Psi}(i\not{\partial} - m)\Psi$ gives $\partial_\mu j^\mu_V = 0$, where $j^V_\mu \sim \bar{\Psi}\gamma_\mu\Psi$ (if Ψ satisfies Dirac equation, i.e., on-shell fermions)

Similar arguments in LP sec 9.7 & PS sec. 5.5 for showing (photon momentum . amplitude) = 0.

$\Rightarrow$ $k_\mu k_\nu$ part of propagator does not seem to contribute $\Rightarrow$

(Also, general argument below PS Eq. 9.58 for $k_\mu k_\nu$ part of photon propagator not contributing)

propagator $\sim 1/k^2$ for $k \gg M_A$

(like for massless propagator)

$\Rightarrow$ theory is renormalizable!

(b). Since gauge boson mass term breaks gauge invariance already, why not couple gauge boson to non-conserved current

(i.e., kind of lost "principle" now)

Indeed, $\partial_\mu j^\mu_A = i 2 m \bar{\psi} \gamma_5 \psi$

(again, if ψ satisfies Dirac equation),

where $j^A_\mu \sim \bar{\psi} \gamma_\mu \gamma_5 \psi$

$\Rightarrow$ If coupling is $A_\mu j^\mu_A$, then $k_\mu k_\nu$ part of propagator is relevant (again, $A_\mu j^\mu_V$ is "safe"): so, spectre of non-renormalizability still present?! (Hypercharge gauge boson in SM is an example)

... (another) However, vertex with gauge boson and axial current gives $\sim \partial_\mu j^\mu_A \propto m_\psi \Rightarrow$ divergence lowered than naive?!

(last!) However, $\partial_\mu j^\mu_A \propto m_\psi$ only if fermions coupled to gauge boson are on-shell (again, Dirac equation was used)

$\Rightarrow$ if fermions are instead internal line, then it's not clear that $\partial_\mu j^\mu_A \propto m_\psi$...

so that could actually obtain naive divergence, i.e., non-renormalizable

[More unambiguous problem comes with non-abelian gauge theory, i.e., gauge boson self-interactions, where gauge boson scattering amplitudes violate unitarity if mass term is explicit: see CL, ch. 11]

$\Rightarrow$ need alternative ("safer" from above behavior of propagator) mechanism to generate gauge boson mass: spontaneous breaking of gauge symmetry / Higgs mechanism...

... to get there, we need to understant spontaneous breaking of global symmetries: these are both interesting topics from pure QFT viewpoint (exemplifying its richness), with applications in condensed matter (ferromagnetism & superconductivity)... here: we'll develop them with above motivation within SM

$\boxed{\text{Extra}}$ note: as we will show later, Higgs mechanism is a renormalizable model for massive gauge bosons. Then, we find that case (a) above, i.e., (massive) gauge boson coupled to conserved current, can indeed be obtained as (suitable) limit of Higgs model...

... so this provides a consistency check for claim above that this case (with bare mass) is renormalizable (based on "$k_\mu k_\nu$/unwanted" part of propagator does not contribute)

— Whereas, case (b), i.e., explicit mass gauge boson coupled to non-conserved current is not a limit of Higgs model; again, a consistency check for result above that case (b) is possibly not renormalizable, since $k_\mu k_\nu$ term in propagator might be relevant

— Also, <u>Stueckelberg</u> mechanism (see Wikipedia for what it is/references) is renormalizable model for massive gauge bosons, but only for abelian/U(1) gauge theory: it could be obtained as limit of Higgs model, thus perhaps related to above arguments