Developing theory of standard Model (SM) ①

- We now have (most of) the QFT "ingredients" needed to "cook" the SM (again, all of the QFT topics covered in this course so far will be relevant in this process)
- Before going into details of each "sector" of SM, let's get "big picture/overview" as follows
- SM describes 3 types of forces [EM, weak (nuclear) & strong (nuclear)] acting between matter particles
- The "moving parts" (particle content) of SM can be divided into 3 classes:
- ① Matter particles [spin-1/2, i.e, fermions]
  - quarks: all 3 forces
  - leptons (e.g. electron): only weak (nuclear) & EM forces, i.e., no strong (nuclear)
- ② Force carriers/gauge bosons [spin-1], whose exchange results in 3 forces between matter particles
  - (a) photon (γ_μ) for EM: gauge group is $U(1)_{EM}$
  - (b) $W^{\pm}_{\mu}$ (& Z: see below) for weak (nuclear) force: gauge group is $SU(2)_L$, where "L" denotes action only on left-handed (LH) fermions.

Note that we had already indicated the need

for a non-abelian symmetry here, since W^+ couples μ to ν_μ (for example), i.e., off-diagonally: why exactly SU(2) will be made clear in a bit. [The gauge group can also be denoted as $SU(2)_W$, i.e., "W" for weak]

(c) Gluons (G^a_μ) for strong (nuclear) force: gauge group is $SU(3)_{color}$ [or simply $SU(3)_C$], where "color" denotes "charge" under SU(3), e.g., quarks come in 3 colors, i.e., transform as triplet (fundamental) representation of $SU(3)_C$. Again, we "need" a non-abelian gauge theory here in order to have asymptotic freedom (i.e., gauge coupling becomes smaller at high energies): why exactly SU(3) will be motivated later [This gauge theory is called quantum chromodynamics (QCD)].

— ③ Higgs field [spin-0, i.e., scalar boson] for making carriers of weak (nuclear) force massive via spontaneous (gauge) symmetry [$SU(2)_L$] breaking [again, maintaining renormalizability of the gauge theory]

— Remarkably (as we will show in detail soon), this process/sector also incorporates "unification" of weak and EM forces into "electro-weak (EW)" gauge group, i.e., we actually start with $SU(2)_L \times U(1)_Y$ [instead of $SU(2)_L$ & $U(1)_{EM}$], where "Y" denotes hypercharge

One combination of T_3 (subgroup/generator of $SU(2)_L$ ③ and $U(1)_Y$ is identified with $U(1)_{EM}$, whereas the orthogonal combination (also an electrically neutral gauge boson) is denoted by "Z"; the other 2 gauge bosons/generators of $SU(2)_L$ [recall that gauge bosons are in adjoint representation, i.e., total of 3 for $SU(2)$] are $W^{\pm}$, i.e., charged carriers of weak force

— So, Higgs VEV breaks $SU(2)_L \times U(1)_Y$ down to $U(1)_{EM}$, i.e., it gives masses to $W^{\pm}$ & Z, but keeps γ massless [in this sense, it "breaks" (EW) unification]

— Finally, we have the 4th force (formally, not part of "SM"), i.e., gravity, which can be considered as "gauging" of general coordinate invariance, i.e., including $x_\mu \to x_\mu - a_\mu$ with a_μ now space-time dependent [Poincare symmetry]. Thus, we have a "gauge theory" for gravity: gauge boson [graviton] couples to conserved charge, i.e., energy-momentum.

— In this sense, gravity is similar to 3 forces of SM (which are all gauge theories). However, the crucial difference is that the "gauge coupling"

for gravity has negative (mass) dimension, i.e., inverse of Planck scale $\sim 1/M_{Pl}$, which is just $\sqrt{G_{Newton}/(\hbar c^5)} \sim 1/(10^{19}\,\text{GeV})$

— So, gravity is (inherently) non-renormalizable (cf. other 3 forces of SM), hence cannot really be handled (predictively) within QFT (so it is not included in SM)

— As an aside (speculation), we can "grand-unify" 3 forces of SM into GUT/single gauge group: SU(5) being simplest possibility.

— So, sort of like EW unification/breaking, we can have

$$SU(5)\ [\text{GUT}] \xrightarrow[\text{new scalar at very high energies}]{\text{SSB due to}} SU(3)_C \times SU(2)_L \times U(1)_Y\ [\text{SM}]$$

[SU(5) gauge boson which are not SM, called X, Y, are massive]

... but, unlike EW unification, this is not a "must", i.e., motivation is more theoretical/even aesthetic (i.e., simplicity: 1 gauge coupling etc.): of course, GUT leads to phenomenological (testable) predictions [hopefully, we'll have time at end of this course to discuss this briefly; also, one of the term papers will be on this topic].

— Without further adieu, let's dig into formulae etc. starting with EW theory, followed by QCD.