Phenomenology of EW theory/sector of SM : part (II) (flavor : observables)

— we saw in the theory discussion of flavor that $\boxed{\text{charged}}$ current (weak) interactions of $\boxed{\text{quarks}}$ (but not for leptons) in the SM can change /mix generation, i.e., $W^{\pm}_{\mu}$ and $\phi_{\pm}$ [(would-be) NGB couplings.

— Whereas neutral current interactions [i.e., involving photon, Z, gluon, H (physical Higgs boson) & G: (would-be) NGB] are generation-preserving $\boxed{\text{both}}$ for quarks & leptons

— Since couplings to $\boxed{\phi_{\pm}}$ are small ($\sim$ Yukawa strength $\sim m_q/v$) for most quarks (except top), we will neglect that source of generation-number violation, $\boxed{\text{focussing}}$ instead on $\boxed{W^{\pm}}$ couplings, i.e.,

$$\mathcal{L}_{\text{mixing}} = -\frac{g}{\sqrt{2}} \overline{(u_L)}_i^{\text{mass}} \not{W}^{+} (V_{CKM})_{ij} (d_L)_j^{\text{mass}} \quad (+\text{ h.c.})$$

creates u_L of "i^{th}" generation; destroys W^+; destroys d_L of "j^{th}" generation

$$= -\frac{g}{\sqrt{2}} \times (+\text{ h.c}) \; \boxed{\bar{u}_L \not{W}^{+} V_{CKM} d_L}$$

(in compact notation and dropping "mass" superscript from here on, i.e., when we say charm quark, it is mass eigenstate of 2^{nd} generation etc.)

Outline of flavor/generation mixing observables

(1) Tree-level charged current, changing flavor/generation by 1 unit: for example $\boxed{\Delta S}$ (strange quark number) $\boxed{=1}$

$$\boxed{\underbrace{K^+}_{\bar{s}u} \rightarrow \underbrace{\pi^+}_{\bar{d}u} \underbrace{\pi^0}_{u\bar{u}}, \pi^0 e^+ \nu}$$

(2) Loop-level $\boxed{\text{neutral}}$ current, changing generation/flavor by $\boxed{2}$ units [featuring extra suppression due to difference of quark masses inside loop]: for example, $\boxed{\Delta S = 2}$

$$\boxed{\underbrace{K^0}_{\bar{s}d} \longleftrightarrow \underbrace{\bar{K}^0}_{s\bar{d}}} \text{ mixing/oscillation}$$

use off-diagonal $\boxed{V_{CKM}}$, with $\boxed{2}$ generations

(3) $\boxed{\text{CP}}$ violation from $\boxed{\text{interference}}$ of tree and loop-level: for example, $\boxed{\Delta B}$ (bottom quark number) $(= \Delta S) =$ 1 (charged tree and neutral/loop current flavor/generation change by 1 unit) $\boxed{\underbrace{B^-}_{b\bar{u}} \rightarrow \underbrace{K^-}_{s\bar{u}} \pi^0}$

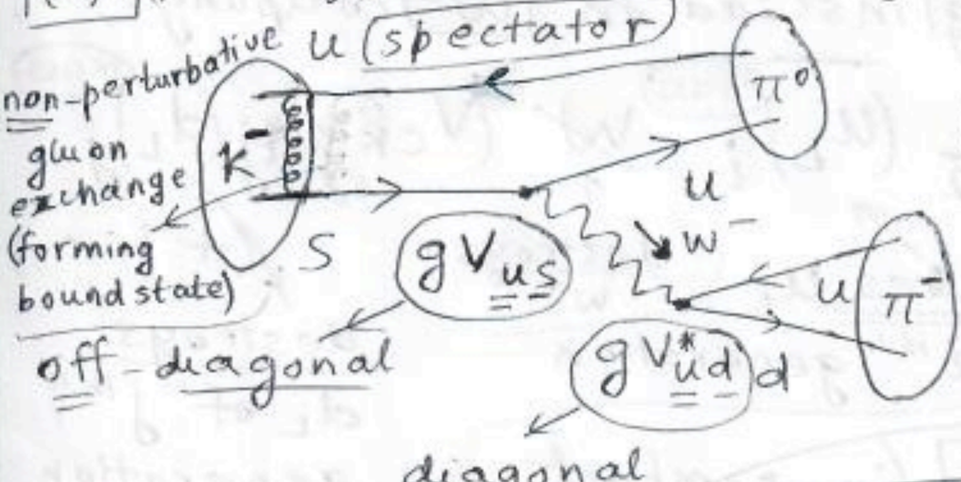
uses $\boxed{\text{complex}}$ (& off-diagonal) V_{CKM} from $\boxed{3}$ generations [and strong/QCD interaction, $\boxed{\text{CP}}$-preserving phase

— Onto details of $\boxed{\text{observables}}$, starting with $\boxed{1^{st}}$ and $\boxed{2^{nd}}$ (lightest two) generations

(1). $\boxed{\text{Tree-level}}$ flavor-changing (FC) charged currents/processes (again, "flavor" is just type of mass eigenstate quark): basic idea here is to use above $W_\mu^\pm$ coupling (containing off-diagonal — in generation space — piece) to "convert" 1 flavor into another (especially of different generation), e.g., changing "strangeness" quantum number by 1 unit ($\Delta S = 1$) in $\boxed{\text{Kaon}}$ (bound state/meson made of 1 strange quark & 1 lighter quark) decays:

$\boxed{(i).}$ (purely) $\boxed{\text{hadronic}}$ decay: $\boxed{K^-(s\bar{u}) \to \pi^0 (u\bar{u}\ldots) + \pi^-(\bar{u}d)}$

(quark) content of K^-



(we drop "CKM" subscript on V_{CKM} for brevity)

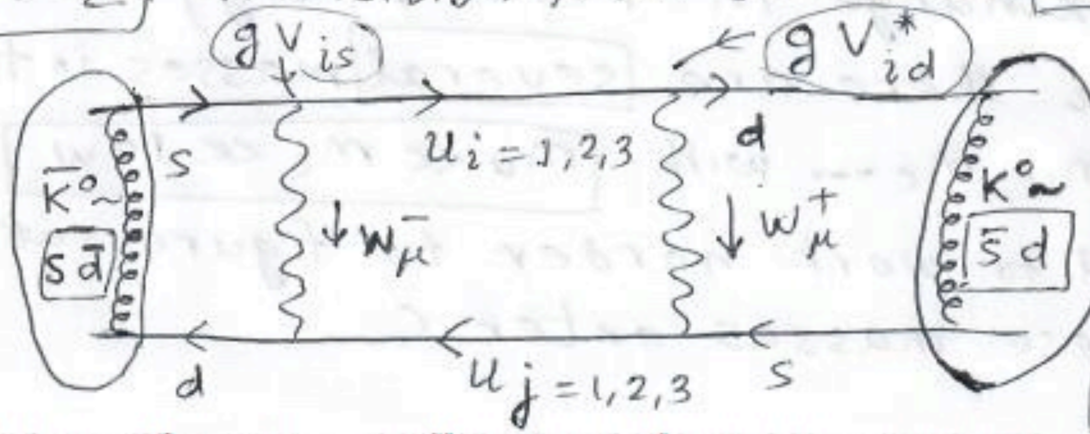
$\boxed{(ii).}$ semi-leptonic decay: $\boxed{K^- \to \pi^0 e^- \bar{\nu}_e}$

Again,
- Both above processes involve $\boxed{s}$ (2nd generation) $\longrightarrow$ $\boxed{u}$ (1st generation) transition via $W^{\pm}_{\mu}$ coupling, i.e., 2 quarks of different charge, (general case for $W^{\pm}_{\mu}$ exchange) and different $\boxed{\text{generation}}$ (due to off-diagonal $\boxed{V_{CKM}}$) mixing
- We can measure $\boxed{V_{us}}$ (called Cabibbo angle or $\boxed{\lambda_c}$) using them to be $\boxed{\simeq 0.22}$
- Note, at tree-level, we do not have flavor changing neutral currents (FCNC), i.e, $\boxed{\text{no}}$ $s \rightarrow \boxed{d}$ etc.

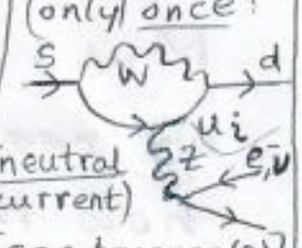
(2). Loop-level FCNC

- The idea is to use charged current flavor change $\boxed{\text{twice}}$: $\boxed{s} \underset{(+W^-_\mu)}{\longrightarrow} u \underset{(+W^+_\mu)}{\longrightarrow} \boxed{d}$, so effectively get neutral current flavor change...

... but then need to "absorb" extra W^+_μ, W^μ_- by doing it again, then closing the "loop", i.e., $\boxed{\Delta S = 2}$ transition from so-called box diagram:



or get $\Delta S = 1$ from $s \rightarrow u \rightarrow d$ (only) once:



(neutral current)

[see process (3)]

— This will give effective $\boxed{\text{Lagrangian}}$:

$\mathcal{L}_{\boxed{\Delta S=2}} \sim C \left(\bar{d}_L\, s_L\right)^2$ (schematically, i.e., dropping details of Dirac, color structures)

i.e., $\boxed{K^0}$ $(\bar{s} d)$ oscillating to (mixing with) $\boxed{\bar{K}^0}$ $(s\bar{d})$ roughly

— Our $\boxed{\text{goal}}$ here is to (roughly) $\boxed{\text{estimate}}$ coefficient $\boxed{C}$ of above operator, showing that there is a suppression relative to "naive" expectation due to so-called Glashow, Iliopoulos, Maiani (GIM) mechanism / cancellation

— It is actually not so difficult to instead "just" calculate C (see, for example, Cheng, Li's section 12.2, page 379 onwards), especially given that it is a finite loop diagram (see below), but it is always good to know what to expect (going-rate) beforehand

— So, here goes (in steps): $\boxed{(a)}$ mass dimension of $\boxed{C}$ is $\boxed{-2}$ (i.e., operator has dimension = 6, being 4 fermionic, like from tree-level, massive W/Z exchange in Fermi theory).

— However there are $\boxed{\text{several}}$ masses in town: M_W; $m_u, m_c \ldots$ with $\boxed{m_u \ll m_c \ll M_W}$, so we need to work harder to figure out how these masses enter C

- let's simplify to only $\boxed{2}$ generations, i.e., ⑤
$\boxed{i, j = 1, 2}$ ($u_{i,j}$ = u or c)

- $\boxed{(b)}$ Rough estimate for loop amplitude is

$$C \sim \int \frac{d^4k}{(2\pi)^4} g^4 \sum_{\substack{i,j \\ =1,2 \\ \text{or } u,c}} V_{is} V^*_{id} V_{js} V^*_{jd} \left(\frac{1}{k^2 - M^2_W}\right)^2$$

loop momentum

2 W propagators (say 't Hooft-Feynman gauge)

$$\times \; \bar{d}_L \gamma^\mu \frac{(\not{k} - m_i)}{(k^2 - m_i^2)} \gamma^\nu s_L \quad (\text{top fermion line})$$

$$\times \; \bar{d}_L \gamma_\nu \frac{(\not{k} - m_j)}{(k^2 - m_j^2)} \gamma_\mu s_L \quad (\text{bottom fermion line})$$

not quark!

- we $\boxed{\text{neglect}}$ external momenta: see step (e) below
[again, dropping signs; factors of 2, i etc.]
(As expected from operator dimension, above expression for C has mass dimension −2)

- $\boxed{(c)}$ We can drop $\boxed{m_{i,j}}$ in numerator of fermionic propagators, since that part "switches" L to R (chirality), but only L interacts with W's,

e.g., $\bar{d}_L \gamma^\mu m_i \gamma^\nu s_L = \bar{d}_L \gamma^\mu m_i \gamma^\nu \underbrace{P_L}_{\frac{1}{2}(1-\gamma_5)} s$

$= \bar{d}_L \gamma^\mu P_{\boxed{R}} m_i \gamma^\nu s$

$= (d_L)^\dagger \gamma_0 P_L \gamma^\mu m_i \gamma^\nu s = (P_L d)^\dagger P_{\boxed{R}} \gamma_0 \gamma^\mu \gamma^\nu s m_i$

$= d^\dagger \underbrace{P_L P_{\boxed{R}}}_{=0} \gamma_0 \gamma^\mu \gamma^\nu s m_i = 0$

– So (again schematically), we have

$$C \sim \int \frac{d^4k}{(2\pi)^4}\, g^4 \underbrace{V^{\text{"}4\text{"}}}_{\substack{\text{we'll return}\\ \text{to flavor structure}\\ \text{below}}} \, \frac{k}{k^2 - m_i^2} \frac{k}{k - m_j^2} \left(\frac{1}{k^2 - M_W^2} \right)^2$$

drop m's

– (d) $\boxed{UV}$ limit, i.e., $\boxed{k \gg M_W}$ clearly gives

$C_{\boxed{UV}} \propto \int d^{\boxed{4}} k / k^{6}$, i.e., finite, while

– (e) IR limit, say $m_c \ll k \ll M_W$ gives (quadratic) "divergence" (which of course is "cut-off" by M_W as above):

$$C_{IR} \propto \int \frac{d^4k}{M_W^4} \left(\frac{1}{k} \right) \left(\frac{1}{k} \right)$$

[it's even more divergent for $k \ll m_{u,c}$:

$$\int \frac{d^4k}{M_W^4} \frac{k^2}{m_i^2 m_j^2} \Big]$$

– So, (d) & (e) combined indicate that leading contribution to $\boxed{C}$ comes from $\boxed{k \sim M_W}$, which justifies neglect of external momenta [of order kaon mass $\lesssim$ GeV vs. $k \sim M_W$], i.e., we can express (using $m_{u,c} \ll M_W$)

$$C \sim \frac{g^4}{16\pi^2} \sum_j V_{js} V_{jd}^* \sum_i V_{is} V_{id}^* \times \left[\frac{a}{M_W^2} + \frac{b\, m_i^{2}}{M_W^4} + \frac{d\, m_j^2}{M_W^4} + \dots \right]$$

(a, b, c are generation-independent)

[clearly, M_W enters "squared" from get-go, i.e., W propagator, while no $m_{i,j}$ was argued (7) above in step (c)] (of course, we will see later this "misses" a $\boxed{\text{crucial}}$ factor!)

— $\boxed{(f)}$. For a "naive" estimate, we can $\boxed{\text{drop}}$ $\boxed{m_{i,j}}$ in above form for c $\boxed{\text{and}}$ use

$$|V_{us} V_{ud}| \sim |V_{cs} V_{cd}| \sim \underset{\substack{\uparrow \\ \text{diagonal entries} \\ \{ V_{ud} \text{ or } V_{cs}}}{1} \times \overbrace{\underset{\substack{\nwarrow \\ V_{us} \text{ or } V_{cd} \\ (\text{off-diagonal})}}{\lambda_c}}^{\text{cabibbo mixing}}$$

since $V_{CKM} = U_L^\dagger D_L$ is unitary matrix, which is $\boxed{1}$ to "zeroth" approximation, i.e., mixings are small (based on observations)

— so, we get $c \sim \dfrac{g^4}{16\pi^2} \dfrac{[\lambda_c \sim 0.22]^2}{M^2_W}$ from $K^- \to \pi^0 e^- \bar{\nu}_e$ decay

$$\sim g^2 G_F \frac{(0.22)^2}{16\pi^2}$$

as mentioned in process (1) above

... which predicts $\boxed{K^0 - \bar{K}^0}$ mixing $\boxed{\text{too}}$ large compared to data

[To be complete, we need to know hadronic matrix element $\langle K^0 | (\bar{d} s)^2 | \bar{K}^0 \rangle$ in order to connect quarks in $\mathcal{L}_{eff}$ to their bound states observed. Now, we can't compute this quantity analytically from 1st principles, given non-perturbative character of strong (nuclear) force binding quarks into hadrons (see next note), but can simply use (naive) dimensional analysis to estimate it (or we can use numerical/ lattice calculation) ⑧

- $\boxed{(g)}$ What we "forgot" above is a more careful use [than done in step (f)] of unitarity of V_{CKM}, i.e.,

$$\underbrace{\sum_{i \,(\text{or } j)} V_{is} V_{id}^{*}}_{(V^{+}V)_{ds}} = \delta_{sd} \text{ (off-diagonal)} = 0!$$

$\nearrow$ or $\mathbb{1}$

so that $\boxed{C}$ actually vanishes in above approximation [step (f)] of $\boxed{m_{i,j} \to 0}$; in fact, $C = 0$ if $\boxed{m_u = m_c}$ ($\neq 0$)., since we can then factor out common $m_{i,j}$ in last 2 terms in estimate of step (f)

- So, we conclude [again, see Cheng, Li for details]

$$C \sim \frac{g^4}{16\pi^2} \frac{(m_c^2 - m_u^2)}{M_W^4} (\lambda_c)^2, \text{ i.e.,}$$

a suppression of $\boxed{\sim m_c^2/M_W^2}$ (given $m_u \ll m$

as compared to naive estimate, which $\boxed{\text{agrees}}$ with data for "suitable" value of $\boxed{m_c}$: indeed, historically speaking, this mechanism (called "GIM") was used to $\boxed{\text{predict}}$ $\boxed{m_c}$ in early 1970's, prior to its (direct production) discovery at colliders also in 1970's!

(For details of $\boxed{\text{phenomenology}}$ of K^0-$\bar{K}^0$ $\boxed{\text{mixing}}$, see Cheng, Li, section 12.2 p 374 onwards

- So far, we focussed on $\boxed{V_{CKM}}$ being off-diagonal
- Next, we will see that $\boxed{V_{CKM}}$ is "complex" (i.e., contains a CP-violating phase), resulting in, for example, particle (specifically, meson) decay width being different than that of its anti-particle

$\boxed{(3).}$ $\boxed{\text{Kobayashi, Maskawa}}$ (Nobel prize in 2008) theory of $\boxed{CP}$ violation

- Recall that

$$\mathcal{L}_{mixing} = -g/\sqrt{2}\, \overline{(u_L)_i}\, W^+ (V_{CKM})_{ij} (d_L)_j + \text{h.c.},$$

which is $-g/\sqrt{2}\, \overline{(d_L)_j}\, W^- (u_L)_i \underbrace{(V_{CKM})^*_{ij}}_{V^\dagger_{CKM\,ji}}$,

whereas $\boxed{CP}$-transformed $\mathcal{L}$ mixing is

$$\mathcal{L}^{(\prime)}_{mixing} = -g/\sqrt{2}\, \overline{(d_L)_j}\, W^- (u_L)_i\, V^{CKM}_{ij} + \text{h.c.},$$

$(\prime) \leftarrow$ denotes new

note

since $\boxed{\text{CP}}$ transformation does $\boxed{\text{not}}$ change coefficient of terms in Lagrangian: again, it's $(V_{CKM})_{ij}$ in 1st term of both $\mathcal{L}_{mixing}$ & $\mathcal{L}'_{mixing}$. Whereas, $\boxed{\text{h.c.}}$ complex conjugate coefficient of Lagrangian term coefficient, so 2nd (h.c.) term in $\mathcal{L}_{mixing} \propto (V_{CKM})^*_{ij}$

$$= (V^\dagger_{CKM})_{ji}$$

– Comparing $\boxed{2^{nd}}$ term of $\mathcal{L}_{mixing}$ with 1st term of $\mathcal{L}'_{mixing}$ (similary other 2 terms), it is clear that CP is $\boxed{\text{violated}}$ (if i.e., $\mathcal{L}'_{mixing} \neq \mathcal{L}_{mixing}$)

$$\boxed{(V_{CKM})_{ij} \neq (V_{CKM})^*_{ij}}$$

i.e., V_{CKM} is complex, which it will in general be [again, only "constraint" from theory is that V_{CKM} is unitary]

– So, let's ask how many $\boxed{\text{phase}}$ factors are contained in V_{CKM}? $\boxed{\text{Claim}}$: using unitarity of V_{CKM} and remnant freedom to do (global) phase transformations on quark fields, we will show that there is (only) one (physical) phase [i.e., there is CP violation] with $\boxed{3}$ generations of SM, but $\underline{\underline{\text{no}}}$ (physical) phase with only $\boxed{2}$ generations (11)

- $\boxed{\text{Proof}}$ A general $(n \times n)$ unitary matrix (n ← number of generations)

$$= \exp\left[i\ (n \times n)\ \text{Hermitian matrix}\right]$$

In turn, $n \times n$ Hermitian matrix has

$$n\ (\text{real, diagonal}) + \underbrace{{}^{n}C_2}_{\substack{\text{number of off-diagonal entries in}\\ \text{upper (or lower) triangular part of matrix} = \frac{n(n-1)}{2}}} \times \underbrace{2}_{\substack{\text{off-diagonal entries are complex}}} = n^2\ \underline{\underline{\text{real}}}\ \underline{\text{parameters}}$$

(again, off-diagonal entries in upper triangular part are complex conjugates of lower triangular part)

[Of course, this "matches" $\boxed{(n^2-1)}$ number of generators of $\underline{\underline{S}}U(n)$ for $SU(n)$: "-1" because we also require tracelessness of $(n \times n)$ Hermitian matrix.]

- $\boxed{\text{At the same time}}$, $(n \times n)$ unitary matrix

$$= (\text{schematically})\ (n \times n)\ (\text{real})\ \underline{\text{orthogonal}}\ \text{matrix}$$
$$\text{"}\times\text{"}\ \boxed{\text{phases}}$$

- Now, $(n \times n)$ $\boxed{\text{orthogonal}}$ matrix can be (completely) parametrized by $\boxed{{}^{n}C_2}$ mixing $\boxed{\text{angles}}$: θ_{ij} (i = 1, 2 ... n; j = 1, 2 ... n) (but order is $\boxed{\text{not}}$ relevant & $i \neq j$)

- So, number of phases (not yet physical) is given by (number of real parameters in $(n \times n)$ unitary matrix) $-$ (number of mixing angles in $(n\times n)$ orthogonal matrix) $= n^2 - \underbrace{\frac{n(n-1)}{2}}_{{}^nC_2} = \frac{n^2+n}{2}$

- However, recalling $\boxed{\overline{(u_L)}_i \, W \, (V_{CKM})_{ij} (d_L)_j}$,

the following U(1)/ phase transformations will $\boxed{(a).}$ keep quark masses real; $\boxed{(b).}$ keep quarks in mass basis (i.e., not re-introduce generational mixing) and $\boxed{(c).}$ only modify $\boxed{V_{CKM}}$ ($W^{\pm}_{\mu}$ coupling), i.e., not affect neutral current interactions, i.e., couplings to photon, Z, gluon, H & G:

$$(u_{L,R})_i \longrightarrow (u_{L,R})_i \, e^{i\alpha_{u\,i}}$$ ← generation-dependent; imaginary

$$(d_{L,R})_i \longrightarrow (d_{L,R})_i \, e^{i\alpha_{d\,i}}$$ → different for d vs. u

- So, naively, we can "remove" $(2n)$ phases (← u & d) from $\boxed{V_{CKM}}$ ...

... but if we choose $\boxed{\alpha_{u\,i} = \alpha_{d\,i} = \alpha}$ for all i (i.e., universal - over u, d and generations - phase transformation), then V_{CKM} (see above form of how V_{CKM} enters Lagrangian) is unchanged.

— Thus, only $\left[2n \, - 1\right]$ ($\xrightarrow{}$ again, universal phase rotation) phases can be removed from V_{CKM}, resulting in

number of $\boxed{\text{physical}}$ phases in $\boxed{V_{CKM}}$ for n generations $= \underbrace{\frac{n^2+n}{2}}_{\text{original number of phases}} - \underbrace{(2n-1)}_{\text{removal of phases}} = \frac{(n^2-3n+2)}{2}$

$$= \begin{cases} 0 & \text{for 2 generations} \\ \boxed{1} & \text{for } \boxed{3} \text{ generations, as in } \boxed{SM} \end{cases}$$

— Because of above freedom of phase rotations, there is no unique way to write/parametrize $\boxed{V_{CKM}}$ [of course, physical result for a decay width or cross-section is independent of form of V_{CKM}].

— Based on data on mixing angles of V_{CKM} [e.g. "1–2" mixing angle, $|V_{us}|$ or $|V_{cd}| \simeq 0.22$ (λ_c), i.e., "small"] and unitarity of V_{CKM}, the following (due to Wolfenstein) parametrization is quite convenient, i.e., phase is only in $\boxed{V_{ub}}$ and $\boxed{V_{td}}$, at leading order in λ_c expansion

↓ V_{id} column

note

$$V_{CKM} = \begin{array}{r} \\ \\ V_{tj}\text{ row} \rightarrow \end{array} \begin{bmatrix} (1-\lambda_c^2/2) & \lambda_c & A\lambda_c^3(\rho - i\eta) \\ -\lambda_c & (1-\lambda_c^2/2) & A\lambda_c^2 \\ A\lambda_c^3(1-\rho-i\eta) & -A\lambda_c^2 & 1 \end{bmatrix} + O(\lambda_c^4)$$

note

where $\boxed{A, \rho, \eta}$ are (nominally) $O(1)$ factors: indeed fit to data "confirms" that expectation.

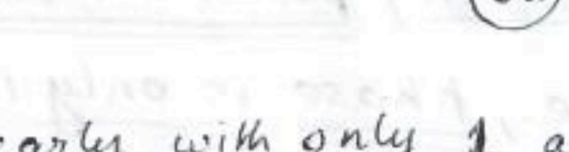
$\boxed{4}$. $\boxed{\text{Observables}}$ for CP-violation

- If CP is violated, then the rates for a process and its conjugate need not be the same
- Here, we will consider a specific example (a general formulation is in HW 10.6):

$$\boxed{\Gamma(B^+ \rightarrow K^+\pi^0) \neq \Gamma(B^- \rightarrow K^-\pi^0)}$$

decay width

- Let's consider $B^- \rightarrow K^-\pi^0$ decay from tree-level diagram:

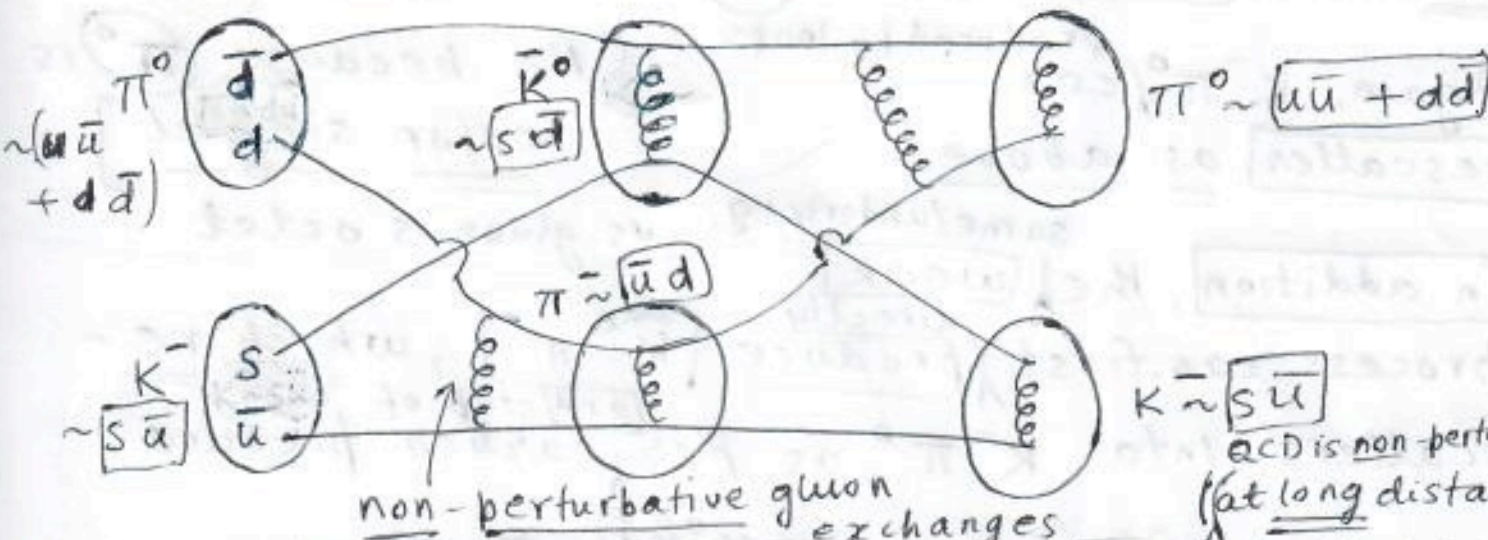


$\boxed{A_{\text{tree}} \propto V_{ub} V_{us}^*}$, where V_{ub} has CP-violating (often called weak) phase in Wolfenstein parametrization of V_{CKM}

- Clearly with only 1 amplitude contribution as above (even if it is complex), we won't get inequality of rates $\propto \boxed{|A|^2}$, i.e, we need (as is familiar from

QM, an interference between 2/more amplitudes with different phases

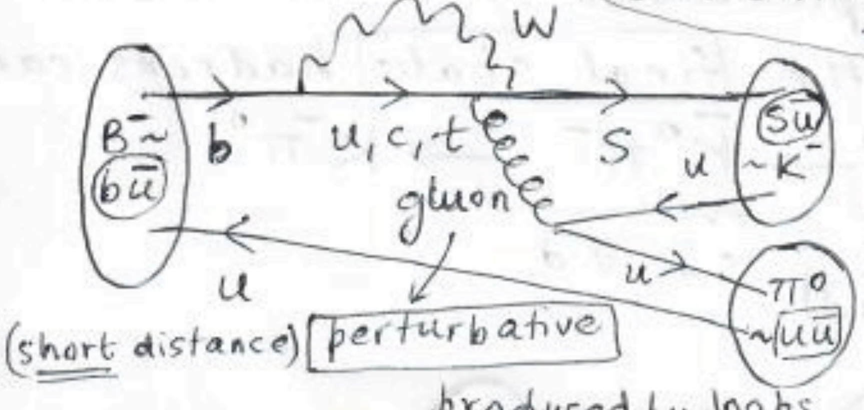
- We will indeed show that there's a loop-level contribution with different phase
- However, let's first complete the form of $\mathcal{A}_{tree}$: the point is that the $\boxed{\text{final-state}}$ hadrons can "re-scatter": $\underbrace{K^-}_{s\bar{u}}\underbrace{\pi^0}_{(u\bar{u}+d\bar{d})} \longrightarrow \underbrace{\bar{K}^0}_{s\bar{d}}\underbrace{\pi^-}_{\bar{u}d} \longrightarrow K^-\pi^0$



QCD is non-perturbative (at long distances)

- Since hadrons interact strongly with each other, this process is unsuppressed. Also, it involves (possibly) on-shell $\boxed{\text{intermediate}}$ states hadrons, i.e., such propagators give $\boxed{\text{imaginary}}$ part to this amplitude from $\sim \frac{1}{(i) m \Gamma}$ (See Cheng, Li section 12.2, page 374 onwards; Donoghue, Golowich, Holstein, section IX-2, page 238 onwards)
- $\boxed{\text{Bottom line}}$: $\boxed{\mathcal{A}_{tree} = |\mathcal{A}_{tree}| e^{i\delta_{CKM}} e^{i\phi_{tree}}}$, where δ_{CKM} comes from V_{ub} (weak part of decay), while $\boxed{\phi_{tree}}$ is from (strong) re-scattering phase

– Onto loop-level diagram (it has "no" GIM suppression, since it involves virtual top quark with $m_t \sim M_W$, cf. $K^0 - \bar{K}^0$ mixing with 2 generations dicussed earlier) [Note: we can similarly do $\Delta S = 1$ neutral current for Kaon decays.]

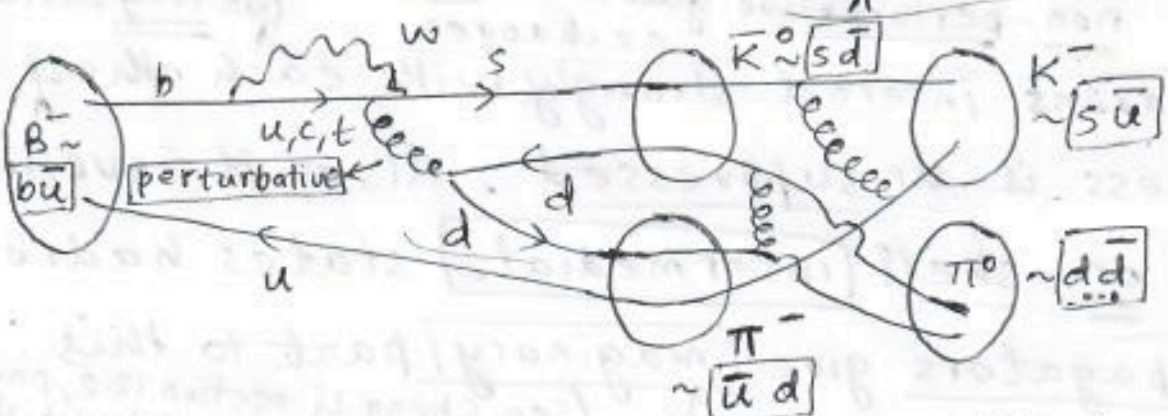


As an aside: can 1 gluon producing $u\bar{u}$ give π^0 instead ?!

[No because π^0 is color singlet] vs. gluon is octet

– Again, $K^- \pi^0$ (produced by loops) can rescatter as above

– In addition, the same/underlying weak process can first directly produce $\bar{K}^0 \pi^-$, which re-scatters into $K^- \pi^0$ as per 2nd step of earlier picture:



i.e., net phase from (strong) re-scattering is different for loop diagram vs. tree

– And, $\mathcal{A}^{\text{(both)}}_{\text{loop}} \propto V_{tb} V^*_{ts}$, with no (CP-violating) phase in same parametrization of V_{CKM}

– Thus, we get: $\mathcal{A}_{loop} = |\mathcal{A}_{loop}| \, e^{i\phi_{loop}}$, with $\phi_{loop} \neq \phi_{tree}$

(in another/ general parametrization of V_{CKM}, both A_{tree} & A_{loop} will have weak phases which are different)

- Combining 2 above contributions to decay amplitude:

$$\boxed{A\left(B^- \to K^- \pi^0\right) = |A_{tree}|\, e^{i\delta_{CKM}} e^{i\phi_{tree}} + |A_{loop}|\, e^{i\phi_{loop}}}$$

- Onto CP-conjugate process, i.e., $B^+ \to K^+ \pi^0$: note that $\boxed{\delta_{CKM}}$ will <u>flip</u> <u>sign</u> (since it's $\boxed{CP}$-<u>violating</u> phase), whereas since <u>strong</u> (QCD) interactions <u>preserve</u> $\boxed{CP}$, $\phi_{tree,\,loop}$ will be $\boxed{\text{same}}$ for a process and its CP-conjugate.

- Thus, we get

note note

$$\boxed{A\left(B^+ \to K^+ \pi^0\right) = A_{tree}\, e^{-i\delta_{CKM}} e^{i\phi_{tree}} + |A_{loop}|\, e^{i\phi_{loop}}}$$

- Using $\Gamma \propto |A|^2$, we can show (see <u>HW</u> 10.6) that indeed

$$\Gamma\left(B^- \to K^- \pi^0\right) \neq \Gamma\left(B^+ \to K^+ \pi^0\right)$$

provided (a) $\boxed{\delta_{CKM} \neq 0}$ (i.e., in general weak/CP-violating phases of 2 contributions to net amplitude are different) <u>and</u> (b) $\boxed{\phi_{loop} \neq \phi_{tree}}$, i.e., strong/CP-preserving phases are also different

- As an <u>aside</u>, another way to obtain $\boxed{CP}$-<u>conserving</u> phases is via <u>time evolution</u>: see Donoghue, Golowich, Holtstein, section XIV-5, page 400 onwards