Overview of Electroweak (EW) theory/unification

- **Weak** (nuclear) force arises in radioactive (β- or neutron) decays, muon decay into electron etc.
- At low ($\ll 100$ GeV) energies, this can be described by the phenomenological Lagrangian (four-fermion, contact/local interaction called Fermi theory):

destroys LH μ^- · creates LH e^-

$$\mathcal{L}_{\mu\text{-decay}} = \frac{G_F}{\sqrt{2}} \left[\bar{\Psi}_{(\mu)} \gamma^\mu (1-\gamma_5) \Psi_{(\nu_\mu)} \right]^\dagger \left[\bar{\Psi}_e \gamma_\mu (1-\gamma_5) \Psi_{(\nu_e)} \right]$$

$(\mu^- \to e^- \nu_\mu \bar{\nu}_e)$ — muon; μ: Dirac index; creates $\bar{\nu}_e$

$$\mathcal{L}_{\text{neutron decay}} = \frac{G_F}{\sqrt{2}} \left[\bar{\Psi}_{(u)} \gamma^\mu (1-\gamma_5) \Psi_{(d)} \right] \left[\bar{\Psi}_e \gamma_\mu (1-\gamma_5) \Psi_{(\nu_e)} \right]$$

$d \to u\, e^- \bar{\nu}_e$ (at quark-level)

charge: $-\frac{1}{3}$ $\;+\frac{2}{3}$ $\;-1$ $\;0$

[with $G_F = 1.17 \times 10^{-5}/(\text{GeV})^2$]

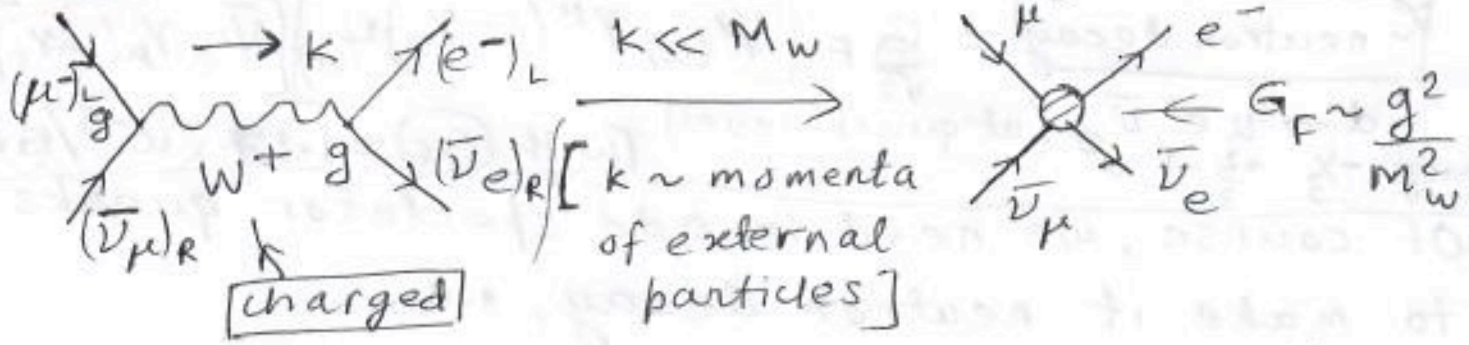
- Of course, we need to add spectator quarks to make it neutron decay, i.e.,

$$\underbrace{\underbrace{(ud)}_{\text{spectator}}\, d}_{\text{neutron}} \longrightarrow \underbrace{(ud)u}_{\text{proton}} + e^- + \bar{\nu}_e$$

- Again, this was proposed based on experimental data, in particular, the **chiral** structure: $(1-\gamma_5)$, i.e., only **LH** e^- (or RH e^+) etc. are involved
- While Fermi theory successfully described all the

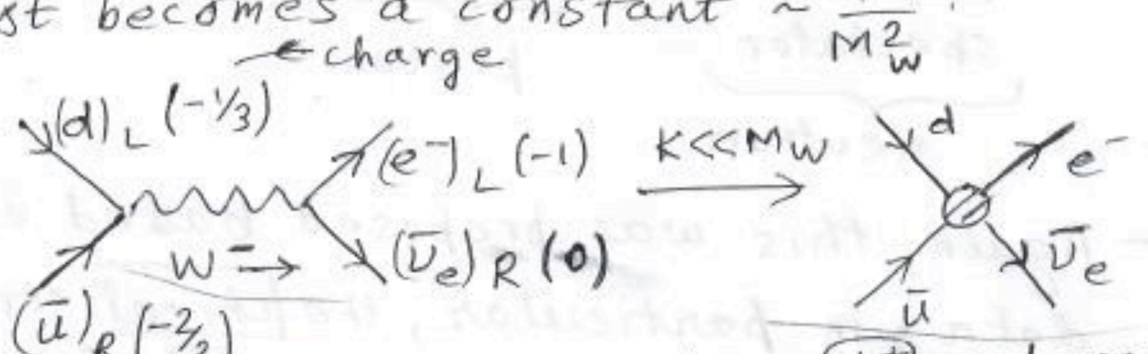
observations, there are theoretical "problems" with it. Namely, this theory is non-renormalizable (due to coupling constant, G_F, having negative mass dimension). In addition, the "universality" of coupling, i.e, same G_F and Dirac structure, for all weak processes, begs an explanation.

- Solution: above Lagrangian can be (effectively) obtained by exchange of massive spin-1 gauge boson exchange, i.e., weak force is described by underlying gauge theory, kind of like QED (apart from mass of gauge boson),



since momentum-dependent full propagator $\sim \frac{1}{k^2 - M_W^2}$ just becomes a constant $\sim \frac{1}{M_W^2}$.

Similarly,



(this is called charged current weak interaction from $W^\pm$ exchange: later, we will describe neutral version from Z exchange)

— Gauge theory "explains" above universality of coupling (like QED) and "hope" was it might be renormalizable (again, like QED, even though gauge boson is massive)

— Of course, if gauge boson mass is put in by hand (explicitly), then the theory is still non-renormalizable

— Spontaneous gauge symmetry breaking (Higgs mechanism) saves the day by (realizing above hope of renormalizability)

— Most of above discussion is review of what was done at start of SSB part of course, i.e., served as motivation for studying this topic

— Next, let us see (schematically for now, with details to follow) what should be gauge group for weak interaction?

— As already indicated earlier, since W^+ couples 2 fermions with different charges, it is clear that we need a non-abelian gauge group

— The simplest possibility is SU(2) gauge theory with $\left[(\nu_e)_L, (e^-)_L\right]^T$ and $\left[(u)_L, (d)_L\right]^T$ being doublets of SU(2): it is clear that this choice will result in above (off-diagonal) vertices from $(\bar{u}\ \bar{d})\gamma_\mu W^\mu_{1,2}(1-\gamma_5)\sigma_{1,2}\begin{pmatrix} u \\ d \end{pmatrix}$ (again, $\sigma_{1,2}$ are off-diagonal)

— Of course, SU(2) has 3 generators/associated gauge bosons : W^+, W^- (anti-particle of W^+) as above + $\boxed{W^3}$ coupled to $(\bar{u}\ \bar{d})\sigma^3\begin{pmatrix}u\\d\end{pmatrix}$ and $(\bar{\nu}_e\ \bar{e})\sigma^3\begin{pmatrix}\nu_e\\e^-\end{pmatrix}$, where $\sigma_3 = \begin{pmatrix}+1 & 0\\0 & -1\end{pmatrix}$

[dropping Dirac structure]

(+1: charge of u & ν_e etc.; −1: charge of d & e^-)

$\Rightarrow$ $\boxed{W^3}$ is (electrically) neutral [it couples u to u etc. : σ_3 is diagonal generator, cf. $\sigma_{1,2}$ associated with $W^\pm$ coupling off-diagonally

— Natural question is whether $\boxed{W_3}$ can be photon (?!) However, it is clear that can't be, since W^3 (unlike photon) couples also to $\boxed{\nu_e}$ [and couples to u, d with opposite charges, cf. electric charges being $+2/3$ & $-1/3$, respectively]

[As an aside, one could embed fermions in other/non-minimal/larger representations of $SU(2)$, i.e., involving fermions beyond e, ν_e etc. in such a way that $\boxed{W^3}$ can indeed be photon : the motivation here being unification of weak & EM forces, i.e., single/minimal gauge group/coupling (see HW 8.5 for details). It turns out this idea works theoretically, but eventually failed experimentally.]

- So, we "live with" (extra) neutral gauge boson (W_3): see its exact fate below, i.e., keep minimal fermion content/representation, i.e., doublet of SU(2)
- To incorporate photon, we can try (again, as simplest possibility) the gauge group:
  $SU(2)_L \times U(1)_Y$ (where "Y" is yet to be fixed charge)
  ← again, only LH fermions couple to $W^{\pm}$

[Just to be clear, the "×" above indicates that the 2 symmetry transformations act independently, i.e., both u_L and d_L are rotated by $\exp(-i\beta g' Y_Q)$ where Y_Q is $U(1)_Y$ charge of entire doublet of SU(2), i.e., both components: $Q = (u_L, d_L)^T$. Recall that we have encountered such independent rotations before, e.g., in the coupling term: $W_\mu^a (\bar{u} \; \bar{d}) \sigma_a \gamma^\mu (1-\gamma_5) \begin{pmatrix} u \\ d \end{pmatrix}$

internal space → σ_a: (2×2) matrix; $\gamma^\mu(1-\gamma_5)$: [4×4] matrix in Dirac space

where σ^a acts in internal space, while $\gamma_\mu(1-\gamma_5)$ operates in Dirac/spinor space. So, when we perform a Lorentz transformation, i.e., rotate Dirac spinor index, we do it identically for u and d. Similarly, SU(2) rotation/mixing between u and d does not touch Dirac/spinor index.]

— Next natural question is whether $U(1)_{EM}$ can really be just $U(1)_Y$ [i.e., "decoupled" from $SU(2)_L$ of weak interaction] (?!)

— Answer is (again) "No", since in this case, u_L & d_L [similarly $(e^-)_L$ & $(\nu_e)_L$] would have same electric charge, cf. if W^3 was identified as photon, then these charges would be opposite of each other (both cases of course are inconsistent with data)

⇒ $U(1)_{EM}$ cannot "commute" with $SU(2)_L$, i.e., $U(1)_{EM}$ must be partially inside $SU(2)_L$; in particular, photon has admixture of W^3 (again, W^3 can't "fully" be photon)

⇒ We must have EW unification: just to belabor this point, it is a more theoretical chain of arguments, i.e., there was no — a priori — experimental motivation to have this unification: in fact, data suggests quite the "opposite," i.e., photon is massless (EM force is long-range) vs. $W^\pm$ is massive (weak force is short-range); more on this below

— Anyway, there is a concrete prediction of this (almost purely) theoretical proposal, i.e., extra neutral gauge boson (called "Z"): it has to be massive (just like $W^\pm$), otherwise it would have led to a long-range force, which has not been seen. So, it gives 4-fermion contact ("neutral weak current") interaction at low energies (analog of charged current interaction from $W^\pm$ exchange), which was experimentally detected a few years afterwards

- So, just to summarize, EW gauge group is $\boxed{SU(2)_L \times U(1)_Y}$ (Y denotes hypercharge: see values later), with photon and $\boxed{Z}$ being (orthogonal to each other) $\boxed{\text{combinations}}$ of $\boxed{U(1)_Y}$ and $\boxed{W_3}$ part of $SU(2)_L$
- Since photon is massless, while $Z, W^\pm$ are massive, we have electroweak symmetry breaking (EWSB) in sort of ② "senses/forms", i.e., gauge symmetry is $\boxed{\text{broken}}$ giving mass to gauge bosons and only some of the original set of gauge bosons have this fate, i.e., EW unification is also "spoilt"
- In order to preserve renormalizability of original gauge theory, this gauge symmetry has to be broken spontaneously
- In the SM, this is achieved by VEV of scalar/ $\boxed{\text{Higgs}}$ field (although there are alternative ways

to implement SSB), giving also a physical scalar/Higgs boson [denoted by "η" in U(1) case]

— In (HW) 5.3 (global) and 7.1.1 (gauged), a simpler version, i.e., $SU(2) \rightarrow U(1)$ was studied, with some symmetries unbroken/gauge bosons massless

— Bottomline picture of bosonic (EW) sector of SM is:

$SU(2)_L \times U(1)_Y$ + Higgs field (3 (+1) massless gauge bosons) [with "negative mass^2"]

$\xrightarrow[\text{VEV}]{\text{Higgs}}$ $U(1)_{EM}$ (massless photon)

+ massive $W^\pm$, Z

+ (massive) Higgs boson

— Finally, let's consider SM fermion mass terms. Since only LH fermions transform under SU(2), i.e., RH fermions are singlets, SM fermion mass terms which "couple" LH & RH fermions break EW symmetry.

— Thus, in order to maintain renormalizability of the EW theory, SM fermion mass terms should also arise from SSB, i.e., via (Yukawa) coupling of fermions to Higgs field (we will show that the same one works for giving both gauge boson & fermion masses).