Phenomenlogy of EW sector: part (I) [details]

①

(1). Neutrino (neutral current: $\boxed{Z}$ exchange) scattering

— 1st observation of Z-exchange (weak neutral current) was via

$$\boxed{\nu + \text{nuclei} \longrightarrow \text{hadrons} + \nu}$$ (inelastic, i.e., net final state $\neq$ initial state)

again, non-observation of e^- in final state implies it's not from $\boxed{W^\pm}$ exchange (charged weak current)

(i.e., that would be $\nu + \text{nuclei} \rightarrow \text{hadrons} + e^-$)

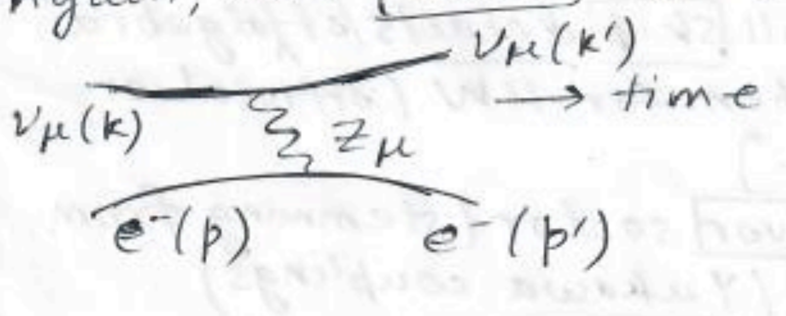
and obviously $\boxed{\nu}$ doesn't couple to photon $\boxed{\Rightarrow}$

$\boxed{\text{mere}}$ observation of above process (above background of course) is evidence for Z-exchange

— Just to avoid complications (in calculation) due to strong (nuclear) force, will instead study here a (purely) leptonic "analog" of above:

$$\boxed{\nu_\mu + e^- \longrightarrow \nu_\mu + e^-}$$ (this is elastic)

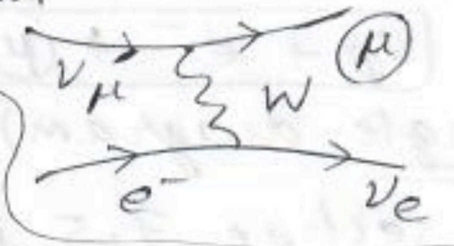
Again, this $\boxed{\text{must}}$ be via Z-exchange



[$\boxed{\text{no}}$ coupling of ν to H (physical Higgs boson) or photon to give other neutral current]

— Note that $\nu_{\mu} + e^- \to \mu^- + \nu_e$ (inelastic)

occurs via $\boxed{W}$ exchange instead: (2)



— Using Feynman rules given earlier, we get

$$i\mathcal{M} = -\big(i \underbrace{g_Z}_{\to\, g/\cos\theta_W}\big)^2 \times \underbrace{\left(\frac{-i g^{\lambda\rho}}{q^2 - M_Z^2}\right)}_{Z \text{ propagator}} \times \Big[\bar{u}(k')\,\gamma_\lambda \big(\tfrac{1}{2}\big) P_L\, u(k)\Big]$$

$\boxed{q = k' - k = p - p'}$

$\frac{1}{2}$: T_3 of ν_L $\qquad$ P_L: no ν_R (like charged weak current)

$$\times \Big[\bar{u}(p')\, \gamma_\rho \underbrace{\left(Q^V_{Z,e} - \gamma_5 Q^A_{Z,e}\right)}_{\substack{\boxed{\text{both}}\ L,\, R \text{ chiralities of } e^- \\ \text{contribute (unlike charged weak current)}}} u(p)\Big]$$

low energies $(q^2 \ll M_Z^2)$

$$\longrightarrow \left(\frac{g}{\underbrace{2 M_Z \cos\theta_W}_{M_W = \frac{1}{2} g v}}\right)^2 \Big[\bar{u}(k')\,\gamma_\lambda L\, u(k)\Big] \times \Big[\bar{u}(p')\,\gamma^\lambda \left(Q^V_{Z,e} - \gamma_5 Q^A_{Z,e}\right) u(p)\Big]$$

which can be obtained from (effective) phenomenological/ $\boxed{\text{4-fermion}}$ (local) interaction (a la charged weak current):

$$\mathcal{L}_{int,Z} = \left(\frac{G_F}{\sqrt{2}}\right) \Big[\bar{\Psi}_{(\nu_\mu)}\, \gamma_\lambda (1 - \gamma_5)\, \psi_{(\nu_\mu)}\Big] \times \Big[\bar{\Psi}_{(e)}\, \gamma^\lambda \underbrace{\left(Q^V_{Z,e} - \gamma_5 Q^A_{Z,e}\right)}_{\text{again, } \boxed{\text{not}} \text{ purely } \boxed{L} \text{ chirality for } e^-} \psi_{(e)}\Big]$$

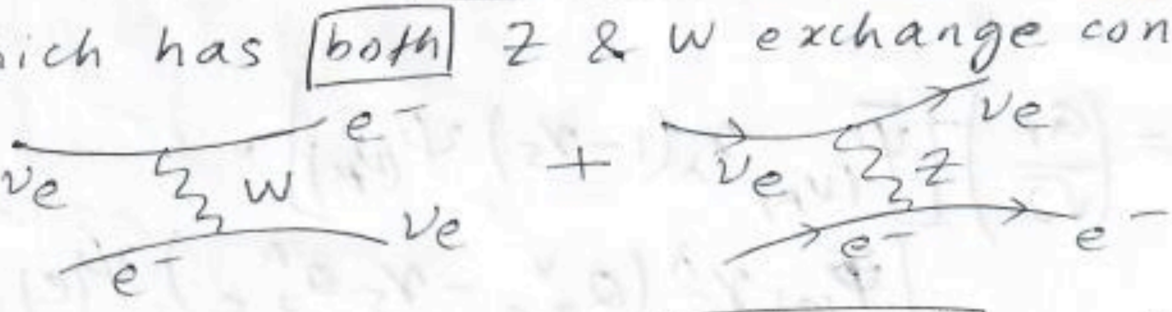
(compare to muon-decay operator)

— $\boxed{\text{Calculation}}$ of cross section is <u>similar</u> ③
to $\boxed{\nu_{\mu} + e^- \to \mu^- + \nu_e}$ (again, both involve <u>single</u> diagram), which is done in detail in section 7.5 (on page 140 onwards) of Lahiri, Pal. We get (see section 15.5.2 on pages 349, 350 of Lahiri, Pal)

$$\sigma = G_F^2 \, \frac{m_e \omega}{2\pi} \left[\underbrace{\left(Q^V_{Z,e} + Q^A_{Z,e}\right)}_{Q_{Z,e_L}}{}^2 + \frac{1}{3} \underbrace{\left(Q^V_{Z,e} - Q^A_{Z,e}\right)}_{Q_{Z,e_{\boxed{R}}}}{}^2 \right]$$

— This measurement [labelled (i)] determines $Q_{Z,e}$; thus, <u>in principle</u>, we can <u>get g', g separately from</u> it and e (actually, a "related" process $\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-$ measures another combination of Q_Z's: (in)formal HW!).

— However, there are clearly <u>discrete</u> <u>ambiguities</u> in above idea, so we need <u>two</u> <u>more</u> $\boxed{\text{measurements}}$ <u>in order to resolve</u> them: <u>for example</u> (see Cheng, Li: section 12.1 on page 364 onwards; especially Fig. 12.3 on page 367) we can use [labelled (ii)] $\boxed{\nu_e + e^- \to \nu_e + e^-}$ (<u>HW</u> 9.1), which has $\boxed{\text{both}}$ Z & W exchange contributions:



thus it measures a $\boxed{\text{different}}$ combination of $\boxed{g, g'}$ than above process

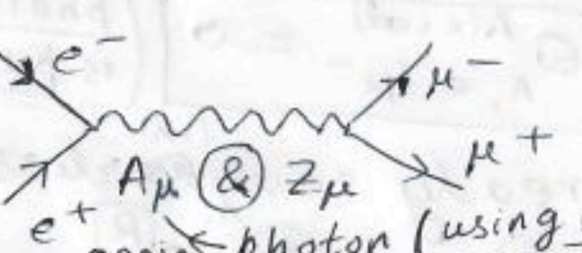
(of course, mere observation of above process is then not quite evidence for Z exchange)

$\boxed{\text{Another}}$ measurement labelled (iii) completing the process of determining g, g' separately is our next topic:

$\boxed{(2).}$ Forward-backward asymmetry in $e^+e^- \to \mu^+\mu^-$

- Neglecting Yukawa couplings ($\sim m_{e,\mu}/v$) of electron & muon (i.e., dropping H, ζ exchange), we get $e^+e^- \to \mu^+\mu^-$ from A, $\boxed{Z}$ exchange

photon



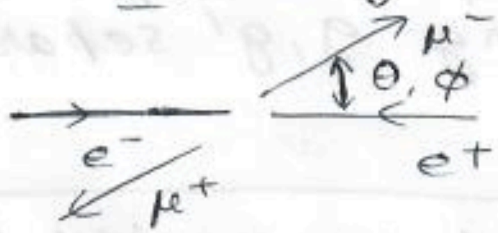
again, photon (using notation from $W_3^\mu - B^\mu$ mixing)

- So, mere observation of this process is obviously not evidence of $\boxed{Z}$ exchange (again, due to the other i.e., neutral $\boxed{EM}$ current), cf. $\nu_\mu e^- \to \nu_\mu e^-$ above

- So, we try to "construct" an $\boxed{\text{observable}}$ whose measured value "simply" being non-zero suffices for confirmation of Z exchange, namely, forward (F)-backward (B) asymmetry, defined as

$$\boxed{A_{FB} \equiv \frac{\int d\phi \left[\int_0^{\pi/2} d\cos\theta \; d\sigma/d\Omega - \int_{\pi/2}^{\pi} d\cos\theta \; d\sigma/d\Omega \right]}{\int d\phi \int_0^{\pi} d\cos\theta \; d\sigma/d\Omega}}$$

(denoted by "A" in Lahiri, Pal Eq. 15.83) where θ, ϕ are polar, azimuthal scattering angles, i.e., between directions of outgoing $\boxed{\mu^-}$ and incoming e^- (say in e^+e^- COM frame)



$\boxed{0 < \theta < \pi/2}$ is forward (F), while $\boxed{\pi/2 < \theta < \pi}$ is backward (B)

– We will show below (and you might have calculated in Phys 624) that

$$\boxed{A_{FB} = 0 \text{ in (pure) QED}} \text{ (only } \underset{\text{photon}}{A_\mu} \text{ exchange)}$$

in turn due to $Q^{(L)}_{A, e/\mu} = Q^{\boxed{R}}_{A, e/\mu}$ or ← photon (i.e., electric charge)

$Q^{V}_{A, e, \mu} = -2$, but $\boxed{Q^{A(xial)}_{A, e^-/\mu^-} = 0}$ (photon coupling is purely vectorial)

– However, as we already saw (in earlier note),

$\boxed{Q^{\boxed{A}}_{(Z), e^-/\mu^-} \neq 0}$ or $Q^{(L)}_{Z, e^-/\mu^-} \neq Q^{\boxed{R}}_{Z, e^-/\mu^-}$

(i.e., (Z) couplings are parity-violating (or have axial part)), we will find

$$\boxed{A_{FB} \neq 0 \text{ (only) when we include } \boxed{Z} \text{ exchange}}$$

i.e., an "in-your-face" verification of EW theory.

– Let's now get into $\boxed{\text{actual}}$ calculation, assuming

$\boxed{(i)}$ $\boxed{m_{e,\mu} \ll \sqrt{s}}$ (which is e^+e^- or $\mu^+\mu^-$ COM energy) so that $\boxed{\text{helicity}}$ eigenstates [i.e., left/right handed (LH/RH)], which are always energy eigenstates $\boxed{\approx}$ chirality eigenstates (denoted simply by (L, R)), which transform independently of each other under Lorentz group, hence can have different gauge interactions (again, as in case of coupling to Z)

$\boxed{\text{(ii)}}$ at the same time, $\boxed{\sqrt{s} \ll M_Z}$: note that such $\sqrt{s}$ is indeed "allowed", i.e.,

$m_e \sim 0.5$ MeV, $m_\mu \sim \boxed{100 \text{ MeV}} \ll \boxed{\sqrt{s}} \ll M_Z \sim \boxed{100 \text{ GeV}}$

Hence, we take the Z exchange contribution to be a (effectively) local 4-fermion interaction, as in process (1) above [and for β or muon decay from W exchange]

– Finally, $\boxed{\text{(iii)}}$ since $\boxed{Z_\mu}$ exchange has "γ_μ" structure like photon (of course, Z_μ exchange also has γ_μ $\boxed{\gamma_5}$, but that's a simple generalization), we will be able to "recycle" some of QED calculation for Z exchange

(For $\boxed{\text{QED}}$ calculation, see section $\boxed{9.6}$ of Lahiri, Pal and sections $\boxed{5.1, 5.2}$ of Peskin, Schroeder)

– We will use the helicity ($\approx$ chirality here) formalism:

$$\sigma(e^+e^- \to \mu^+\mu^-) = \sigma\left[(e^-)_L (e^+)_R \to (\mu^-)_L (\mu^+)_R\right]$$ (7)
$$+\ \sigma\left[(e^-)_L (e^+)_R \to (\mu^-)_R (\mu^+)_L\right]$$
$$+\ \sigma\left[(e^-)_R (e^+)_L \to (\mu^-)_L (\mu^+)_R\right]$$
$$+\ \sigma\left[(e^-)_R (e^+)_L \to (\mu^-)_R (\mu^+)_L\right]$$

i.e., there is no interference between the above $\boxed{4}$ helicity amplitudes (cross-sections simply add up), since these are energy eigenstates (different initial/final states).

- However, $\boxed{\text{why}}$ does $(e^-)_L$ annihilate only with $(e^+)_R$ and $(\mu^-)_L$ is produced with only $(\mu^+)_R$ [i.e., there is no $(e^-)_L (e^+)_L$ or $(\mu^-)_L (\mu^+)_L$] etc. ?!
- Because [as already indicated at start of EW theory (fermionic sector)] gauge interactions "connect" L to L (or R to R) due to γ_μ [or $\gamma_\mu \gamma_5$] structure of coupling to gauge field (A_μ): again L/R for this point is chirality, but that's same as helicity (due to $m_{e,\mu} \ll \sqrt{s}$), which underlies above expression for cross-section
- $\boxed{\text{Explicitly}}$, we have [taking Z coupling as example: photon is obtained simply by setting $Q^{A(xial)} = 0$]:

$$\bar{\Psi}_{(e)} \gamma_\mu \left(Q^V_{Z,e} + \gamma_5 Q^A_{Z,e}\right) \underbrace{P_L \Psi_e}_{\text{destroys } \boxed{e^-_L}}$$ $\boxed{\text{inserted}}$

$= \overline{\psi}_{(e)} \gamma_\mu \left(Q^V_{Z,e} P_L - P_L \gamma_5 Q^A_{Z,e}\right) \psi_{(e)}$ since $\gamma_5 \underbrace{P_L}_{\frac{1}{2}(1-\gamma_5)} = - P_L \gamma_5$

$= \overline{\psi}_{(e)} \boxed{P_R} \gamma_\mu \left(Q^V_{Z,e} - \gamma_5 Q^A_{Z,e}\right) \psi_{(e)}$

use $\gamma_\mu \gamma_5 = (-)\gamma_5 \gamma_\mu$

$= \left(\psi^\dagger_{(e)}\right)(\gamma_0) P_R \gamma_\mu \left(Q^V_{Z,e} - \gamma_5 Q^A_{Z,e}\right) \psi_{(e)}$

$= \psi^\dagger_{(e)} P_L \gamma_0 \gamma_\mu \left(Q^V_{Z,e} - \gamma_5 Q^A_{Z,e}\right) \psi_{(e)}$

$= \overline{\left[P_L \psi_{(e)}\right]} \gamma_\mu \left(Q^V_{Z,e} - \gamma_5 Q^A_{Z,e}\right) \psi_{(e)}$

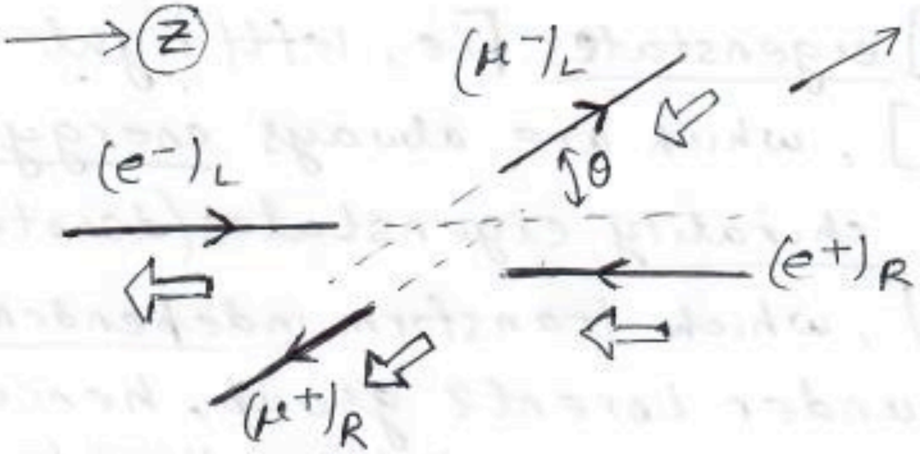
Since $P_L \psi_{(e)}$ destroys $(e^-)_L$ and creates its antiparticle [i.e., $(e^+)_{\boxed{R}}$], it is clear that $\overline{\left[P_L \psi_{(e)}\right]}$ destroys $(e^+)_{\boxed{R}}$. So, $(e^-)_L$ in initial state pairs with $(e^+)_R$ etc.

— Now, we have from $\boxed{\text{QED}}$ calculation,

$$\left|\mathcal{M}^{QED}\left[(e^-)_L (e^+)_R \to (\mu^-)_L (\mu^+)_R\right]\right|^2 = \underbrace{e^4 (1+\cos\theta)^2}_{(\theta = \pi)},$$

i.e., $\boxed{\text{vanishes}}$ in backward direction: this is expected (as follows) based on angular momentum conservation (justifying, albeit not quite proving, above angular dependence)

— Namely, orientation of $\boxed{\text{spins}}$ for this helicity amplitude looks like:
(where direction of incoming e^- is taken to be z, while that of outgoing μ^- is z')



(⇦ denotes spin of particle)

Note that e^- & e^+ (similarly μ^- & μ^+) can have an "impact parameter", i.e., non-zero orbital angular momentum [$\underline{\underline{L}}$ need not be 0], but clearly its component along direction of e^-, e^+ (or μ^-, μ^+) vanishes, i.e.,

$$J_{z \to e^- \text{ direction}} \text{ initial (total)} \overset{L_z=0}{=} S_z \text{ initial (total)} = \underbrace{-1/2}_{\text{from } (e^-)_L} + \overbrace{(-1/2)}^{\text{from } (e^+)_R} = -1$$

– Similarly, $J_{z'} \text{ final (total)} = S_{z'} \text{ final (total)}$, with z': μ^- direction,

$$\overset{L_{z'}=0}{=} \underbrace{(-1/2)}_{\text{from } (\mu^-)_L} + \underbrace{(-1/2)}_{\text{from } (\mu^-)_R} = -1$$

– So, for $\boxed{\theta = \pi}$, i.e., $z' = -z$, <u>angular momentum</u> can<u>not</u> be <u>conserved</u>, i.e., $\mathcal{M}$ vanishes, "consistent" with $\boxed{(1 + \cos\theta)^2}$ factor

– <u>Analogous</u> <u>argument</u> for: $(e^-)_L (e^+)_R \to (\mu^-)_{\boxed{R}} (\mu^+)_L$ shows that this amplitude <u>vanishes</u> in <u>forward</u> direction ($\theta = 0$): indeed explicit

" calculation shows $\boxed{(1 - \cos\theta)^2}$ factor:

$$\left|\mathcal{M}^{(QED)}\left[(e^-)_L (e^+)_R \rightarrow (\mu^-)_{\boxed{R}} (\mu^+)_L\right]\right|^2 = e^4 (1-\cos\theta)^2$$

(The other $\boxed{2}$ helicity amplitudes look similar.)

– Clearly, including Z exchange is then rather straightforward, i.e., $\boxed{\text{"}e\text{" of QED} \rightarrow g_Z Q_Z^{L,R}}$ as appropriate

(we drop "e/μ" subscript on $Q_Z^{L,R}$, since e & μ have identical coupling to Z) and

$\boxed{1/s \text{ (from photon propagator)} \longrightarrow \frac{1}{s - M_Z^2}}$ for same "reason"

– So, we get net result:

$$\left|\mathcal{M}^{(A+Z)}\left[(e^-)_L (e^+)_R \rightarrow (\mu^-)_L (\mu^+)_R\right]\right|^2 = \overbrace{(1+\cos\theta)^2}^{\text{same as QED}} \times \left\{ e^2 + \underbrace{\frac{s}{s - M_Z^2}}_{\substack{\text{ratio of} \\ \text{photon, Z} \\ \text{propagators}}} g_Z^2 \underbrace{\left[Q_Z^{L}\right]^2}_{\substack{\text{due to } (e^-)_L \\ \& (\mu^-)_L}} \right\}^2$$

$\mathcal{M}^{(A+Z)}$: photon & $\boxed{Z}$ exchange (full/EW theory)

Similarly,

$$\left|\mathcal{M}^{(A+Z)}\left[(e^-)_L (e^+)_R \rightarrow (\mu^-)_{\boxed{R}} (\mu^+)_L\right]\right|^2 = (1-\cos\theta)^2 \times \left\{ e^2 + \frac{s}{s - M_Z^2} g_Z^2 \underbrace{Q_Z^{L}}_{\text{due to } (e^-)_L} \underbrace{Q_Z^{\boxed{R}}}_{\text{due to } (\mu^-)_{\boxed{R}}} \right\}^2$$

- Turning the crank, we get

$$\frac{\text{total } |\mathcal{M}|^2 \text{ (sum over final spins) for } (A+Z) \text{ exchange}}{4 \text{ (average over initial spins)}} = \boxed{e^4\left[(1+\cos^2\theta)(1+a_1) + a_2 \cos\theta\right]},$$

where $a_{1,2}$ come from Z-exchange (i.e., QED result corresponds to $a_{1,2} \to 0$):

$$a_2 = 4 f_Z^{\boxed{1}} (Q_Z^A)^2 + 8 f_Z^{2} (Q_Z^V)^2 (Q_Z^A)^2 \;\&$$

$$a_1 = 2 f_Z^{\boxed{1}} (Q_Z^V)^2 + f_Z^{2} \left[(Q_Z^A)^2 + (Q_Z^V)^2\right]^2, \text{ with}$$

$\boxed{1^{st}}$ terms in $a_{1,2}$ come from photon-Z interference, while 2^{nd} terms are purely Z-exchange. Also,

$$\boxed{f_Z \text{ (as in Lahiri, Pal)} = \underbrace{\frac{s}{s - M_Z^2}}_{\substack{\text{ratio of}\\ \text{photon, } Z\\ \text{propagators}}} \times \underbrace{\frac{(g_Z/2)^2}{e^2}}_{\substack{\text{ratio of } Z\text{, photon}\\ \text{(gauge) couplings}}} = \frac{s}{s - M_Z^2}\,\frac{1}{\sin^2 2\theta_W}}$$

(using $g_Z = g/\cos\theta_W$, $e = g\sin\theta_W$)

- So, $\boxed{A_{FB} \neq 0}$ originates from $\boxed{a_2}$; it is easy to show that $\boxed{A_{FB} = \frac{3}{8}\frac{a_2}{1+a_1}}$

i.e., $\boxed{(\cos\theta)^1}$ piece, which needs $\boxed{Z}$ exchange

($A_{FB} = 0$ in QED) [explicitly, $\cos\theta \begin{cases} > 0 : & 0 \le \theta \le \pi/2 \text{ (F)} \\ < 0 : & \pi/2 \le \theta \le \pi \text{ (B)} \end{cases}$]

- Also, $\boxed{a_2 \propto \left[Q_Z^A\right]^2}$, i.e., it's $\boxed{\text{axial}}$ coupling to $\boxed{Z}$ that gives $A_{FB} \neq 0$

(again, photon coupling is purely vectorial)

- Once again, above two (and other similar) effects of Z exchange are obtained even at $\boxed{\text{energies} \ll M_Z}$, even if suppressed by $\sim \sqrt{s}/M_Z$ (a numerical example of A_{FB} is in HW 9.2 combined; i.e.. (i), (ii) & (iii))
- Recall that these neutral current measurements pinned down (along with charged current weak interaction and EM data) $\boxed{g, g', v}$ separately.
- Thus, $\boxed{M_{W,Z} \sim g, g_Z\, v}$ could be predicted (again, in $\boxed{1970\text{'s}}$), based on which $\boxed{\text{UA1/UA2}}$ experiments at CERN targetted W, Z direct production/discovery via

$\boxed{p\bar{p}}$, i.e., $\underset{\text{from } p}{u \,(\text{or } d)} + \underset{\text{from } \bar{p}}{\bar{d}\,(\text{or } \bar{u})} \longrightarrow W^{\pm}$ (check that charges match)

(similarly, $u\bar{u}$ or $d\bar{d} \to Z$)

followed by $W^- \to e^-/\mu^- + \bar{\nu}_e/\bar{\nu}_\mu$ (or $Z \to e^- e^+$), succeeding in $\boxed{1980\text{'s}}$ [obviously by going to COM energies $\boxed{\text{above}}$ $M_{W,Z}$]

- Then, in $\boxed{\text{early}}\,\boxed{1990\text{'s}}$, $\boxed{\text{LEP1}}$ experiment at CERN produced Z $\boxed{\text{resonance}}$ in $\boxed{e^+e^-}$ collisions $e^+e^- \to \boxed{Z} \to \boxed{f\bar{f}}$ (where f = all SM fermions), doing $\boxed{\text{precision}}$

measurements of Z mass, couplings

— So, $\boxed{\text{LEP 1}}$ operated at $\boxed{\sqrt{s} \simeq M_Z^2}$, but then the $\boxed{Z}$ propagator used in above (lower energy) processes, i.e., $\boxed{\frac{1}{s - M_Z^2}}$, is $\boxed{\text{singular}}$/ ill-defined...

(as follows)

... what saves the day is inclusion of $\boxed{Z}$ boson decay width (Γ_Z), i.e., Z boson is unstable

— The $\boxed{\text{next}}$ calculation we will undertake is that of $\boxed{\Gamma_Z}$, but before we do that, let us see $\boxed{\text{how to include } \Gamma_Z \text{ in propagator}}$ (this is a rather heuristic discussion, following section 15.6 of Lahiri, Pal : a more precise treatment might have been given in Phys 624, based on, for example, Peskin, Schroeder's pages 101 and 236-237

— The rough idea/intuition is that a decaying particle with $\boxed{\text{rest}}$ frame decay width $\boxed{\Gamma}$ (thought of as — per unit time — a probability) looks like a suitably modified plane wave, i.e.,

$$\underbrace{\phi(x)}_{\substack{\text{scalar}\\\text{particle}}} \sim \exp\Big(\underbrace{-iEt + i\bar{p}\cdot\bar{x}}_{\substack{\text{for usual, "stable"}\\\text{plane wave}}} - \frac{m}{2E}\Gamma t\Big)$$

so that we get the usual exponential decay in $\boxed{\text{time}}$, i.e.,

$$(\text{probability})\ |\phi(x)|^2 \sim \exp\Big(-\underbrace{\frac{m}{E}}_{\boxed{\text{time dilation}} \text{ factor in general reference frame}} \underbrace{\Gamma}_{\text{probability per unit time for decay (in rest frame) (i.e., inverse of lifetime)}} t\Big)$$

— Now, we can "hand-wave" that for a proper particle interpretation of quanta of field, we need $\boxed{\Gamma \ll m}$ (which is satisfied for Z: see in a bit)

[In detail, if we wish to assign a unique/well-defined mass, i.e., energy in rest frame, to a particle, then we have to require that the uncertainty in its $\boxed{\text{energy}}$ measurement, $\boxed{\Delta E \ll m}$. However, using the usual uncertainty relation, $\boxed{\Delta t}$ (time taken for achieving such small error in E) $\boxed{\gg 1/m}$: in turn, (this better be smaller than lifetime of particle [i.e., we need to measure accurately particles energy (= mass) before it decays], i.e., $\boxed{\Delta t \ll 1/\Gamma}$. Combining the 2 bounds on Δt, we get $\Gamma \ll m$.]

- Back to $\boxed{\text{modified}}$ plane wave: the $\boxed{\text{EOM}}$ satisfied by it is (neglecting Γ^2/m^2 terms):

$$\Big(\underbrace{\Box + m^2}_{\text{usual Klein-Gordon piece}} - \overbrace{i m \Gamma}^{\text{new}}\Big)\phi(x) = 0$$

Fourier transforming which gives $\boxed{\text{propagator}}$

$$= \frac{1}{p^2 - m^2 + \boxed{i m \Gamma}\ (\text{extra})}$$

- Similarly for gauge boson/fermion.
- For $\boxed{\text{both}}$ $p^2 \gg m^2$ (as in last 2 processes/we will study), i.e., high-energy, and $p^2 \ll m^2$ (as in low energy processes discussed above), we can drop "$m\Gamma$" in propagator
- Whereas, for $\boxed{p^2 \approx m^2}$ ($\sqrt{s} \approx M_Z^2$ for $e^+e^- \to Z$), we must keep $\boxed{m\Gamma}$ so as to make propagator finite, but still (much) larger than $\underset{M_Z^2}{1/s}$, i.e., $\frac{1}{M_Z \Gamma_Z} \sim \frac{1}{M_Z^2} \frac{1}{\frac{\Gamma_Z}{M_Z} (\ll 1)}$

So, $\boxed{f_Z}$ $\underset{\text{used in } A_{FB}}{\sim}$ $s/(s^2 - M_Z^2) \longrightarrow \frac{s \simeq M_Z^2}{M_Z \Gamma_Z} \sim \frac{M_Z}{\Gamma_Z}\ \boxed{\gg 1}$

- Hence, at $\boxed{\text{LEP(I)}}$ (again $\sqrt{s} \simeq M_Z^2$), we can actually $\boxed{\text{neglect}}$ photon exchange, i.e., keep

Z exchange only to find

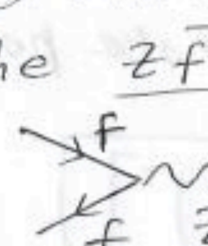
$$A_{FB} \ \underline{\text{at}} \ \boxed{Z\text{-pole}} \simeq \boxed{\frac{3\left(Q_Z^V\right)^2\left(Q_Z^A\right)^2}{\left[\left(Q_Z^V\right)^2+\left(Q_Z^A\right)^2\right]^2}}$$

(Recall, Q_Z^V for $e^-, \mu^- = -1/2 - 2\sin^2\theta_W$ &
$Q_Z^A = -1/2$)

Onto calculation of Γ_Z (numerical example in HW 9.3)

[3]. Decay width of Z boson

- Obviously, W/Z being massive can decay into matter particles (cf. photon is stable)
- Only decay modes of Z (similary for W) are into pair of SM fermion-anti fermion $(f\bar{f})$, since EW gauge couplings: W^+W^-Z & W^+W^-A ← photon (there is no purely Z, photon coupling) can't give W, Z decay (recall $2M_W > M_Z > M_W$). ← see HW 10.1
  Similarly, W^+W^-H & ZZH can give (heavy) Higgs decay only (we will see later that there is no ZHH coupling)
- The Zff Feynman rule given earlier, i.e.,



$$: (-)i\frac{g_Z}{2}\left(Q_{Z,f}^V\gamma_\mu - Q_{Z,f}^A\gamma_\mu\gamma_5\right)$$

new vs. $\phi \to f\bar{f}$

gives

$$\mathcal{M}(Z \to f\bar{f}) = -i \frac{g_Z}{2} \underbrace{\bar{u}(p)}_{f \text{ created}} \gamma^\mu \left(Q^V_{Z,f} - \gamma_5 Q^A_{Z,f}\right) \overbrace{v(p')}^{\bar{f} \text{ created}} \varepsilon_\mu(k)$$

polarization vector of decaying $\boxed{Z}$ [new as compared to $\phi \to f\bar{f}$ as in Lahiri, Pal section 7.2.1, page 117 onwards] → scalar

— We sum over fermion spins (as in $\phi \to f\bar{f}$) and average over polarizations of Z (for decaying Z being unpolarized), giving

$$|\mathcal{M}(Z \to f\bar{f})|^2 = \frac{g_Z^2}{4} \times \underbrace{\frac{\sum_r \varepsilon^r_\mu(k)\, \varepsilon^{r*}_\nu(k)}{}}_{\text{sum over polarizations of } Z}$$

average → 3 (2 transverse + 1 longitudinal)

sum over fermion spins

$$\times \, \mathrm{Tr}\left[\not{p}\, \gamma^\mu (Q^V_Z - Q^A_Z \gamma_5)\, \not{p}'\, \gamma^\nu (Q^V_Z - Q^A_Z \gamma_5) \right]$$

(Trace over 4 γ-matrices vs. 2 for $\phi \to f\bar{f}$)

— For polarization sum, we use (from SSB discussion):

$$\sum_r \varepsilon^r_\mu(k)\, \varepsilon^{r*}_\nu(k) = \left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{M_Z^2} \right)$$

— Turning the crank (see separate note posted), we get (for each f):

$$\boxed{\Gamma(Z \to f\bar{f}) = g_Z^2/(48\pi)\, M_Z \left[(Q^A_Z)^2 + (Q^V_Z)^2 \right]}$$

- To get total Γ_Z, we <u>sum</u> over $\boxed{f}$ appropriately, <u>e.g.</u>, including neutrinos, color factor for quarks, 3 generations (but dropping <u>too</u> heavy f): we indeed find $\boxed{\Gamma_Z \ll M_Z}$ (mainly due to $g_Z < 1$)
- Again, $\boxed{\text{precision}}$ measurements of Γ_Z, A_{FB} etc. (thus $Q^{V,A}_{Z,f}$) were performed at <u>LEP1</u>, verifying SM predictions...

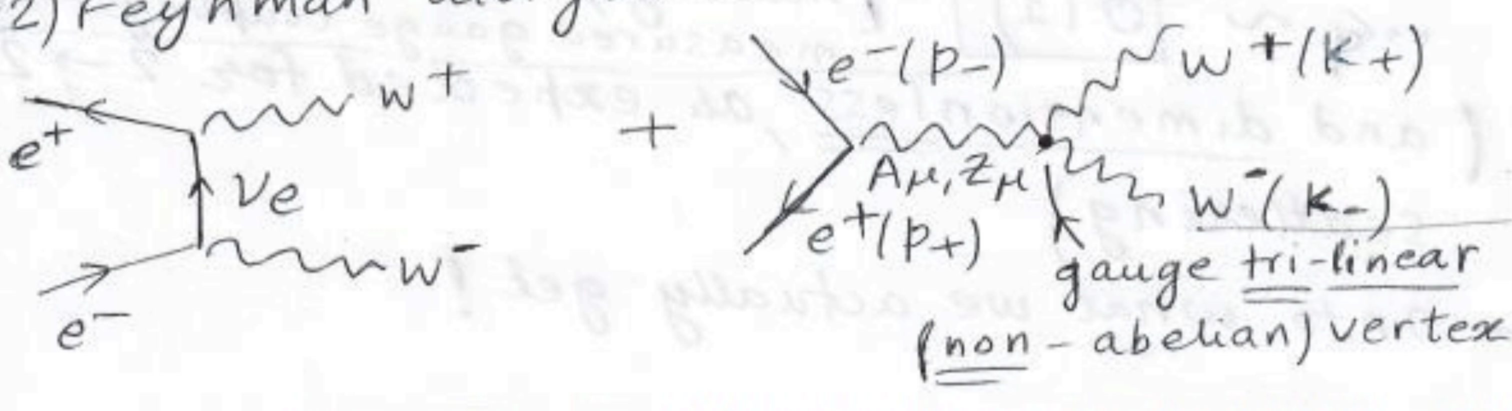
... onto <u>even</u> higher energies, i.e.,

$\boxed{(4).}$ $\boxed{e^+ e^- \longrightarrow W^+ W^-}$ at <u>LEP2</u> (CERN)

for $\sqrt{s} \gtrsim 2 M_W \simeq \underline{180}$ GeV (vs. $\sqrt{s} = M_Z \simeq \underline{90}$ GeV at LEP1) in <u>late</u> <u>1990's</u>, <u>early</u> <u>2000's</u>

(obviously, $\sqrt{s} \gg m_e$ so that helicity/energy eigenstates of e^- are also chirality eigenstates, i.e., $e^-_{L,R}$)

- Neglecting Yukawa coupling of e^- ($\sim \frac{m_e}{v} \ll 1$), i.e., H, φ exchange, we get the following (1+2) Feynman diagrams:



- We will $\boxed{\text{simplify}}$ the calculation by assuming

$\boxed{(i)\ e^- \text{ is only RH}}$

so that we neglect $\boxed{\nu_e}$ exchange diagram, since only e^-_L couples to W, and e^+ being annihilated is then LH (in photon + Z exchange), like discussed for A_{FB} calculation

- Before calculating (photon + Z) exchange diagrams, let us see what we (naively) expect:

$$\boxed{\mathcal{M}\left[e^+_L e^-_R \to W^+W^-\right]} \sim (\text{gauge coupling})^2 \times \underbrace{(\varepsilon_+ \varepsilon_-)}_{\text{polarization vectors for } W^+, W^-}$$

$$\times \left(1/s\right) (\text{photon, Z propagator})$$

$$\times \left[2 \text{ spinors } (e^+, e^-), \text{ net } \sim (\sqrt{\text{energy}})^2 \sim \sqrt{s} \right]$$

$$\times \left(E \sim \sqrt{s}\right) \text{ from derivative at gauge } \boxed{\text{tri-linear}} \text{ vertex}$$

$$\sim \boxed{(\text{gauge couplings})^2 \times (\varepsilon_+ \varepsilon_-)}, \text{ i.e.,}$$

$E \sim \sqrt{s}$ factors (roughly) $\boxed{\text{cancel}}$

$\ldots \sim \boxed{O(1)}$ [naively, $\varepsilon_\pm \sim O(1)$ and measured gauge couplings $\sim O(1)$]

(and dimensionless, as expected for $2 \to 2$ scattering)

- Onto what we actually get!

$i\mathcal{M}[(e^+)_L(e^-)_R \to W^+W^-$ from photon + Z exchange$]$

$$= \Big[\underbrace{\bar{v}(p_+)}_{(e^+)\text{ destroyed}}\ \gamma_\rho \ \overset{\text{restrict to }(e^-)_R}{P_R}\ \underbrace{u(p_-)}_{e^-\text{ destroyed}}\Big] \times \underbrace{\left(-i g^{\lambda\rho}\right)}_{\text{from gauge boson propagator in 't Hooft-Feynman gauge }(\xi=1)}$$

$$\times \underbrace{V_{\mu\nu\lambda}}_{\text{from tri-linear vertex (see below)}} \times \underbrace{\varepsilon^{\mu}_{-}(k_-)\ \varepsilon^{\nu}_{+}(k_+)}_{W^\pm\text{ polarization vectors}}$$

$$\times \Big[\underbrace{ie}_{e^+e^-\text{ photon vertex}} \times \underbrace{ie}_{\text{photon-}W^+W^-\text{ vertex (see below)}} \times \underbrace{\frac{1}{s}}_{\text{from photon propagator}}$$

$$+ \underbrace{(-i g_Z)(+\sin^2\theta_W)}_{(e^+)_L\, e^-_R - Z\text{ vertex}} \times \underbrace{(i g\cos\theta_W)}_{Z W^+W^-\text{ vertex (see below)}} \times \overbrace{\frac{1}{s - M_Z^2}}^{\text{from }Z\text{ propagator}}\Big]$$

- again, $\boxed{R}$H e^- destroyed implies $(e^+)_L$ also destroyed (as per discussion for A_{FB}) ← more on this in HW 9.4
- Explanation of tri-linear gauge vertex factors: from the discussion/Feynman rule of tri-linear gauge coupling done for non-abelian gauge theories $\boxed{\text{in general}}$, we get $\boxed{\text{tensor}}$ structure:

$$\boxed{V_{\mu\nu\lambda}(k_-, k_+) = (k_- - k_+)_\lambda g_{\mu\nu} + (2k_+ + k_-)_\mu g_{\nu\lambda} - (2k_- + k_+)_\nu g_{\lambda\mu}};$$

where momentum conservation has been used to obtain photon, Z momentum in terms of those of $W^\pm$, i.e., $k_\pm$

— Also, the $\boxed{\text{gauge coupling}}$ starts out being $\sim g \boxed{W_1 W_2 W_3}$ [i.e., $\propto \varepsilon_{abc}$, the structure constants of $SU(2)$] : we first re-write $W_{1,2}$ in terms of $W^\pm$ to get $\sim i g \, \underline{W^+ W^- W_3}$; then $\boxed{W_3^\mu} = Z^\mu \cos\theta_W + \underbrace{A^\mu}_{\text{photon}} \sin\theta_W$ to find (upto any overall sign, which is included in $V_{\mu\nu\lambda}$ above) $ig \sin\theta_W = \boxed{ie}$ for photon-W^+W^- vertex vs. $\boxed{ig \cos\theta_W}$ for $Z W^+ W^-$ vertex (as used above)

— Remarkably / Somewhat surprisingly (although, in hindsight, as expected: see below), the gauge coupling factors inside last [...] in photon vs. $\boxed{Z}$ exchange end up being equal in magnitude, but opposite in sign, i.e., $\boxed{-e^2}$ for photon exchange vs.

$+ (g_Z \sin^2\theta_W)(g \cos\theta_W) = \boxed{+e^2}$ (using $g_Z = \frac{g}{\cos\theta_W}$ and $e = g \sin\theta_W$) for $\underline{Z}$-exchange

— Of course, photon & Z propagators are not

quite equal, unless we go to $\sqrt{s} \gg M_Z$
(we will actually take that limit in a bit)

— Anyway, for general $\sqrt{s}$, we then get

$$i\mathcal{M}^{(A+Z)}\left[(e^-)_R (e^+)_L \to W^+ W^-\right] = e^2\left[\bar{v}(p_+)\gamma^\lambda P_R u(p^-)\right] \times V_{\mu\nu\lambda}\, \varepsilon_-^{\mu}\, \varepsilon_+^{\nu} \underbrace{\left(\frac{1}{s} - \frac{1}{s - M_Z^2}\right)}_{= \frac{-M_Z^2}{s(s-M_Z^2)} \,\simeq\, -M_Z^2/s^2 \text{ for } \sqrt{s} \gg M_Z}$$

— Recall that $V_{\mu\nu\lambda} \sim E$ of $W^\pm$ $\sim \sqrt{s}$, which $\boxed{\text{each}}$ of u, v spinors (for $e^\pm$) $\sim \sqrt{E} \sim (\sqrt{s})^{1/2}$

so that in the <u>high-energy</u> limit of $\boxed{\sqrt{s} \gg M_Z}$, we get

$$\mathcal{M}^{(A+Z)}\left[(e^-)_R (e^+)_L \to W^+ W^-\right] \sim \left(e^2\right)\left(\varepsilon_+ \varepsilon_-\right) \underbrace{\left(\frac{M_Z^2}{s}\right)}_{\text{"extra" factor vs. naive estimate of before}}$$

$\boxed{\text{From now on}}$, we will make this ($2^{nd}$) <u>simplification</u>, i.e., (ii) $\boxed{\sqrt{s} \gg M_Z}$: we have 2 cases

$\boxed{(a)}$ $\boxed{\text{Transverse}}$ W^+W^-, for which $\boxed{\varepsilon^+, \varepsilon^- \sim O(1)}$

so that above $\boxed{\mathcal{M}} \ll 1$ ($\to 0$) in the <u>high energy limit</u>?!

— Indeed, this result was expected as follows. Roughly speaking, for $\boxed{\sqrt{s} \gg M_W, M_Z}$, we can "neglect" EWSB, i.e., $\boxed{\text{think}}$ (back) in terms of massless $W^{\pm}$ and $\boxed{W_3}$, B (and not their combinations photon & $\boxed{Z}$)

— However, only W_3 (not B) couples to $\boxed{W^+ W^-}$ in final state [via SU(2) tri-linear] vertex, while $\boxed{W_3}$ does not couple to $(e^-)_R$ [$SU(2)_L$ singlet] in initial state. in initial state

— Similarly, $\boxed{B}$ does couple to $(e^-)_R$ in initial state [again, both L, R e^- couple to B, unlike to W's], but (again) not to $\boxed{W^{\pm}}$ in final state

— And, $\boxed{W^{\pm}}$ being discussed here have only transverse polarizations in this massless limit (again, in hindsight, which is always 20-20!)

— Hence, it is clear from above argument why $\mathcal{M}^{(A+Z)}\left[(e^-)_R (e^+)_L \to W^+_T W^-_T\right] \to 0$ for $\sqrt{s} \gg M_{W,Z}$ (again, there is simply "no diagram") ← $\boxed{\text{transverse}}$ specifically (see below)

— At the same time, in this high energy limit (again, gauge couplings are finite, but we neglect v, compared to $\sqrt{s}$), would-be

NGB's, i.e., $\boxed{\zeta \text{ and } \phi_\pm}$, which are taken to be unphysical usually, sort of become physical, i.e., they are $\boxed{\text{actually}}$ NGB's. 24

- And, $\boxed{B_\mu}$ couples both to $\boxed{(e^-)_R}$ in initial state and $\boxed{\phi_\pm}$ in final state (in an unsuppressed manner: again both $(e^-)_R$ and Φ have $Y \neq 0$), so

$$\boxed{(e^-)_R (e^+)_L \rightarrow \phi_+ \phi_-}$$ goes through,

which in unitary gauge is

$$(e^-)_R (e^+)_L \longrightarrow W^+_{\boxed{L}} \, W^-_{\boxed{L}}$$

$\boxed{\text{longitudinal}}$ polarization

(again, $\phi_\pm$, ζ are eaten by $W^\pm_\mu$, Z_μ becoming their extra longitudinal polarizations)

- So, we $\boxed{\text{expect}}$ $\mathcal{M}^{A+Z}\left[(e^-)_R (e^+)_L \rightarrow W^+_{\boxed{L}} W^-_{\boxed{L}}\right]$ to be not suppressed, i.e., have going $\boxed{O(1)}$ rate (even though W_{T} amplitude vanishes)

- Indeed, above explicit calculation confirms this prediction: $\boxed{\varepsilon^\mu_{\boxed{L}}(k)} \simeq k^\mu / M_W \boxed{\gg 1}$ in high-energy limit [cf. $\varepsilon^\mu_{T}(k) \sim O(1)$], so we can show [again, this is case (b): $\boxed{\text{longitudinal}}$ $W^+ W^-$]

$$V_{\mu\nu\lambda}\, \varepsilon_L^\mu \varepsilon_L^\nu \simeq \frac{s}{2M_W^2}(k_+ - k_-)_\lambda \text{ (in COM frame)}$$

i.e., $\boxed{\varepsilon_L^\mu \varepsilon_L^\nu}$ factors precisely "cancel" suppression of $\sim M_Z^2/s$ seen earlier (causing amplitude for $W^{\pm}_{T}$ to vanish for $\sqrt{s} \gg M_Z$) to give

$$\mathcal{M}^{(A+Z)}\left[(e^-)_R (e^+)_R \to W^+_{\boxed{L}} W^-_{\boxed{L}}\right]$$

$$= \frac{-e^2}{2\, s \cos^2\theta_W} \left[\bar{v}(p_+) \gamma^\lambda P_R u(p^-)\right](k_+ - k_-)_\lambda$$

$\sim O(1)$ rather $O\left[(\text{gauge coupling})^2\right]$

[again, using each of v, u spinors $\sim \sqrt{E} \sim (\sqrt{s})^{1/2}$ and momentum factor from tri-linear gauge vertex $\sim \sqrt{s}$]

[HW 9.5 lets you work more on this amplitude: checking precisely equivalence theorem & A_μ, Z_μ vs. $W^3{}_\mu, B^\mu$ exchanges]

– Finally, we consider currently operating highest energies, i.e.,

(5). Higgs boson production & decay

– Higgs production can occur at $\boxed{e^+e^-}$ collider experiments: naively, we might expect pair production, i.e.,

$$e^+e^- \to \text{virtual } Z \to H\,H$$

since/after all, H seems to have $Q_Z^{(H)}$ (Higgs)

$= T_3 \text{ (of H)} - (Q \text{ of } H = 0) \sin^2\theta_W = -\tfrac{1}{2}$

(kind of like ν)

– However, there is $\boxed{\text{no}}$ Z(HH) vertex:

as can be explicitly shown starting from EW sector Lagrangian [HW 10.3(i)] and argued on general grounds, based on Bose-Einstein statistics and angular momentum conservation [HW 10.3(ii)]

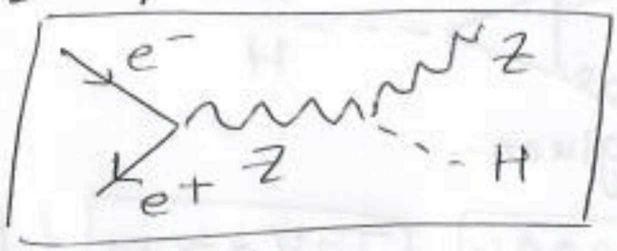
— there $\boxed{\text{is}}$ however $H\,ZZ$ (and $H\,W^+W^-$) coupling from

$$(D_\mu \Phi)^\dagger (D^\mu \Phi) \sim \left[\text{pick } Z_\mu \text{ (or } W^\pm\text{) from } \underline{\text{both}}\ D_\mu\text{'s}\right]$$

$$g_Z^2\, Z_\mu (v+H)\, Z^\mu (v+H)$$

$$\sim g_Z^2\, Z_\mu Z^\mu v H \sim g_Z M_Z Z_\mu Z^\mu H$$

which leads to



i.e., $\boxed{\text{"Higgs" strahlung}}$, ala radiating photon (bremstrahlung)

— Indeed, in early $\boxed{2000\text{'s}}$ $\boxed{\text{LEP2}}$ (e^+e^- collider) experiments operating at $\simeq 200$ GeV CoM energy did $\boxed{\text{not}}$ find $\boxed{\text{Higgs}}$ boson, setting a lower limit : $\boxed{M_H \gtrsim 110 \text{ GeV}}$ (just based on kinematics, i.e., $M_H + M_Z \gtrsim 200$ GeV)
$M_Z \simeq 90$ GeV

[Again, LEP2 did produce $\boxed{W, Z}$ pairs : latter from

, since 200 GeV (CoM energy) $\gtrsim 2\, M_{W,Z}$]

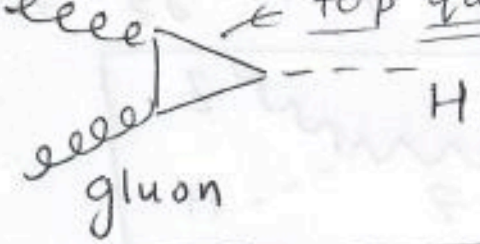
- Higgs boson can also be produced at $\boxed{\text{hadron}}$ colliders:

  pp or $\boxed{p\bar{p}}$ gives $u\bar{u}$ or $d\bar{d} \to$ (virtual) $\boxed{Z}$
  $\to \boxed{Z}H$ (like in $\boxed{e^+e^-}$ collisions)

  and $u\bar{d}$ (or $\bar{u}d$) $\to$ (virtual) W
  $\to WH$ (absent at $\boxed{e^+e^-}$ colliders)

  at tree-level, while at loop-level, we can have gluon fusion:

  ← top quark loop dominates due to largest Yukawa coupling



- In $\boxed{2000\text{'s}}$, $\boxed{\text{TeVatron}}$ ($p\bar{p}$) collider at Fermilab searched for Higgs boson, mostly via Higgsstrahlung, finding some evidence for it, but $\boxed{\text{not}}$ resulting in discovery

- Of course, $\boxed{\text{LHC}}$ in $\boxed{2010\text{'s}}$ discovered (to begin with) Higgs boson at $\boxed{125 \text{ GeV}}$ mass via $\boxed{\text{gluon}}$ fusion instead

  [because LHC is $\boxed{pp}$ collider, probability for Higgsstrahlung is relatively suppressed -vs.- $p\bar{p}$ collisions at TeVatron - by having to extract $\bar{u}$ or $\bar{d}$ out of p (vs. $\bar{p}$ at TeVatron)]

- Onto Higgs boson $\boxed{\text{decay}}$: we can "invert" production processes (Higgsstrahlung & gluon fusion) to get

$$\boxed{H \to W^+W^-, \; ZZ, \; gg \text{ and } \underbrace{2 \text{ photons}}_{\text{replacing gluons in loop diagram by photons}}}$$

see HW 10.5

- In fact, $H \to$ di-photon was the first discovery mode for Higgs boson

- In addition, we have $\boxed{H \to \bar{f} f}$ via Yukawa coupling ($\sim m_f/v$) (like $\phi \to e^+e^-$ studied in Lahiri, Pal, section 7.21 on page 117 onwards) : see HW 10.4 also

- As an aside, another process studied at these high energy $\boxed{\text{hadron}}$ collider experiments (Tevatron first, then LHC) has been (direct) production & decay of the $\boxed{\text{top}}$ quark (t), the heaviest particle of SM ($\simeq 175$ GeV) : $p\bar{p}$ or pp gives $q\bar{q}$ or $gg \to t\bar{t}$ (many diagrams with virtual gluon / top quark exchange), followed by $t \to bW$ (decay) : see HW 10.2 for latter calculation, including application of equivalence theorem

- $\boxed{\text{Future}}$ e^+e^- colliders operating at $\sqrt{s} \simeq 250$ GeV can also produce (Z+) Higgs boson, since $\sqrt{s} > (m_H + m_Z)$ (LEP2 didn't satisfy it)