Detailed Electroweak (EW) theory/ unification: part (II) (fermionic sector)

- Next, we add SM fermions to above (purely) bosonic sector (i.e., gauge & Higgs fields), including couplings between the 2 sectors
- We will consider only one generation (out of ③) of SM fermions, postponing study of full generations/story (including mixing between them) to the phenomenlogy part of this course
- Begin with EW gauge interactions of SM fermions: recall that these are chiral in Dirac/spinor space, cf. QED coupling being purely vector-like
- Since above is a (relatively) new feature, let's elaborate on this point (at the risk of repetition!): recall that 4-component Dirac spinor (ψ) is a reducible representation of Lorentz group, i.e., its left/right (L/R) chiralities [obtained by projection operator $\frac{1}{2}(1 \mp \gamma_5)$ on ψ], denoted by $\psi_{L,R}$, transform independently under Lorentz group [Note, in so-called Weyl/chiral

basis for Dirac/$\underline{\underline{\gamma}}$- matrices, $\psi_{L,R}$ are
simply <u>upper</u>/<u>lower</u> 2 components of ψ.]

– So, $\boxed{\psi_{\textcircled{L},\textcircled{R}}}$ <u>can</u> have $\boxed{\text{different}}$ <u>gauge</u> interactions : this does not happen in QED, but that's the case with EW gauge interactions

– Before getting into full EW theory, let's just "warm-up" with $\boxed{U(1)}$ case; QED first, then its chiral version

– $\mathcal{L}_{\psi \text{ only}} = i\bar{\psi}\gamma_\mu \partial \psi - m\bar{\psi}\psi$

$$= i\bar{\psi}_{\textcircled{L}} \not\partial \psi_{\textcircled{L}} + i\bar{\psi}_{\boxed{R}} \not\partial \psi_{\boxed{R}} - m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$$

i.e., kinetic term "couples" $\textcircled{L}$ to $\textcircled{L}$ (and R to R), while mass term connects $\textcircled{L}$ to $\boxed{R}$

– We add coupling to photon (A_μ), which is <u>same</u> for both $\textcircled{L}$, $\boxed{R}$, i.e., $\partial_\mu \to D_\mu = \partial_\mu - ieA_\mu Q$, so that

$$\mathcal{L}_{\psi \text{ only}} + \mathcal{L}_{\psi-A} = i(\bar{\psi}_L \not{D}\psi_L + \bar{\psi}_R \not{D}\psi_R) - m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L),$$

i.e., photon also couples $\textcircled{\text{L to L}}$ <u>and</u> $\boxed{\text{R to R}}$, that too <u>identically</u> (vector-like). Also, (bare) mass term is gauge-invariant

– Onto $\boxed{\text{chiral}}$ version of <u>U(1)</u> [(while discussing U(1) Higgs mechanism, we had considered such a theory as "toy" model for EW theory, whose gauge group is more involved]:

$$\mathcal{L} \ni i(\bar{\psi}_L \textcircled{$\not{D}$} \psi_L + \bar{\psi}_R \boxed{\not\partial} \psi_R) - h\underset{+1}{\phi}\,\underset{-1}{\bar{\psi}_L}\,\underset{0}{\psi_R} + h.c. \quad [U(1) \text{ charges}]$$

i.e., only $\psi_{\text{Ⓛ}}$ couples with charge +1 to gauge boson (A_μ), so $\psi_{\boxed{R}}$ only has "ordinary" kinetic term. Also, a bare mass term for ψ will not be gauge-invariant [again, since only ψ_L is charged under U(1)], hence has to arise after spontaneous (gauge) symmetry breaking, i.e., from coupling to ϕ VEV (as shown above)

- The EW theory generalizes above to $SU(2)_L \times U(1)_Y$. Here are the EW gauge quantum numbers/representations of SM fermions [of each quark carries also "color", i.e., charge under QCD/$SU(3)_C$, but we will suppress that here for simplicity]:

(1). Leptons:
$$\boxed{\text{Ⓛ} \equiv \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L : \Big(\underset{\substack{\uparrow \\ \text{doublet} \\ \text{of } SU(2)_L}}{2} \,,\, \underset{\substack{\uparrow \\ Y}}{-1/2} \Big)} \;\&\; \boxed{e_R : \Big(\underset{\substack{\nearrow \\ \text{singlet of} \\ SU(2)_L}}{1} \,,\, \underset{\nearrow}{-1} \Big)}$$

(2) Quarks:
$$\boxed{\begin{pmatrix} u \\ d \end{pmatrix}_L \equiv \text{Ⓠ} : \left(2, +1/6\right)} ; \boxed{u_R : \left(1, +\frac{2}{3}\right)} \;\&\; \boxed{d_R : \left(1, -\frac{1}{3}\right)}$$

- Explanation: $SU(2)_L$ representations are clear based on weak charged current interaction, i.e., neutron and muon decays etc. involving only Ⓛ chiralities of both quarks & leptons

- Y assignments are derived by requiring electric charge, $\boxed{Q = T_3 + Y}$, since, as shown in previous note (based on abelian/neutral part of general D_μ), massless combination of W_3^μ & B^μ (aka photon) couples to $(Y + T_3)$: as sanity check, we do get observed electric charges:

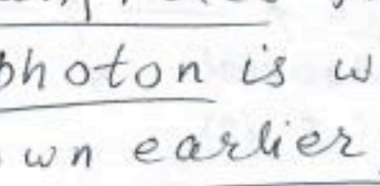
Q of $\boxed{e_L^-} = \overset{T_3}{-1/2}\ \overset{Y}{-1/2} = -1$; Q of $\boxed{e_R^-} = \overset{T_3}{0}\ \overset{Y}{-1} = -1$

[i.e., T_3 is different for L, R, but so is Y, thus electric charge is same] & Q of $\boxed{\nu_e} = \underset{T_3}{+1/2}\ \underset{Y}{-\frac{1}{2}} = 0$

[i.e., Y is same for ν_{e_L}, e_L, but $\boxed{T_3}$'s are different, thus electric charges also differ]

Similarly, Q of both $u_{L,R} = +2/3$ & Q of both $d_{L,R} = -\frac{1}{3}$

– Onto Feynman rules for SM fermion-gauge couplings: photon is well-known already, while as shown earlier, (massive) Z_μ has gauge coupling $\boxed{g_Z = \sqrt{g^2 + g'^2}}$ and "charge", $\boxed{Q_Z = T_3 - Q \sin^2\theta_W}$. So, we get

Feynman rule for 

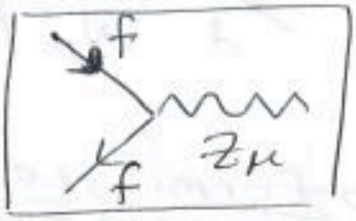
is $\boxed{-i\, g_Z \underbrace{\left(T_{3H} - Q\sin^2\theta_W\right)}_{Q_Z^H} \times \gamma_\mu \times P_{L,R}}$

where f denotes fermion (quark/lepton) of charge Q; $H = L, R$ chirality, $\boxed{P_H = 1/2\,(1 \mp \gamma_5)}$ being corresponding projection operator

— Note that $\boxed{Z_\mu}$ coupling is $\boxed{\text{different}}$ for Ⓛ vs. $\boxed{R}$ chiralities (due to T_3 part being different, while Q is same for both charalities), i.e., it is not (purely) vector-like

— Also, $\boxed{\nu_e}$ does couple to Z_μ (even though not to the other neutral gauge boson, i.e., photon)

- It is convenient to combine Z_μ coupling to $\boxed{L, R}$ chiralities :



$$: \boxed{-ig_Z\left(\frac{Q_Z^V}{2}\gamma_\mu - \frac{Q_Z^{\textcircled{A}}}{2}\gamma_\mu \textcircled{γ_5}\right)}$$

$$= -\frac{ie}{\sin 2\theta_W}\left(Q_Z^V \gamma_\mu - Q_Z^A \gamma_\mu \gamma_5\right)$$

where in (1st) line $\boxed{Q_Z^V} = Q_Z^L + Q_Z^R$

$$\boxed{= T_{3\textcircled{L}} - \textcircled{2}\, Q \sin^2\theta_W}$$

and $\boxed{Q_Z^A = Q_Z^L - Q_Z^R = T_{3\textcircled{L}}}$ are the vector-like (i.e., involving only γ_μ) and axial [coupling to $\gamma_\mu \textcircled{$\gamma_5$}$] parts of coupling to Z_μ.

In $\boxed{2^{nd}}$ line above, we have used $g_Z = g/\cos\theta_W$ and $e = g \sin\theta_W$

- For example, $\boxed{Q_Z^{\textcircled{V}}}$ for $\textcircled{$e^-$} = \overbrace{-1/2}^{T_{3\textcircled{L}}} + 2\sin^2\theta_W$ & $\boxed{Q_Z^{\textcircled{A}}} = -\frac{1}{2}$

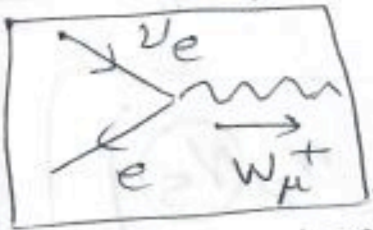
— Onto charged current weak interaction, i.e, really <u>non</u> - <u>abelian</u> part of D_μ:

$$\mathcal{L}_{\text{charged, weak}} = -\left(\bar{\nu}_{e_L}\ \bar{e}_L\right)\gamma^\mu g \frac{\sigma_{1,2}}{2} W_\mu^{1,2}\begin{pmatrix}\nu_{e_L}\\ e_L\end{pmatrix}$$

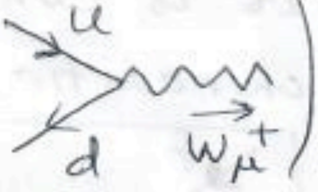
$$= -g/\sqrt{2}\left[\bar{\nu}_{e_L}\gamma^\mu \underbrace{e_L}_{\frac{1}{2}(1-\gamma_5)e} W_\mu^+ + \bar{e}_L\gamma^\mu W_\mu^- \nu_{e_L}\right]$$

where $\boxed{W_\mu^\pm} = \frac{W_1 \pm i W_2}{\sqrt{2}}$

(similarly for quarks) giving Feynman rules

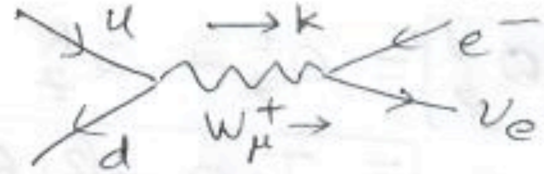


: $\boxed{\frac{-ig}{\sqrt{2}}\gamma^\mu \underbrace{P_L}_{\frac{1}{2}(1-\gamma_5)}}$ (also for



)

— We can then precisely "match" EW theory to Fermi's, i.e., amplitude for underlying process in neutron (similarly, muon) decay:



is given by

$$i\mathcal{M}_{\beta\,\text{decay}} = \left(-ig/\sqrt{2}\right)^2 \underbrace{\left(\frac{-ig^{\lambda\rho}}{k^2-M_W^2}\right)}_{W_\mu^+\text{ propagator}} \left(\bar{u}_{\nu_e}\gamma^\lambda P_L \underset{\uparrow}{v_e}\right)\left(\bar{u}_d \gamma_\rho P_L u_u\right)$$

spinor (<u>not</u> up-quark!)

low energies ($k^2 \ll M_W^2$)

$$\approx \underbrace{\left(\frac{g^2}{8M_W^2}\right)}_{\text{identify with } \boxed{G_F/\sqrt{2}}}\left[\bar{u}_{\nu_e}\gamma^\lambda(1-\gamma_5)v_e\right]\left[\bar{u}_d\gamma_\lambda(1-\gamma_5)u_u\right]$$

[again, above form - in terms of G_F - was deduced experimentally]

using $M_W = \frac{1}{2} g v$

So, we get $\boxed{G_F = \frac{1}{\sqrt{2}\, v^2}} = 1.17\times 10^{-5}/(\text{GeV})^2 \Rightarrow \boxed{v \approx 250 \text{ GeV}}$

- As an aside (for now), we also have $\boxed{e(\simeq 0\cdot 3) = gg'/\sqrt{g^2+g'^2}}$. So, we need $\boxed{1}$ more measurement to determine separately $\boxed{g \text{ and } g'}$, i.e., this part of EW theory has ③ parameters (v, g, g'): we have used 2 data (e & G_F) thus far in order to fix $\boxed{v}$ and one combination of g, g'.
- This ③rd measurement must involve neutral current weak interaction ($\boxed{Z_\mu}$ exchange): it was performed in $\boxed{\text{1970's}}$ (the EW theory itself was proposed in $\boxed{\text{1960's}}$).
- Then, $M_{W,Z}$ ($\sim [g, g_Z]\, v$) values could be predicted; these gauge bosons were discovered in $\boxed{\text{1980's}}$ (We will return to these issues in the phenomenology part of the course.)
- Onto, $\boxed{\text{SM } \boxed{\text{fermion}} \text{ masses}}$: as already indicated, bare mass terms will break $SU(2)_L$ gauge symmetry [and $U(1)_Y$] since L, R chiralities have different gauge charges/interactions
- So, mass terms for SM fermions must come from (Yukawa) coupling to Higgs field (VEV), again, like in toy chiral U(1) model discussed before
- However, since gauge group/charges are more involved than in U(1) case, it is useful to take a "group theory detour"

Detour on group theory [how to form SU(n)-invariant objects] (see, for example, sections 4.1 – 4.3 of Cheng, Li)

- Suppose a field $\boxed{\Psi}$ (which could be bosonic!) transforms in a representation of SU(n) group of dimension $\boxed{d_\psi}$ [i.e., Ψ can be thought of as a column vector with d_ψ rows], with generators T_a $(a = 1, 2 \ldots n^2-1)$ [$d_\psi \times d_\psi$ traceless, Hermitian matrices, but not necessarily a complete set, i.e., that's the case only for fundamental representation]

— So, under an SU(n) transformation (with parameters β_a), we have

$$\boxed{\Psi'} \text{(i.e., "new" } \Psi) \boxed{= \overset{\text{matrix}}{U} \overset{\text{column}}{\Psi}}, \text{ where } U = \exp(i\beta_a T_a)$$

— Then $\boxed{\Psi^*}$ (still thought of as column vector) transforms in conjugate representation:

$$(\Psi^*)' = (\Psi')^* = \left[\exp(i\beta_a T^a)\Psi\right]^* = \exp(\ominus i\beta_a T^{a*})\Psi^*,$$

i.e., $[(-)(T^a)^*]$ are also generators [of conjugate representation] of SU(n), since they satisfy the SU(n) Lie algebra:

$$\left[(-)T^{a*}, (-)T^{b*}\right] = i f^{abc}\left((-)T^{c*}\right)$$

which, in turn, follows from taking complex conjugate of

$$[T^a, T^b] = i f_{abc} T^c$$

[The conjugate representation is often denoted by a "bar" on top of original one, e.g., if Ψ is 3 of SU(3) – as for quarks under QCD, then Ψ^* is $\bar{3}$ (i.e, anti-quarks)]

– $\boxed{\text{If}}$ there exists a matrix $\boxed{S}$ such that

$$S T_a S^{-1} = (-) T_a^* \text{ (for all } a\text{'s)}$$

of course, we need different S's for different representations

then $\boxed{T_a \text{ and} (-) T_a^*}$ are said to be equivalent and such a representation is called "real"

– Onto, 2 ways of forming $\boxed{SU(n)}$-invariant objects

$\boxed{(1)}$ Suppose $\boxed{\text{two}}$ [in general, different: each being fermionic or bosonic] fields transforming in the same representation: in component form,

$$\boxed{\phi_{i = 1 \ldots d_\phi}} \text{ and } \boxed{\psi_{i = 1 \ldots d_\psi}}, \text{ with } \boxed{d_\psi = d_\phi}$$

Then, the following product is SU(n)-invariant

1st way

$$\boxed{\mathbb{1}_1 \equiv \phi_i^* \psi_i}$$

singlet

[similarly $\psi_i^* \phi_i^*$: repeated indices being summed as usual]

This is easy to show if we "think" of ϕ^* as row-vector, i.e., $\boxed{\Phi^\dagger}$ ["dagger" in SU(n)-space matrix sense, although we'll do it for corresponding quantum $\boxed{\text{operator}}$ as well later]

$$= (\phi_1^* \phi_2^* \ldots \phi_{d\phi}^*)$$

so that $\mathbb{1}_1 = \Phi^+ \Psi$, where $\Psi' = U\Psi$ and $\Phi' = U\Phi$ implies $(\Phi^+)' = (\Phi')^+ = \Phi^+ \boxed{U^+}$,

giving $(\Phi^+)'\Psi' = \Phi^+ \underset{\mathbb{1}}{\boxed{U^+ U}}\, \Psi = \Phi^+ \Psi$

i.e., $\phi_i^* \psi_i$ is $\boxed{\text{singlet}}$ of SU(n)

← component form

$\boxed{(2)}$ If $\Psi, \Phi \ldots \chi_{i=1,2\ldots n}$ are $\boxed{n}$ fields each

column vectors

transforming as $\boxed{\text{fundamental}}$ (i.e., n-dimensional) representation of SU(n), then (proof is left as-informal-HW!)

$$\overset{\text{2nd way}}{\mathbb{1}_2} \equiv \boxed{\left(\psi_{i_1} \phi_{i_2} \ldots \chi_{i_n}\right) \underbrace{\left(\boxed{\epsilon}^{\,i_1 i_2 \ldots i_n}\right)}_{\text{completely anti-symmetric}}}$$

is another SU(n)-invariant.

- Onto $\boxed{SU(2)}$ specifically ($n = 2$), where all representations are real. For example for fundamental representation (i.e, doublet), $\boxed{S = \sigma_2}$ relates $T^a = \frac{\sigma^a}{2}$ to $-T^{a*}$, i.e., $\tilde{\Psi} \equiv \boxed{i \sigma_2 \Psi^*}$ transforms like Ψ itself (i.e., doublet or "2") and not like Ψ^* (i.e., "anti"-doublet or "$\overline{2}$")

... now, $\boxed{\text{back}}$ to SM fermion masses!

(Yukawa) Coupling of SM fermions to Higgs field, $\Phi : (2, \overset{\leftarrow Y}{\oplus \tfrac{1}{2}})$

↳ doublet of SU(2)

- Begin with leptons [recall that $L = \begin{pmatrix} \nu_{e_L} \\ e^-_L \end{pmatrix}$ is $\left(2, -\frac{1}{2}\right)$, while e^-_R is $(1, -1)$] :

coupling constant (a la Lahiri, Pal)

$$\mathcal{L}_{\underline{\text{lepton}}\ \underline{\underline{\text{Yukawa}}}} = -h_e \underbrace{\left(\bar{L}_i \,\phi_i\right)}_{\substack{\boxed{SU(2)_L}\ \text{singlet a la } \textcircled{1st} \text{ way} \\ (i = 1, 2,\ \text{i.e., doublet index})}} e_R + \text{h.c.}$$

[just to be clear, "bar" on "L" is as in Dirac space, i.e., $L^{\dagger} \gamma_0$, but here it also stands for "$\bar{L}$" being anti-doublet.]

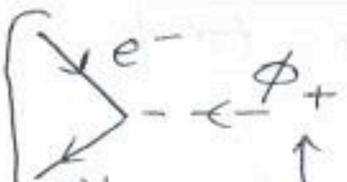
$\boxed{U(1)_Y}$ charges add: $(\underset{\substack{\uparrow \\ Y = (-)1/2 \\ \text{for } L}}{+1/2} \ \underset{\substack{\uparrow \\ \Phi}}{+ 1/2} \ \underset{\substack{\uparrow \\ e_R}}{-1})$ upto $\boxed{0}$

- After SSB, i.e, setting $\Phi = \begin{pmatrix} \phi_+ \\ \frac{v + H + i\xi}{\sqrt{2}} \end{pmatrix}$ (and arguing that $\boxed{h_e}$ can be chosen to be real without loss of generality (see later however when we go to 3 generations), we get (after some algebra)

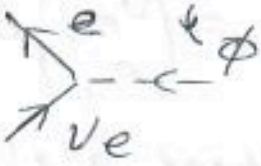
$$\mathcal{L}_{\text{lepton Yukawa/mass}} = -h_e \Big[v/\sqrt{2} \left(\bar{e}_L e_R + \bar{e}_R e_L\right) + \bar{\nu}_{e_L} e_R \phi_+ + \bar{e}_R \nu_{e_L} \phi_- + \frac{1}{\sqrt{2}} \left(\bar{e} e H + i \bar{e} \underset{\substack{\uparrow \\ \text{note}}}{\gamma_5} e \xi\right) \Big]$$

i.e, $\boxed{m_e = h_e v/\sqrt{2}}$ from 1st term and following Feynman rules for (Yukawa) interactions from other terms:

charged

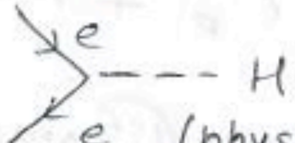


(would-be NGB) : $-i\sqrt{2}\,\frac{m_e}{v}P_R = -\frac{ig\,m_e}{\sqrt{2}M_W}P_R$ ← e^-_{R} is annihilated

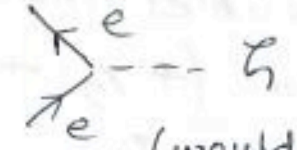


: $-i\sqrt{2}\,\frac{m_e}{v}P_L = -\frac{ig\,m_e}{\sqrt{2}M_W}P_L$ ← ν_{e_L} is annihilated

neutral



(physical) : $-i\frac{m_e}{v} = -\frac{ig}{2\cos\theta_W}\frac{m_e}{M_Z}$



(would-be NGB) : $\frac{m_e}{v}\gamma_5 = \frac{g}{2\cos\theta_W}\frac{m_e}{M_Z}$

- What about $\boxed{2^{nd}}$ way of forming SU(2)-invariant out of L and Φ (does it give different/additional Lagrangian terms) $\boxed{?!}$

"No": the 2 ways of forming SU(2)-invariant objects are actually equivalent / give <u>same result</u> due to doublet representation being real. Explicitly

(1) $\boxed{\bar{L}_i\phi_i}$ (that we already used above)

(~) [<u>not</u> keeping track of $\boxed{\text{Dirac}}$ structure, i.e., focussing only on SU(2)/internal space]

$\left(\nu_{e_L}^{\dagger}\ e_L^{-\dagger}\right)\begin{pmatrix}\phi_+\\ \phi_0\end{pmatrix} = \boxed{\nu_{e_L}^{\dagger}\phi_+ + (e^-)_L^{\dagger}\phi_0}$, which

$\dagger$ → acting on operator

when coupled to e^-_R and Φ set to VEV & fluctuations gave the above bottomline

$\mathcal{L}$ lepton Yukawa/mass

note e^-, ν_e "flipped" due to σ_2

$$(2).\ \boxed{\underbrace{\left(i\sigma_2\, L^\dagger\right)}_{}{}_i\ \phi_j\ \varepsilon^{ij}} = \left[(e^-_L)^\dagger\ \underset{\uparrow}{(-)}(\nu_e)_L^\dagger\right]\underbrace{\begin{pmatrix}0 & +1\\ (-)1 & 0\end{pmatrix}}_{\varepsilon\ \text{"matrix"}}\begin{pmatrix}\phi_+\\ \phi_0\end{pmatrix}$$

note (−1)

$$= \boxed{(e^-)_L^\dagger\, \phi_0 \oplus (\nu_e)_L^\dagger\, \phi_+}$$

doublet: still a column-vector (i.e., $\tilde{L}$), since "dagger" here acts on operators [not quite in SU(2) matrix space]

(i.e., 2, not $\bar{2}$)

i.e., the 2 invariants are identical

Finally (in lepton sector still), let's consider fate of **neutrino** in terms of mass term. There is **no** $(\nu_e)_R$ particle/field in SM, since none has been observed (at least directly) thus far (cf. all other fields/particles). So, there is no (Yukawa) coupling of neutrino to Higgs VEV (cf. for electron), giving $\boxed{m_\nu = 0}$ (again, with SM renormalizable Lagrangian).

- However, there is evidence for $\boxed{m_\nu \neq 0}$ (albeit $m_\nu \ll m_e$) from observations of neutrino **oscillations** (i.e., conversion of neutrino of 1 type/generation into another): we need to go **beyond** SM in order to accommodate $m_\nu \neq 0$ (see term paper?!)

Onto $\boxed{\text{quark}}$ sector, where masses come from

$$\boxed{U(1)_Y}: \quad \mathcal{L}_{\text{quark Yukawa}} = -h_d \underbrace{\left(\overset{-1/6}{\bar{Q}}_i \; \overset{+1/2}{\phi_i}\right)}_{\text{SU(2) singlets}} \overset{-1/3}{d_R} - h_u \underbrace{\overset{-1/6}{\bar{Q}}_i \; \overset{-1/2}{\tilde{\Phi}}_i} \overset{+2/3}{u_R} + \text{h.c.}$$

[Again, $\boxed{U(1)_Y}$ charges add up to $\boxed{0}$ & doublets form $\boxed{SU(2)_L}$ singlets.]

In more detail,

(a) down-type quark coupling involves Φ/just like leptons/, giving $\boxed{m_d}$ (mass term) $\rightarrow$ $(h_d/\sqrt{2}\, v)$ and (Yukawa) interactions like for leptons

(b) Subtlety for $\boxed{\text{up}}$-type quark: we can't couple

$$\underbrace{\bar{Q}\, u_R}_{\substack{Y = -1/6 + 2/3 \\ = +1/2}} \text{ to } \underbrace{\Phi}_{Y = +1/2} \text{ since } U(1)_Y \text{ charges won't sum to } 0,$$

i.e., we have to use $\Phi^* (Y = -1/2)$, which can be done in 2 ways (giving same result, like shown for leptons above):

"dagger" for operator

$$\text{(i)} \quad \bar{Q}_i \underbrace{\left(i\sigma_2 \Phi^*\right)_i}_{\tilde{\Phi}} \sim \left(u_L^{+} \;\; d_L^{+}\right) \begin{bmatrix} \phi_0^* \\ (-1)\phi^- \end{bmatrix} = u_L^{+} \phi_0^* \ominus d_L^{+} \phi_-$$

(used above)

[Again, we can't use simply Φ^* (i.e., without $i\sigma_2$), since that doesn't transform as doublet (it's a $\bar{2}$), but $i\sigma_2 \Phi^*$ does: see $\boxed{HW}$ 8.3]

(ii) (just for sanity check/practice!)

$$\underbrace{\bar{Q}_i}_{\uparrow}\ \underbrace{\phi^*_j}_{\uparrow}\ \boxed{\varepsilon^{ij}} \overset{\text{matrix/notation}}{\sim} (u_L^+\ d_L^+) \overbrace{\begin{pmatrix}0 & +1\\ \underset{\overset{\uparrow}{\text{note}}}{-1} & 0\end{pmatrix}}^{\textcircled{\varepsilon}\ \text{"matrix"}} \begin{pmatrix}\phi_+^*\\ \phi_0^*\end{pmatrix}$$

both are anti-doublets ($\bar{2}$)

$$= u_L^+ \phi_0^* \ominus d_L^+ \underbrace{\phi_-}_{\phi_+^*}$$

i.e., same as (i)

- The above story with up-type quark is not a big deal in the SM, but it is when we go to its $\boxed{\text{supersymmetric}}$ version (see term paper), where (in short) we add (super)partners of SM particles with spin differing by ½ from SM. It turns out that $\boxed{\text{coupling}}$ $\tilde{\Phi}$ (or ϕ^*) as above to $\bar{Q}\, u_R$ breaks supersymmetry (SUSY). So, we must introduce $\boxed{2}$ Higgs doublet fields: $\Phi_{\textcircled{d}}$ coupled to leptons $(\bar{L}\ \&\ \bar{e}_R)$ and to d_R & $\bar{Q}$ (like Φ in SM), while $\Phi_{\boxed{u}}$ (with opposite hypercharge to $\Phi_{\textcircled{d}}$) couples to $(\bar{Q}\ u_R)$: in SUSY Φ_u plays role of $i\sigma_2\Phi^*$ in SM, but again, we can't use $i\sigma_2\Phi^*_{\textcircled{d}}$ here.

- Anyway, it is easy to see that h_u coupling gives $\boxed{m_{\textcircled{u}}}$ (mass term) and (Yukawa) interactions (like for leptons & down-type quark)