QCD phenomenlogy : details of 3 processes

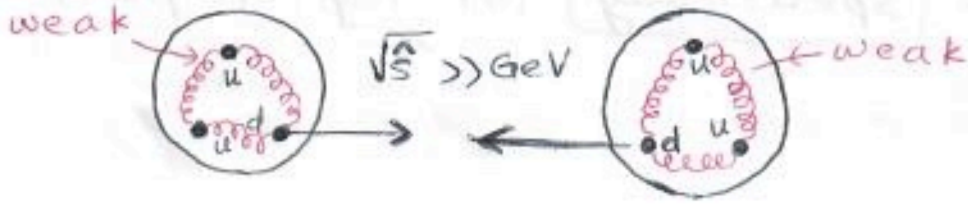
(3). Hadron-hadron collisions (hard scattering part)

— Most of the hadron-hadron collisions involve soft interactions (i.e., momentum transfer $\lesssim$ GeV) between the constituent quarks & gluons; this gives a cross-section of scattering with "geometric" size: $\sigma_{soft} \sim 1/(\text{GeV})^2$, which is not calculable in perturbation theory, since relevant α_s is large.

— However, occasionally (i.e. with a smaller cross-section/probability), (some) collisions involve instead two quarks/gluons (one from each colliding hadron) exchanging hard [large momentum transfer ($\gg$ GeV), that too $\perp$ to collision axis] gluons or EW gauge bosons

— Thus, the underlying/parton-level interactions take place very rapidly, i.e., time-scale (inverse of virtuality of gluon/EW gauge boson exchanged, which is momentum transferred) $\ll 1/\text{GeV}$. Comparatively, time-scale of hadronization (wavefunctions of hadrons) $\sim 1/\text{GeV}$ is too slow: again hard scattering occurs at short distances, much smaller than size of hadrons.

— So, just like in $e^+e^- \to$ all hadrons & DIS discussed earlier, we can start with lowest-order QCD prediction, but now involving (product of) two PDFs [vs. 1 in DIS] since we have 2 hadrons in initial state:



e.g. (if) hard/parton-level process is $q_f \bar{q}_f \to Y$, then (2)

$$\sigma[p(P_1) + p(P_2) \to (Y) + X] = \int_0^1 dx_1 \int_0^1 dx_2 \times$$

(p = proton; P = proton momentum; X = any hadronic state/debris)

$$\sum_{f = u, d \ldots} f_f(x_1) \sum_{\bar{f} = \bar{u}, \bar{d} \ldots} f_{\bar{f}}(x_2) \quad \text{(2) PDF's}$$

$$\times \, \sigma[q_f(x_1 P) + \bar{q}_f(x_2 P) \to (Y)] \quad \text{underlying/parton-level cross-section}$$

($x_1 P$: momentum of incoming quark of flavor f)

[Note that (q_f) could come from either of [2] protons, with $\bar{q}_f$ from the other one, which is why we didn't specify $P_{1 \text{ or } 2}$ in last line above.]

[Note also that [PDF's] appearing above are same as in DIS, i.e., are "universal"... except that only (1) PDF, hence $\int dx$, appeared in DIS cross-section due to 1 hadron in initial state.]

— Again, above is a good approximation for cross-section if momentum transferred in underlying $q_f \bar{q}_f$ reaction is [$\gg$ GeV]

— Like for DIS, it is convenient to [convert]/massage above general formula into a differential form written directly in terms of quantities which can be measured: this involves "getting rid of" $\int\int dx_1 dx_2$ etc...

... we'll do it (specifically) for lepton-pair production via photon (or Z) exchange in proton-proton collisions [called Drell, Yan (DY) process]

- Details are given in a separate note: here, we will just give the result:

$$\frac{d^2\sigma}{dM\,dY}\left(pp \rightarrow e^+e^-X\right) = \sum_f x_1 f_f(x_1)\, x_2 f_{\bar{f}}(x_2) \Big\} \text{PDF's}$$

$$\times \frac{1}{3} Q_f^2 \frac{4\pi\alpha^2}{3M^4} \Big\} \text{underlying/ QED cross-section}$$

where $\boxed{M^2} \equiv$ (virtual photon momentum [denoted by q])^2

= (sum of (4)-momenta of leptons)^2

(called invariant mass(2) of lepton pair)

with $\boxed{Y}$ (called rapidity) defined by writing

$M^2 =$ (again) $[q \text{ (virtual photon momentum)}]^2 = q^{0\,2} - q_{\parallel}^2$

$= M^2[\cosh^2(Y) - \sinh^2(Y)]$

(q^0: component in lab frame; $q_\parallel$: longitudinal (virtual) photon momentum i.e., ∥ to collision axis)

and $\boxed{x_{(1)}} = \frac{M}{\sqrt{s}} e^{Y}$, $\boxed{x_{(2)}} = \frac{M}{\sqrt{s}} e^{-Y}$ (neglecting momenta of partons ⊥ to collision axis @ $O[\alpha_s^{(0)}]$)

[s being COM energy of 2 protons]

- Thus, (as promised) $\int dx$'s... have "disappeared" and (differential) cross-section is expressed in terms of measurable quantities, i.e., (ultimately) the energy-momentum of two leptons detected

- Again, PDF's to be used above can be obtained from DIS measurements
- Also, there are perturbative QCD, i.e., $O(\alpha_s)$, corrections to above formula from 2 sources as in DIS, i.e., hard gluon emissions/exchanges modifying (i). PDF's, e.g., giving partons significant momentum transverse to collision axis and (ii). underlying $q\bar{q} \to$ virtual photon $\to \ell^+\ell^-$ cross-section

Note on running of α_s (applicable to most of above (3) processes):

- α_s entering these cross-section formulae must be defined at some renormalization point/scale, M due to presence of UV divergence (as in QED discussion at beginning of this course), cf. we talked about IR divergence cancellation between real gluon emission and virtual exchange for $e^+e^- \to$ all hadrons.
- Now, in QED, on-shell renormalization (either for photon or electron) was used, but here we would like to avoid using on-shell quarks (i.e., at energy/momentum $\lesssim$ GeV), since these are strongly-coupled
- Instead, we "define" α_s by renormalization condition imposed at $M \gg$ GeV, where QCD is weakly-coupled
- We can then use above α_s to predict cross-sections for processes at other (large) momentum transfer