QCD: $\boxed{\text{overview}}$ of theory (color structure) phenomenology (strong/weak coupling interplay) (1)

- Here is the $\boxed{\text{Lagrangian}}$ for quarks & gluons:

$$\mathcal{L}_{\text{pure gauge/gluon}} = -\frac{1}{4} G_{\mu\nu}^{a=1,2\ldots8} G^{\mu\nu}_{a}, \text{ where}$$

$G^a_{\mu\nu}$ is gluon field strength {transforming as adjoint representation [i.e., we have 8 ("a" index) gluons] under local $\boxed{SU(3)_{\text{color}}}$ symmetry}.

$\mathcal{L}_{\boxed{\text{quark}}}$ (focussing on gluon interaction/color structure)

$$= \bar{q}^{\,\alpha \leftarrow 1,2,3\ (\text{color})}_{\underset{\uparrow\ \text{flavor}\ (=u,d,c,s,t,b)}{i=1,2\ldots6}}\ i \Big\{ \underbrace{\delta_{ij}}_{\mathbb{1}\text{ in flavor space}} \Big[\not{\partial} + i \underset{\uparrow\ \text{QCD coupling constant}}{g_s} G^a_\mu \gamma^\mu \underbrace{\left(\frac{\lambda_a}{2}\right)_{\alpha\beta}}_{\text{SU(3) generators in triplet representation ("Gell-Mann" matrices)}} \Big]$$

(purely) vector-like Dirac structure

$$+ \underbrace{m_i}_{\text{flavor-dependent}} \delta_{ij} \overbrace{\delta_{\alpha\beta}}^{\text{identity in color space}} +$$

$$\underbrace{\delta_{\alpha\beta}}_{\mathbb{1}\text{ in color space}} \times \left(\underline{\text{EW}} \text{ gauge interactions} \right) \Big\} q^\beta_j$$

from earlier discussion

$\boxed{\text{not}}$ "δ_{ij}", since $W^\pm_\mu$ couple up to $\boxed{\text{down}}$-type quarks, that too of $\boxed{\text{different}}$ generations (due to V_{CKM}) + both axial and vector-type (L, R couplings are different)

- Just to be clear, $\boxed{q^{\alpha}_{i}}$ denotes quark of $\boxed{i^{th}}$ flavor ($i = 1, 2 \ldots 6$, corresponding to u, d, c, s, t, b) and $\alpha (= 1, 2, 3)$ color index: it is a mass eigenstate [of mass m_{i}]
- Also, $\boxed{\text{QCD}}$ interactions of quark (coupling to gluon) are flavor-independent (i.e. flavor "i" index goes along for the ride), but obviously $\boxed{\text{non}}$-trivial in color "α" space
- Similarly, EW gauge interactions don't affect color "α" index, but $\boxed{W^{\pm}_{\mu}}$ couplings mix-up flavor "i" index, since they couple up to down-type quarks, that too of different generations (due to V_{CKM}). Neutral current (photon & Z_{μ}) exchanges of course also preserve flavor, i.e. $\boxed{\propto \delta_{ij}}$
- Finally, $\boxed{\text{QCD}}$ couplings of quarks are (purely) vector-like, just like $\boxed{\text{photon}}$, while $\boxed{W^{\pm}_{\mu}}$ and $\boxed{Z_{\mu}}$ interactions are chiral [i.e., different for L, R, due to axial-vector & vector components]
- Representations (full SM) of quarks, Higgs doublet & leptons are shown as:

$$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix} : \Big(\underset{\substack{\uparrow \\ \text{color triplet}}}{3}, 2, +\tfrac{1}{6}\Big); \; u_R : \left(3, 1, \tfrac{2}{3}\right); \; d_R : \left(3, 1, -\tfrac{1}{3}\right)$$

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} : \Big(\underset{\substack{\uparrow \\ \text{color singlet}}}{1}, 2, -\tfrac{1}{2}\Big); \; e_R : (1, 1, -1); \; \Phi : \left(1, 2, +\tfrac{1}{2}\right)$$

[dropping generation index]

— Onto $\boxed{\text{overview}}$ of QCD phenomenology: recall that we have 2 energy regimes (due to asymptotic freedom), i.e, $\boxed{E_{UV} \gg \Lambda_{QCD} \sim \text{GeV}}$ (binding energy of hadrons), where QCD coupling constant is $\boxed{\text{weak}}$ (perturbation theory is valid) vs. $\boxed{E_{IR} \lesssim \text{GeV}}$, where QCD is strongly-coupled/non-perturbative.

— So, we will compute (obviously/as usual) in perturbative regime $\boxed{E \gg \text{GeV}}$ in terms of quarks & gluons, but will ultimately have to "enter" 2^{nd} regime in order to "convert" quarks/gluons into (observed) hadrons: the challenge will then be how to "circumvent/handle" this last step (strong coupling in) (details in next note)

— Here's the $\boxed{\text{outline}}$ of $\boxed{3}$ types of processes that we will study [again, all at initial/COM $\boxed{E} \gg$ GeV]

$\boxed{(1).}$ $\boxed{e^+e^-} \rightarrow$ all hadrons (i.e., inclusive), i.e., $\boxed{\text{no}}$ hadrons in initial state: the leading-order, underlying (purely EW) process is $e^+e^- \rightarrow$ (virtual) photon/Z_μ $\rightarrow q\bar{q}$. $\boxed{\text{QCD}}$ will then enter in $\boxed{2}$ ways:

$\boxed{(a).}$ short-distance effect (calculable analytically/in perturbation theory) will modify cross-section for underlying/parton-level process by $\boxed{O\left(\frac{\alpha_s}{4\pi} \ll 1\right)}$; whereas, at

$\boxed{(b).}$ long-distances/non-perturbatively, QCD will (mostly) "convert/dress-up" $q, \bar{q}$ into hadrons: since we $\boxed{\text{sum}}$ over all hadrons in $\boxed{\text{final}}$ state, probability for this step is "simply" 1 (basically that's how we can still predict)

[2]. (Deep) inelastic scattering (DIS) at energy transfer >> GeV, ~~i.e.,~~

e^- (or ν) + nucleus $\longrightarrow$ e^- (or ν) + hadrons

[i.e., one hadron in initial state] (i.e., nucleus breaks-up)

— Once again, underlying process is photon/Z_μ/$W_\mu^\pm$ exchange between incoming lepton (e^- or ν) and (weakly-coupled) constituent (parton) of nucleus, i.e., quark, [(a).] which is "corrected" perturbatively by short-distance QCD effects and [(b).] scattered quark materializes (along with remnants (other partons of nucleus) as (final-state) hadrons due to long-distance / non-perturbative QCD processes [like for process (1)]

- Additionally [cf. process (1)], we have (one) hadron in initial state. So, [(c).] we need to know probability to find a parton (in this case quark) inside (initial) hadron with a given fraction of total momentum of hadron, called probability / parton distribution function (PDF)

Obviously, PDF's cannot be computed analytically / perturbatively starting from QCD (i.e., these are long distance effects). So, (rough) idea is we "extract" PDF's from one set of data ("input"); then, use these to predict other observables

[3]. Hadron-hadron collisions : underlying (sub-)process is scattering of [2] partons (quarks or gluons)

into a $\boxed{\text{final}}$ state which may (or not) be hadrons [cf. processes (1) & (2) above]. So, in addition to above $\boxed{3}$ ways QCD effects come in, i.e., (a). QCD short-distance corrections to underlying process; (b). dressing-up of final state – $\boxed{\text{if}}$ they are partons – into hadrons [both being like processes (1) & (2)] and (c). inclusion of PDF's due to hadronic initial state [like in process (2)], we have 2 (or 3) "new" features, i.e., $\boxed{(c)'}$. use PDF's twice [vs. once in process (2)] due to $\boxed{2}$ hadrons in initial state; $\boxed{(d).}$ underlying process could be $\boxed{\text{gluon}}$ exchange, in addition to EW gauge boson exchange (again, since we can have (2) quarks / gluons (in initial state): note that QCD short distance effects come in for $\boxed{\text{both}}$ cases and $\boxed{(e).}$ as already mentioned above, final state could be $\boxed{\text{leptons}}$, e.g., $\boxed{q\,\bar{q}} \rightarrow$ virtual photon/ Z_μ / W_μ exchange [called Drell, Yan $\boxed{DY}$ production] $\longrightarrow \boxed{e^+ e^-}$ or $\boxed{e^- \bar{\nu}}$ so that there is no dressing-up by non-perturbative QCD effects for final state: we will actually study such a process due to its relative simplicity [In general, the discussion of details of above 3 processes that follows will be somewhat "sketchy": for $\boxed{\text{more}}$ precise statements, see $\boxed{\text{chapter 17}}$ of Peskin, Schroeder.]