$\boxed{\text{QCD}}$ phenomenology : details of (1)

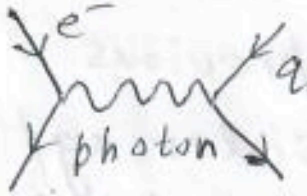
$\boxed{3}$ types/examples of processes

$\boxed{(1).}$ $\boxed{e^+e^- \to \text{hadrons } \boxed{(\text{inclusive}}}$ @

$$\underbrace{m_q, \sim \text{GeV} \ll \boxed{\sqrt{s}}}_{\text{needed}} \; \underbrace{\ll M_Z}_{\text{for simplicity}},$$

— clearly, $\boxed{\text{underlying}}$ process (given purely leptonic initial state) is $\boxed{\text{EW}}$ interaction : for simplicity, we choose $\boxed{\sqrt{s} \ll M_Z}$ so that we keep $\boxed{\text{only}}$ photon exchange (drop Z_μ exchange, although it is rather straightforward to include it, which will not qualitatively modify discussion below of QCD effects)

—(with QCD "turned-off") $\boxed{\text{Leading order}}$ (in EW couplings) process is $e^+e^- \to$ (virtual) photon $\to q\bar{q}$:



, giving $\boxed{\sigma(e^+e^- \to q\bar{q}) = \sigma_0 \times 3 \times \sum Q_f^2}$, $f = u, d\ldots$

(quark) color factor

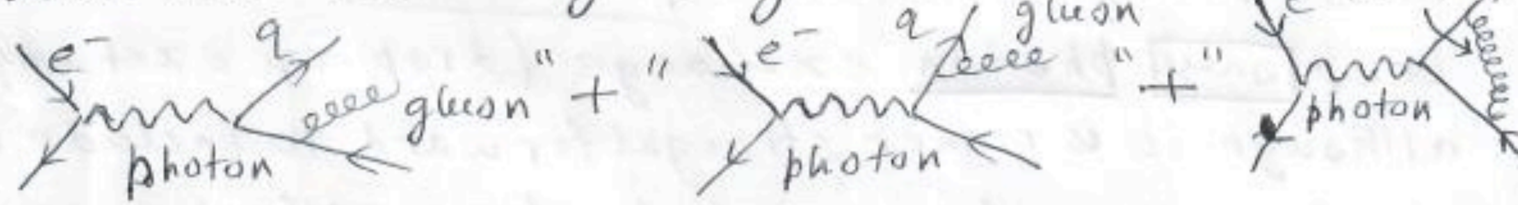
sum over quark favors with $m_q \ll \sqrt{s}$

where we simply re-cycle (with appropriate modifications as shown) the $e^+e^- \to \mu^+\mu^-$ calculation, i.e.,

$\boxed{\sigma_0}$ here $= \sigma(e^+e^- \to \mu^+\mu^-) \boxed{= \frac{4\pi\alpha^2}{3s}}$

(assuming $\boxed{\sqrt{s} \gg m_q}$)

- Of course, observed $\boxed{\text{final}}$ states are hadrons (color singlet bound states: mesons & baryons), so in order to connect to data, we $\boxed{\text{must}}$ "turn-on" $\boxed{\text{QCD}}$ effects (which will convert each quark into a collection of hadrons (called a "jet") and will correct probability to produce $q\bar{q}$ in the first place (see more below).
- At leading-order (LO) in $\boxed{\text{QCD}}$ coupling (g_s), we have the following diagrams:



- Note that we have to include real emission of gluons as in 1st, 2nd diagrams above, even though it is a different final state than $\boxed{3^{rd}}$ diagram (which is same final state as the one without QCD corrections shown on previous page. The reason is that we are considering/ calculating cross-section of $\boxed{\text{inclusive}}$ process, i.e., $e^+e^- \to \boxed{\text{all}}$ hadrons, i.e., we count 2 jets (coming from $q\bar{q}$ final state) and 3 jets (from $q\bar{q}g$) at least $\to$ gluon
- We will also choose $\boxed{\sqrt{s} \gg \text{GeV}}$ so that part of $\boxed{\text{QCD}}$ effects are $\boxed{\text{calculable}}$ (which still leaves "room" for $\sqrt{s} \ll M_Z$, already assumed above such that we can neglect Z exchange)

— In particular, $\sqrt{s} \gg$ GeV implies that virtuality of $\boxed{\text{photon}}$ exchanged is $\gg$ GeV so that (using uncertainty principle), $\boxed{\text{time scale}}$ of $q\bar{q}$ $\boxed{\text{creation}}$ (i.e, how long virtual photon "lives") $\boxed{\ll \frac{1}{\text{GeV}}}$

— $\boxed{\text{Thus}}$, QCD effects (before delving into details) can be nicely $\boxed{\text{categorized}}$ into (as already outlined before)

$\boxed{\text{(a).}}$ Short-distance / energy/momentum transfer $\gg$ GeV) (perturbative, hence calculable) effects can operate on a time scale $\boxed{\frac{1}{\sqrt{s}} \ll \frac{1}{\text{GeV}}}$, thus $\boxed{\text{will}}$ modify (again, in a controlled, perturbative manner) the probability to create $\boxed{q\bar{q}}$ pair (again, since time scale for that process from photon exchange is similar)

$\boxed{\text{(b).}}$ $\boxed{\text{Long}}$-distance / energy-momentum transfer $\lesssim$ GeV (non-perturbative) contributions have time scale $\boxed{\gtrsim 1/\text{GeV}}$, i.e., (much) longer than time taken to create $q\bar{q}$ pair from photon exchange. So, this part cannot modify probability to create $q\bar{q}$ pair. Of course, $\boxed{\text{once}}$ $q\bar{q}$ is created, it is these long-distance effects / soft gluon exchanges which dress-up/ hadronize $q, \bar{q}$ (so are obviously relevant!)... but this process happens with "unit probability" (as long as we $\boxed{\text{sum}}$ over all hadrons), i.e. (again) $\boxed{\text{that}}$ cross-section is not modified. On the other

$\boxed{\text{if}}$ we want to know $\boxed{\text{exclusive}}$ cross-sections, say for $e^+e^- \to \pi^+\pi^-$ vs. K^+K^- etc., then we need to know relative $\boxed{\text{probabilities}}$ for hadronic final states, which are set by long distance effect. Hence, such exclusive rates are not calculable [again, for inclusive cross-section, only net $\boxed{\text{probability}}$ to hadronize matters, which — by "definition" (i.e., each q has to end up in that form) — is unity. So, we "circumvent" calculation of long-distance effects by asking only a "suitable" question, i.e., what is inclusive cross-section!]

— Onto $\boxed{\text{details}}$ of $\boxed{(a)}$, i.e., short-distance modification of cross-section: we find (see Peskin, Schroeder, final project of part I, pages 259-262) from calculation of $\boxed{3}$ diagrams above $\left[\text{LO in QCD, i.e., } O(\alpha_s)\right]$

$$\sigma(e^+e^- \to \text{all hadrons}) = \underbrace{\sigma_0 \left(3 \sum Q_f^2\right)}_{\substack{\text{again, pure} \\ \text{QED result} \\ \text{at Leading-} \\ \text{order in} \\ \text{that coupling}}} \left[1 + \underbrace{\frac{\alpha_s}{\pi}}_{\substack{\text{LO} \\ \text{in QCD}}} + \underbrace{O(\alpha_s^2)}_{\substack{\text{Next-to} \\ \text{LO (NLO)} \\ \text{in QCD}}}\right]$$

— A few $\boxed{\text{comments}}$ (subtle/hidden points) on it:

$\boxed{(i).}$ Since $\boxed{1^{st}, 2^{nd}}$ diagram ($\boxed{\text{real}}$ gluon emission) are — (even if) in $\boxed{\text{principle}}$ — a different final state, we $\boxed{\text{add}}$ that cross-section, i.e.,

$\sigma(e^+e^- \to q\bar{q}(g))$, $\boxed{\text{to}}$ $\sigma(e^+e^- \to q\bar{q})$, i.e., $\boxed{\text{no}}$ real emission; latter 5

corrected by $\boxed{3^{rd}}$ diagram above (i.e, virtual gluon exchange. [Again, in $\boxed{\text{practice}}$, the 2 final states are "similar", i.e, hadrons, so we $\boxed{\text{do}}$ add σ's.]

$\boxed{\text{(ii)}}$ Relatedly, the above $\boxed{\text{two}}$ types of correction to QED result appear at same order in $\boxed{\alpha_s}$:

$\sigma(e^+e^- \to q\bar{q}\,g)$ ← $\boxed{\text{real}}$ emission $/ \sigma(e^+e^- \to q\bar{q})$ (schematically)

$$\sim \underbrace{(g_s \text{ extra in amplitude})^{2}}_{\text{to go to } \sigma \ldots} \times \underbrace{\frac{1}{(2\pi)^3}}_{\text{extra } \boxed{3^{rd}} \text{ body (final state) phase-space factor}} \times \underbrace{2\pi}_{\text{extra angular integration}} \sim g_s^2/\pi^2 \sim \boxed{\frac{\alpha_s}{\pi}}$$

while $\delta[\sigma(e^+e^- \to q\bar{q})]$, i.e., correction to $\boxed{q\bar{q}}$ final state (no extra gluon) from $\boxed{\text{virtual}}$ gluon exchange $\sim \boxed{\text{interference}}$ of loop & tree-level diagram + "(loop)^2" $\sim$ (tree-level cross-section) $\times$

$$\left\{ 1 + \underbrace{\left[g_s^2/(16\pi^2) \right]}_{\text{tree - loop } \boxed{\text{interference}}} + \underbrace{\left[g_s^2/(16\pi^2) \right]^2}_{\boxed{(\text{loop})^2}} \right\}$$

[again, loop amplitude $\sim g_s^2/(16\pi^2) \times \boxed{\text{tree}}$]

i.e., $\delta\sigma(e^+e^- \to q\bar{q})/\sigma(e^+e^- \to q\bar{q}) \sim \boxed{\alpha_s/\pi}$

from virtual 1 loop gluon ($\boxed{3^{rd}}$ diagram) (iii). Actually, the $\boxed{\text{real}}$ emission part (1st, 2nd diagrams) of correction above has $\boxed{\text{IR}}$ divergence, i.e, for $\boxed{\text{soft}}$ gluon (emission) or as gluon mass $\to 0$.

Similarly, $\boxed{\text{virtual}}$ exchange of gluon (3rd diagram) has IR divergence as virtual gluon momentum $\to 0$ (again, this is often regulated by finite gluon mass, as discussed in HW $\boxed{2.1}$).

- The point is that these 2 $\boxed{\text{IR}}$ divergences cancel each other in the above sum so that $\boxed{\text{net}}$ result for $O(\alpha_s)$ correction to QED cross-section has $\boxed{\text{no}}$ IR divergence.

[Indeed, a similar phenomenon happens with (pure) $\boxed{\text{QED}}$, i.e, $\boxed{\text{IR}}$ divergence cancels between real photon emission & virtual photon exchange: see more on this below or for more details, section 12.9.3 of Lahiri, Pal and sections 6.4, 6.5 of Peskin, Schroeder]

- The above $\boxed{\text{cancellation}}$ of IR divergence can be intuitively "understood" (or was "expected to happen") as follows (based on earlier picture of classification of QCD effects. Consider $\boxed{\text{real}}$ emission of soft (collinear) gluons, i.e, $\boxed{p_{T,g} \lesssim \text{GeV}}$ ($\leftarrow$ transverse), which implies that $\boxed{\text{virtual}}$ q (or $\bar{q}$) $\boxed{\text{prior}}$ to emission is off its mass-shell by $\sim p_{T,g}$, i.e., $\boxed{\text{virtual}}$

$\boxed{q}$ (or $\bar{q}$) "lives" for (time) $\sim \frac{1}{p_{T,g}} \gtrsim \frac{1}{\text{GeV}}$.

However, this is much $\boxed{\text{longer}}$ than time/scale of $q\bar{q}$ creation (as estimated earlier from virtuality of photon exchanged) $\sim \frac{1}{\sqrt{s}} \ll \frac{1}{\text{GeV}}$

So, such a "slow" soft (collinear) gluon emission cannot affect probability of $q\bar{q}$ $\boxed{\text{creation}}$ in first place: it will affect which hadrons $\boxed{\text{specifically}}$ $q\bar{q}$ evolve into ... but won't modify $\boxed{\text{net}}$ probability for hadronization (of 1), i.e., this is just part of (or like) "dressing-up" of $q\bar{q}$ (long-distance effects discussed above. So, we expect this IR divergence to "disappear" in the end (and it does cancel as above).

- Again, only $\boxed{\text{correction}}$ to probability of $\boxed{q\bar{q}}$ creation come from "fast" processes, e.g., $\boxed{\text{hard}}$ real emission: $p_{T,g} \sim \sqrt{s}$, in which case they are (perturbatively) calculable
- In summary (after IR divergence cancellation etc.) left-over $\boxed{O(\alpha_s)}$ corrections are from gluon momenta $\boxed{\gg \text{GeV}}$, whether real (tree-level) emission or virtual (loop) exchange (hence these effects are calculable)
- A $\boxed{\text{comparison}}$ of above $\boxed{O(\alpha_s)}$ correction to

$\sigma(e^+e^- \to$ all hadrons) with $O(\alpha_{QED})$ corrections to, say, $\boxed{\sigma(e^+e^- \to \mu^+\mu^-)}$ in pure QED is in order here (as follows).

— In $\sigma(e^+e^- \to \mu^+\mu^-)$ (exclusive process, i.e., no extra "photon": see below), we do not include $\boxed{\text{hard}}$, real $\boxed{\text{photon}}$ emission, since in practice (let alone in principle!) it is a different final state, cf. in $\sigma(e^+e^- \to$ all hadrons), we did add $\boxed{\text{hard}}$ real gluon emission (even though final state is different in principle, like in pure QED) because of $\boxed{\text{inclusive}}$ final state [in turn, that question was asked so that long-distance effects could be "handled", as explained earlier.]; again, "in practice" real gluon emission is "same" final state

— However, even in $\boxed{\sigma(e^+e^- \to \mu^+\mu^-)}$, i.e., "exclusive" process, we do "include" $\boxed{\text{soft}}$ photon emission (just like for QCD corrections above), since in practice it is "same" final state (i.e., soft photon is below detector resolution). Of course, since in principle it is a different final state, we add it at cross-section level ... where it cancels IR divergence from virtual (1 loop) soft photon exchange (again, as outlined above for QCD effects).