(1)

Proton-Proton collision with Hard Scattering of Partons

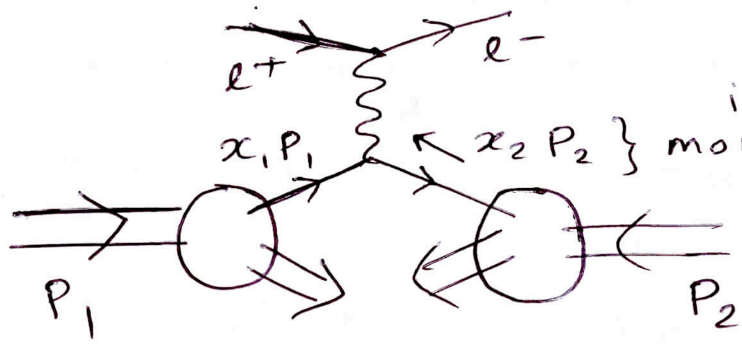
from general formula for cross section to more convenient [and specific to Drell-Yan (DY) process] one (based on Peskin & Schroeder: section 17.4)

— Recall general formula for quark-antiquark (hard) scattering into final state Y:

$$\sigma\left[p(P_1)+p(P_2)\to Y+\underbrace{X}_{\text{any hadron...}}\right] =$$

$$\underbrace{\int_0^1 dx_1 \int_0^1 dx_2}_{\substack{\text{longitudinal}\\ \text{fractions for each}\\ \text{parton}}} \underbrace{\sum_f}_{\text{over } u,d,\bar{u}\ldots} f_f(x_1) f_{\bar{f}}(x_2)\cdot\sigma\left[q_f(\underbrace{x_1 P_1}_{\substack{\text{momentum}\\ \text{of quark}}})+\bar{q}_f(x_2 P_2)\to Y\right]$$

— $\boxed{\text{DY}}$ process, i.e., Y = high-invariant mass $\ell^+\ell^-$ via photon exchange:



— Parton-level cross-section is related to that for $e^+e^-\to q\bar{q}$: instead of sum over quark colors, use average (factors of $1/3$): $\sigma(q_f\bar{q}_f\to\ell^+\ell^-)=\frac{1}{3}Q_f^2\frac{4\pi\alpha^2}{3\hat{s}}$

— Next (just like for deep inelastic scattering), we convert cross-section formula into a more convenient / standard form

— First, if both final-state lepton momenta are observed, then can reconstruct 4-momentum q of virtual photon : denote

$$\boxed{M^2} \equiv q^2 = (\text{invariant mass of lepton pair})^2$$

— since initial partons have small transverse (to collision axis) momentum, so does photon ... but photon's longitudinal momentum will not be small (in general) : define $\boxed{Y}$ (rapidity) by

$$q^2 (\equiv M^2) = \underbrace{q^{0\,2} - q_{\parallel}^2}_{} \equiv M^2 (\cosh^2 Y - \sinh^2 Y)$$

longitudinal photon momentum

(as an aside: rapidity is additive under successive boosts)

— One can determine (as follows) longitudinal fractions of initial quark and antiquark : do it here in terms of M^2 and Y (both are directly measurable/observable)

— In pp center-of-mass frame, proton momenta (neglecting masses) are

$$P_1 = (E, 0, 0, E) \quad \text{and} \quad P_2 = (E, 0, 0, -E)$$

with $s = 4E^2$ ... and constituent quark and antiquark momenta are $x_{1,2}$ times $P_{1,2}$ (neglecting small transverse parton momenta) $\Rightarrow$

$$q \,(\text{momentum of photon}) = \underbrace{x_1 P_1}_{\text{quark momentum}} + x_2 P_2$$

$= [(x_1 + x_2) E, 0, 0, (x_1 - x_2) E]$ so that

$$M^2 = q^2 = x_1 x_2 s$$

Similarly, Y can be written in terms of $x_{1,2}$:

$\cosh Y = x_1 + x_2 / (2\sqrt{x_1 x_2})$ (comparing above 2 formulae for q)

$\Rightarrow e^Y = \sqrt{x_1/x_2}$

and inverting these expressions for M^2 & Y in terms of $x_{1,2}$ gives $\boxed{x_1 = \frac{M}{\sqrt{s}} e^Y}$ and $\boxed{x_2 = \frac{M}{\sqrt{s}} e^{-Y}}$

(as desired)

— Finally, convert $\int dx_{1,2} \ldots$ in general formula for cross-section into integral over M^2, Y (again, observables related to lepton momenta): Jacobian is

$$\frac{\partial(M^2, Y)}{\partial(x_1, x_2)} = \det \begin{pmatrix} x_2 s & x_1 s \\ \frac{1}{2x_1} & -\frac{1}{2x_2} \end{pmatrix} = s = \frac{M^2}{x_1 x_2}$$

$$\Rightarrow \frac{d\sigma}{dM^2 dY} (pp \to e^+ e^- + X) = \sum_f x_1 f_f(x_1)\, x_2 f_{\bar{f}}(x_2) \cdot \frac{1}{3} Q_f^2 \cdot \frac{4\pi\alpha^2}{3M^4}$$

where $x_{1,2}$ are given as above in terms of M^2, Y

i.e., cross-section for DY process can be determined using (PDF) information obtained from deep inelastic scattering

... but $O[\alpha_s(M)]$ corrections to above formula — again, from hard gluon exchange/emission (calculable in perturbation theory) are numerically large...

i.e., expect to be small due to $\alpha_s(M)$, but, in the end, not so...