$\boxed{\text{Non-abelian}}$ gauge theories (classical)

— $\boxed{\text{Outline}}$: add gauge field $\boxed{A_\mu}$ to make earlier global non-abelian symmetry local (space-time dependent parameters $\boxed{\beta_a}$ appearing in transformation)

— $\boxed{\text{Do it}}$ in "steps" [following what is done for QED, i.e, abelian/U(1), case]

— Matter kinetic term made invariant under local non-abelian "phase" rotation by coupling to A_μ, which has to transform in a suitable way [define covariant derivative, $\boxed{D_\mu}$]

$\Rightarrow$ $\boxed{A_\mu}$ itself transforms [in adjoint representation], even for global phase rotation, cf. photon in QED/U(1) doesn't

— Kinetic term for A^a_μ ← adjoint index: gauge invariance then automatically leads to [i.e., without asking for them] self-interactions [$\partial A A^2$ & A^4] for gauge fields

$\Rightarrow$ upon quantization, get asymptotic freedom [gauge coupling becomes weaker in UV] from loops of (only) gauge bosons

— $\boxed{\text{Higgs}}$ mechanism: subset of gauge bosons massive

Non-abelian

Covariant derivative for $\boxed{\text{matter}}$ fields

- Warm-up with $\boxed{QED/U(1)}$: $\psi' = e^{-ieQ\theta(x)}\psi$

$$\Rightarrow \mathcal{L}_{\psi'} = \bar{\psi}'(i\partial\!\!\!/ - m)\psi' = \bar{\psi}(i\partial\!\!\!/ - m)\psi \underbrace{+ eQ\,\partial_\mu\theta(x)\bar{\psi}\gamma^\mu\psi}_{\text{extra term}}$$
$$\neq \mathcal{L}_\psi$$

- So, $\boxed{\text{add}}$ coupling to $\boxed{A_\mu}$:

$$\mathcal{L}_{\psi-A} = -eQ\,\bar{\psi}\gamma^\mu A_\mu\psi, \text{ with } A'_\mu = A_\mu + \partial_\mu\theta(x)$$

so that extra term "cancels", i.e.,

$$\mathcal{L}_{\psi'-A'} = -eQ\,\bar{\psi}'\gamma_\mu A^{\mu\prime}\psi' = -eQ\bar{\psi}A_\mu\gamma^\mu\psi - eQ\bar{\psi}\gamma_\mu\partial^\mu\theta(x)\psi$$

$$\Rightarrow \mathcal{L}_{\psi'} + \mathcal{L}_{\psi'-A'} = \mathcal{L}_\psi + \mathcal{L}_{\psi-A}, \text{ i.e., } \underline{\text{locally}} \text{ invariant}$$
$$= \bar{\psi}(i\underline{D\!\!\!/} - m)\psi, \text{ where } \boxed{D\!\!\!/ = \partial\!\!\!/ + ieQ\gamma^\mu A_\mu}$$

- Then, make A_μ dynamical by including (gauge invariant) kinetic term (quadratic in A_μ with derivatives), i.e, $\mathcal{L}_A = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$,
where $\boxed{F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu}$ such that $\boxed{F'_{\mu\nu} = F_{\mu\nu}}$

- Essentially, repeat above procedure for $\boxed{SU(n)}$ non-abelian case, i.e., $\Psi' = U\,\Psi$, where Ψ e.g., n-column for fundamental representation

$$\boxed{U = \exp\left[-i\,\beta_a(x)\,T^a\,g\right]} \quad \ldots (1)$$

$\beta_a(x)$: like $\theta(x)$ in QED; T^a: like Q in QED, e.g., $n\times n$ Hermitian, traceless matrices; g: like e in QED

so that $\mathcal{L}_{\Psi'} = \bar{\Psi}'(i\partial\!\!\!/ - m\mathbb{1})\Psi$

$$= \bar{\Psi}(i\partial\!\!\!/ - m)\Psi + \bar{\Psi}U^{-1}i\gamma^\mu(\partial_\mu U)\Psi \neq \mathcal{L}_\Psi$$

— $\boxed{\text{Add}}$ $\underline{A_\mu\text{'s}}$: $\overset{a=1\ldots n^2-1}{n^2-1}$ of them corresponding to number of generators (T_a's) or parameters β_a, i.e., $\mathcal{L}_{\Psi - A^a_\mu} = \bar{\Psi}\left(i \not{D} - m\right)\Psi$, where

$$\boxed{D_\mu = \partial_\mu + i g T_a A^a_\mu} \text{ (like for QED)} \ldots (2)$$

— Fix transformation of $\boxed{A_\mu}$ by requiring gauge invariance of $\mathcal{L}_{\Psi - A^a_\mu}$, i.e., transformation of A_μ (Ψ-coupling) cancels extra term in $\mathcal{L}_{\Psi'}$ vs. $\mathcal{L}_\Psi$:

$$(-)\underbrace{\left[i\bar{\Psi}U^{-1}\gamma_\mu(\partial^\mu U)\Psi\right]}_{\text{extra term } ①} = \mathcal{L}_{\Psi' - A^{a\prime}_\mu} - \mathcal{L}_{\Psi - A^a_\mu}$$

can't set to 1 due to T^a in middle

$$= -g \underbrace{\bar{\Psi} U^{-1}}_{\bar{\Psi}'} \gamma_\mu T_a \underbrace{U\Psi}_{\Psi'} \underbrace{A'^a_\mu}_{\text{to be determined}} \quad ②$$

$$+ g\bar{\Psi}\gamma_\mu T_a A^\mu_a \Psi \quad ③$$

$$\Rightarrow \underset{②}{U^{-1} T_a A^{a\prime}_\mu U} = \underset{③}{T_a A^a_\mu} + \underset{①}{i/g\; U^{-1}(\partial_\mu U)} \quad \text{or}$$

$$\boxed{T_a A^{a\prime}_\mu = \underset{(1st)}{U\left(T_a A^a_\mu\right)U^{-1}} + \underset{(2^{nd})}{i/g\,(\partial_\mu U)U^{-1}}} \ldots (3)$$

Interpretations:

(1). Recall that in U(1)/QED case, the "point" of defining $\boxed{D_\mu}$ (covariant derivative) is that it itself transforms exactly like matter field under gauge (or local phase) transformation:

$$\left[D_\mu(\underline{\text{Field}})\right]' = e^{-ieQ_{\text{field}}\theta(x)}\left[D_\mu(\underline{\text{Field}})\right] \text{ (cf. } \partial_\mu \text{ Field has "extra" term)}$$

Similarly, for the non-abelian case here, we can show (see HW 6.1) that

$$\boxed{\underbrace{(D_\mu \Psi)'}_{\text{non-trivial representation of } SU(n)} = U(x)\, D_\mu \Psi} \quad \ldots (4)$$

[again, $U(x)$ is simply generalization of $e^{-i\theta(x) e Q}$]

And, analogosly for scalar fields (see HW 6.2)

$\boxed{2}$ Onto local/gauge transformation of $\boxed{A_\mu^a}$ (gauge field), which has 2 terms (see Eq. 3)

$\boxed{(a).}$ 2^{nd} term is inhomogeneous, involves $\partial_\mu \beta^a(x)$, this is simply generalization of abelian case (see more below)

$\boxed{(b).}$ 1^{st} term, i.e., $U(T_a A_\mu^a) U^{-1}$, is new structure with respect to abelian case; note that U on left side doesn't commute with T_a, so doesn't "cancel" U^{-1} on right. Indeed in abelian case, such a cancelation occurs, since U's, T_a's are numbers (or 1 D matrices): photon field is invariant globally.

In fact, in order to focus on 1^{st} term, we can go to a global non-abelian transformation, i.e., set β^a = constant and use compact notation $\mathcal{A} \equiv T_a A_\mu^a \ldots (5)$, i.e., $(n \times n)$ matrix for Ψ being in fundamental representation [recall we have (n^2-1) A_μ fields] so that

(6) $\ldots \mathcal{A}' = U \mathcal{A} U^{-1}$, under global $SU(n)$ transformation

which shows that A_μ^a transform in an $\boxed{\text{adjoint}}$ representation of $SU(n)$: see $\boxed{\text{HW 5.2.2}}$, where this is shown for an arbitrary spin field

- Of course, this was perhaps "expected" given (again) that we have $\boxed{(n^2-1)}$ gauge fields, i.e., same as dimensionality of adjoint representation.
- The infinitesimal version of the general gauge transformation (see also HW 5.2.2) is:

$$\left(A_\mu^a\right)' = \underbrace{A_\mu^a + ig\,\beta_b(x)\,A_\mu^c\underbrace{\left(-if_{abc}\right)}_{\left[T_b^{(\text{adj.})}\right]_{ac}}}_{(1^{st})} + \underbrace{\partial_\mu \beta_a(x)}_{(2^{nd})} \quad \ldots (7)$$

- So, we see (again) in Eq. 7 that in 2^{nd} (inhomogeneous term, the "a" index simply goes along for the ride, i.e., this piece is not so interesting (being same as for abelian): in particular, "g" doesn't appear here)

And, 1^{st} term has structure constants of $SU(n)$, i.e., generators of adjoint representation, showing (again) that there is a non-trivial, homogeneous transformation / rotation of A_μ^a's into each other.

- $\boxed{\text{Bottomline}}$, non-abelian gauge fields are really "charged" under (even global) $SU(n)$ [cf. photon in QED only transforms inhomogeneously], so we can expect gauge self-interactions among gauge fields to arise when we incorporate gauge-invariant kinetic terms for $\boxed{\text{gauge}}$ fields [just like happened with matter fields above]

Onto $\boxed{\text{Pure}}$ $\boxed{\text{gauge}}$ Lagrangian

— Recall that for $\boxed{\text{abelian}}$ case, we add gauge fields (A_μ) to first make matter kinetic terms locally / gauge invariant; then, we make gauge field dynamical by including a kinetic (i.e., containing derivatives); gauge invariant term for A_μ (by itself), i.e., $\mathcal{L}_A \ni -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$, where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is gauge invariant

— We would like to do it similarly for $\boxed{\text{non}}$-abelian (see end of this note) gauge fields: I can just tell you the answer, but it's good to get some $\boxed{\text{insight}}$ into how it is obtained, so here goes.

— We can begin with the "trial" $\boxed{f^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu}$ ...(8) [reserving "$F_{\mu\nu}$" for the actual / final answer]: it is easy to see that (schematically), upon gauge transformation of A_μ given in Eq. 7, we will get

$$f'^a_{\mu\nu} \sim f^a_{\mu\nu} + \underbrace{(\partial_\mu \beta_{..})(A^{..}_\nu)}_{1^{st}} + \underbrace{(\beta_{..})(f^{..}_{\mu\nu})}_{2^{nd}} \quad ...(9)$$

We see that there is $\boxed{\text{no}}$ inhomogeneous (i.e., involving only β) term in above transformation of $f^a_{\mu\nu}$ [unlike for A^a_μ itself in Eq. 7 and similar to $F_{\mu\nu}$ for abelian case]

— Indeed above transformation of $f^a_{\mu\nu}$ looks like that of kinetic term of a matter field (ϕ^a) in adjoint representation, i.e, for $\phi'^a = \phi^a - i\beta_b(x)\underbrace{(T^b_{adj})_{ac}}_{-if_{acb}}\phi^c$

$(\partial_\mu \phi^a)' \sim \partial_\mu \phi^a + (\partial_\mu \beta_{..})(\phi^{..}) + (\beta_{..})(\partial_\mu \phi^{..})$

— So, just like for $\partial_\mu \phi^a$, we can "take care of" $\boxed{1^{st}}$ term in Eq. 9 by converting "∂_μ" in definition of $f_{\mu\nu}$ into a suitable <u>covariant derivative</u> "D_μ". Similarly, <u>2^{nd}</u> term in Eq. 9, which is just <u>a rotation</u> of $f^a_{\mu\nu}$'s into each other, can be handled (as we show later) by "squaring" the <u>new</u> $f^a_{\mu\nu}$ (which will then be gauge invariant, thus can be included in Lagrangian to make A^a_μ dynamical)

— Here's $\boxed{\text{one}}$ <u>systematic</u> way to obtain correct $F^a_{\mu\nu}$, i.e, "D_μ" A^a_ν [a la Lahiri, Pal: another method is in HW 6.3.2]. Let's go back to $U(1)$ case, where covariant derivative for matter field:

$D_\mu \psi = \partial_\mu \psi + ieQA_\mu \psi$ (infinitesimal)

is "obtained" from transformation of matter field:

$\psi' = \psi - ieQ\theta(x)\psi \quad \dots (10)$

by adding derivative on 1^{st} term on RHS of Eq. 10 <u>and</u> replacing $\theta(x)$ by $-A_\mu(x)$ in 2^{nd} term

— Indeed, same procedure works for gauge field, i.e, we start with transformation of gauge field:

$A'_\nu = A_\nu + \partial_\nu \theta(x) \quad , \dots (11),$

do the above 2 replacements, on RHS of Eq. (11) done for ψ to get

"D_μ" $A_\nu = \partial_\mu A_\nu + \partial_\nu(-A_\mu)$ ... which is, of course the usual $\boxed{F_{\mu\nu}}$ so that kinetic term for photon field; i.e, $-\frac{1}{4} F_{\mu\nu}F^{\mu\nu}$ can be written as $-\frac{1}{4}(\text{"}D_\mu\text{"} A_\nu)(\text{"}D^\mu\text{"} A^\nu)$ so that it looks similar to kinetic term of scalar (i.e., matter) field

— Onto non-abelian gauge theories, where story with matter field is similar to $U(1)$ case, i.e., $D_\mu \psi = \partial_\mu \psi + i g T_a A^a_\mu \psi$, i.e., Eq. 2 applied to ψ, is "obtained" from

$\psi' \approx \psi - i\beta_a(x) T^a g \psi$ (infinitesimal form of Eq. 1)

by adding derivative on 1st term in infinitesimal form of Eq. 1 and $\boxed{\beta_a(x) \to -A^a_\mu(x)}$ in 2nd term

— So, we do similarly for A^a_μ, i.e., our candidate (which as we show will work!) is ...(12)

$$\boxed{F^a_{\mu\nu} = \text{"}D_\mu\text{"} A^a_\nu \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f_{abc} A^b_\mu A^c_\nu}$$

[again, applying recipe for ψ to transformation for A^a_μ in Eq. 7: note $\beta(x)$ appears in 2 terms on RHS of Eq. 7, giving last 2 terms on RHS of Eq. 12 : compared to "trial" in Eq. 8, 3rd term in Eq. 12 is "extra"]

— Note that for $U(1)$ case, $F_{\mu\nu}$ transforms locally like A_μ under global symmetry (i.e., both are actually invariant)

- So, for <u>non</u>-abelian symmetry, we expect $F^a_{\mu\nu}$ to transform <u>locally</u> like A^a_μ <u>globally</u>, homogeneously [that too, as an adjoint rotation]

[As an aside, note that situation is slightly different for matter field, i.e., $D_\mu \psi$ transforms <u>locally</u> like ψ itself does <u>locally</u> ("D_μ goes along for the ride"), but for gauge field case, there is an extra <u>inhomogeneous</u> piece in <u>local</u> transformation of A^a_μ, i.e., $F^a_{\mu\nu}$ does <u>not</u> quite transform <u>locally</u> like A^a_μ <u>locally</u>]

- Indeed, Lahiri, Pal around Eqs. 14.21, 14.22 [with remaining steps in Exercise 14.3, which is assigned as HW 6.3.1] show that

$$\boxed{\left(T_a F^a_{\mu\nu}\right)' = U \left(T_a F^a_{\mu\nu}\right) U^{-1}} \text{ (again locally)} \ldots (13)$$

which says that $F^a_{\mu\nu}$ transforms in <u>adjoint</u> representation: see discussion around Eq. 6 & HW 5.2.2

- Finally, in order to get a Lorentz-invariant term, we "square" $F^a_{\mu\nu}$ <u>and</u> to make it gauge invariant, we take a "trace", i.e.,

$$\boxed{tr}\left(T_a F'^a_{\mu\nu} T_b F'^{\mu\nu}_b\right) = tr\left[\left(U T_a F^a_{\mu\nu} U^{-1}\right)\left(U F^{\mu\nu}_b T_b U^{-1}\right)\right]$$

$$\ldots (14) \quad = tr\left[T_a F^a_{\mu\nu} T_b F^{\mu\nu}_b\right] \text{ (i.e., all } U\text{'s cancel)}$$

- We can choose T_a's to be generators of fundamental representation <u>and</u> $tr(T_a T_b) = \frac{1}{2}\delta_{ab}$, so

$$\mathcal{L}_{\text{pure gauge}} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a \quad \ldots (15)$$

[In HW 8.1.1, you will generalize above point by showing that $\phi_a \phi'_a$ is locally invariant for ϕ, ϕ' transforming in adjoint representation (locally)]

— Based on form of $F^a_{\mu\nu}$ in Eq. 12, we see that Eq. 15 ($\mathcal{L}_{\text{pure gauge}}$) contains not only kinetic term for A_μ^a [i.e., quadratic, like for U(1) case], but also $\boxed{\partial .. A^{a\,3}_\mu}$ and $\boxed{A^{a\,4}_\mu}$ (schematically), which (upon quantization) leads to (as promised!) self-interactions of $\boxed{\text{gauge}}$ bosons [so, in a sense, we get this non-trivial feature by simply following our nose, i.e., without actually asking for it: note we only "required" Lorentz & gauge invariant kinetic, i.e., quadratic, term for A_μ ... but self-interactions inevitably followed!]

— Note also that coupling constant (i.e., apart from f_{abc} factors) in these self interactions is "g", i.e., same as for coupling of gauge field to matter (again, all this is dictated by gauge invariance): of course, coupling to matter also depend's on latter's representation, i.e., T_a's, but that's in a sense a "discrete" choice (cf. abelian case, where Q can be arbitrary)

— "Modification" of classical EOM and conserved current due to gauge boson self-interactions: the full Lagrangian then is

$$\mathcal{L} = -\tfrac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + \bar{\Psi} i \gamma^\mu D_\mu \Psi + (D^\mu \Phi)^\dagger (D_\mu \Phi)$$

+ mass terms, other interactions ... (16)
(no derivatives) for Ψ, Φ

[Note that since D_μ(field) transforms just like Field under gauge symmetry, it is clear that $(D_\mu \text{ Field})^\dagger$, like $(\text{Field})^\dagger$, transforms oppositely: that's why the ψ, ϕ quadratic/kinetic terms shown in Eq. 16 are gauge-invariant]

— We can construct $\boxed{\text{Noether}}$ current as in abelian/U(1) case, except that transformation parameter (β) has index "a": we get

$$J_a^\mu = \underbrace{f_{abc} F_b^{\mu\nu} A_\nu^c}_{\substack{\text{new (vs. abelian),} \\ \text{since gauge fields} \\ \text{are also "charged"}}} + \underbrace{\bar{\Psi} \gamma^\mu T_a \Psi + \left[i (\partial^\mu \phi) T_a \phi + \text{h.c.} \right]}_{\text{matter part}} \quad \dots (17)$$

— Similarly, Equation of motion (EOM) for A_μ^a is

$$\partial^\mu F^a_{\mu\nu} = J^a_\nu \quad \dots (18)$$

which is similar to U(1) case [but J contains A as in Eq. 17]: that's the advantage of above definition of $F^a_{\mu\nu}$

— Alternately (HW 6.3.3 deals with equivalence of these 2 forms), we can keep only matter fields on RHS (i.e., same as for abelian), but then ∂_μ on LHS becomes D^μ (again, since A_μ^a is charged also)

$$\boxed{D^\mu} F^a_{\mu\nu} = j^a_\nu \text{ (only } \Psi, \Phi) \quad \ldots (19)$$

— Onto Higgs mechanism for non-abelian gauge theories (Peskin, Schroeder pages 692-699)

— Big picture (gauge boson self-interactions not relevant here): scalar field VEV's make gauge bosons massive (by eating NGB's), but not all of them

— Consider Lagrangian of scalar fields which is invariant under symmetry group with generators T_a, i.e., $\phi_i \to (1 + i \beta_a T^a)_{ij} \phi_j$ (infinitesimal version)

— It is convenient to re-write in terms of real ϕ_i, i.e., each complex $\phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}}$ ($\phi_{1,2}$ real)

$\Rightarrow$ T_a's are purely imaginary (so that shift in ϕ is also real) and anti-symmetric (since T_a's are hermitian)

e.g. 3×3 matrices $T_{a=1,2,3}$ acting on $\Phi = (\phi_1 \, \phi_2 \, \phi_3)^T$, i.e., pions (in pion-nucleon Lagrangian/system) $\quad \ldots (20)$

— So, we can write $\boxed{(T_a)_{ij} = i\, (t_a)_{ij}}$, where $\boxed{t_a\text{'s}}$ are real, still antisymmetric e.g. generators of $SO(3)$ for $\phi_1, \phi_2\, \phi_3$ [i.e., $\boxed{e^{+\beta_a t^a}}$ are orthogonal matrices]

- As usual, we promote global symmetry to local using $D_\mu \Phi = (\partial_\mu - ig A_\mu^a T^a)\Phi$
$$= (\partial_\mu + g A_\mu^a t^a)\Phi,$$
resulting in gauge boson masses from kinetic terms for Φ (like for abelian case):

$$\mathcal{L} \ni \underbrace{\tfrac{1}{2}}_{\phi\text{'s real}} (D_\mu \phi_i)^2 = \tfrac{1}{2}(\partial_\mu \phi_i)^2 + g A_\mu^a (\partial_\mu \phi_i t^a_{ij} \phi_j) \quad \ldots (21)$$
$$+ \tfrac{1}{2} g^2 A_\mu^a A^{\mu b} (t^a \Phi)_i (t^b \Phi)_j$$

where we expand ϕ_i's about their VEV's, $\boxed{\langle 0|\phi_i|0\rangle \equiv (\phi_0)_i}$ so that last term in Eq. 21 gives
← denotes VEV (as in SSB global discussion)

$$\Delta \mathcal{L} = \tfrac{1}{2} m^2_{ab} A_\mu^a A^{\mu b} \quad (A\text{'s are real fields})$$
with $$\boxed{m^2_{ab} = g^2 \sum_i (t^a \Phi_0)_i (t^b \Phi_0)_i}$$

- Now, this is a positive (semi-)definite (mass)^2 matrix, since diagonal elements in (any basis)
$$m^2_{aa} \text{ (no sum over } a) = g^2 (t^a \Phi_0)^2 \geq 0$$
$\Rightarrow$ gauge bosons get positive (mass)^2 in general, with exception being $\boxed{\text{if}}$ $(t_a \Phi_0) = 0$.
↑ VEV
i.e., some generator $\boxed{t^a}$ annihilates vacuum (corresponding transformation leaves vacuum invariant: see HW 5.3.2)
- In this case, the generator doesn't contribute

to m^2_{ab} above, i.e., that gauge boson remains massless

- Finally, let's check eating of NGB's: 2nd term in Eq. 21 with Φ set to VEV is

$$\boxed{\Delta \mathcal{L}_{\text{mix}} = g^2 A^a_\mu \; \partial_\mu \phi_i \left(t^a \Phi_0\right)_i}$$

i.e., it's a (quadratic-level) mixing of gauge boson and a combination of ϕ_i's given by "vector" in field (ϕ-) space: $(t^a \Phi_0)$...

... which reminds us of $\boxed{\text{identity}}$ of NGB's in ϕ-space (Peskin, Schroeder pages 351-352, also done in lecture). Namely, we said then that if shift of $\phi_a = \alpha \Delta^a(\phi)$ ← all of them, then NGB vector is $\Delta^a(\phi_0)$ → VEV: there "a" was index for ϕ, which here is denoted by "i" with "a" being reserved for generator index, i.e., shift of $\phi_i = \beta_a (t^a \Phi_0)_i$ so that
under generator a → "α" earlier
NGB vector is indeed $\boxed{t^a \Phi_0}$ (in notation here)

$\Rightarrow$ just like for U(1) case that we worked out explicitly, we use gauge-fixing term to "get rid of" above mixing such that $\xi \rightarrow \infty$ (unitary gauge) makes these vectors in ϕ-space [i.e., NGB's] "disappear" (or eaten by massive gauge bosons)

— Same phenomenon is seen using radial representation that we used for U(1) case, but now suitably generalized (see pages 246–247 of Cheng, Li), i.e., we re-write (schematically):

$$\phi_i = \sum_{j \,(\text{not over } i)} \exp\left\{ i\, T_{ij}^{\text{broken only}}\, \zeta_{\text{broken only}}(x) / v \right\} \times \left[v_j + \eta_j(x) \right]$$

[i.e., $T_{ij}^{\text{broken}} v_j \neq 0$ (these T's do not annihilate vacuum), whereas $T_{ij}^{\text{others/unbroken}} v_j = 0$]

— As we did for U(1) case, a gauge transformation:

$$\boxed{\beta_a(x) = -i\, T^{\text{broken only}}\, \zeta_{\text{broken only}}(x) / v}$$

will remove $\zeta^{\text{broken only}}(x)$, leaving only η (i.e., scalar fields orthogonal to broken ζ) (and of course mass terms for gauge bosons) (where above notation is more clear)

[Concrete examples are in HW 7.1.1 and 7.1.2, where the global SU(2) theories for which SSB was studied in HW 5.3 & 5.4 are now gauged.]