Chapter 2

Quark-Gluon Plasma and the Early Universe

There is now considerable evidence that the universe began as a fireball, the so called "Big-Bang", with extremely high temperature and high energy density. At early enough times, the temperature was certainly high enough ($T > 100$ GeV) that all the known particles (including quarks, leptons, gluons, photons, Higgs bosons, W and Z) were extremely relativistic. Even the "strongly interacting" particles, quarks and gluons, would interact fairly weakly due to asymptotic freedom and perturbation theory should be sufficient to describe them. Thus this was a system of hot, weakly interacting color-charged particles, a quark-gluon plasma (QGP), in equilibrium with the other species.

Due to asymptotic freedom, at sufficiently high temperature the quark-gluon plasma can be well-described using statistical mechanics as a free relativistic parton gas. In this Chapter, we explore the physics of QGP, perhaps the simplest system of strong-interaction particles that exists in the context of QCD. As the universe cooled during the subsequent expansion phase, the quarks, antiquarks, and gluons combined to form hadrons resulting in the baryonic matter that we observe today. The transition from quarks and gluons to baryons is a fascinating subject that has been difficult to address quantitatively. However, we will discuss this transition by considering the basic physics issues without treating the quantitative details. At present there is a substantial effort in theoretical physics to address this transition by using high-level computational methods known as lattice gauge theory. This subject is somewhat technical and we will discuss it only very briefly. However, the general features that have emerged from lattice studies to date are rather robust and can be discussed in some detail.

The relatively cold matter that presently comprises everything around us is actually a residue of the annihilation of matter and anti-matter in the early universe. The origin of the matter-antimatter asymmetry which is critical for generating the small amount of residual matter is still a major subject of study, and we discuss this topic at the end of this Chapter.

Another major thrust associated with the transition between the QGP and baryonic matter is the experimental program underway to study observable phenomena associated with the dynamics of this interface. This experimental program involves the collision of relativistic heavy ions that should produce (relatively) small drops of QGP. Large particle detector systems then enable studies of the products of these collisions, which can (in principle) yield information on the transition to the baryonic phase and the QGP itself. The program of experiments and the present state of the experimental data will be discussed in Chapter XX.

2.1 Thermodynamics of A Hot Relativistic Gas

At very high-temperature such that the particles have energy much larger than their rest mass, we may describe them using relativistic kinematics and ignore their masses. Thus these energetic weakly interacting particles form a system that is, to an excellent approximation, a hot relativistic free gas. Since particles and antiparticles can be created and annihilated easily in such an environment, their densities are much higher than their differences. Therefore the chemical potential μ can be neglected. The number densities of the partons (species i) are then described by the quantum distribution functions

$$n_i = \int \frac{d^3p_i}{(2\pi)^3}\frac{1}{e^{\beta E_i} \pm 1} \ , \tag{2.1}$$

where $\beta = 1/k_{\rm B}T$ and the $-$ sign is for bosons and the $+$ is for fermions. For relativistic particles, $p_i = E_i$. For $E_i\beta < 1$, the exponential factor is small and there is a large difference between fermions and bosons. For $E_i\beta \geq 1$ the ± 1 becomes increasingly unimportant, and the distributions become similar. Integrating over the phase space, one finds,

$$n_i = \begin{cases} \zeta(3)/\pi^2 T^3 & \text{(boson)} \\ (3/4)\zeta(3)/\pi^2 T^3 & \text{(fermion)} \end{cases} \tag{2.2}$$

where $\zeta(3) = 1.20206...$ is a Riemann zeta function. The T^3-dependence follows simply from dimensional analysis (the Boltzmann constant k_B can also be taken to be 1).

The energy density for a free gas can be computed from the same quantum distribution functions:

$$\begin{aligned} \epsilon_i &= \int \frac{d^3p_i}{(2\pi)^3}\frac{E_i}{e^{\beta E_i} \pm 1} \\ &= \begin{cases} \pi^2/30T^4 & \text{(boson)} \\ (7/8)(\pi^2/30)T^4 & \text{(fermion)} \end{cases} \end{aligned} \tag{2.3}$$

where the fermion energy density is 7/8 of that of boson.

These expressions are valid for each spin/flavor/charge/color state of each particle. For a system of fermions and bosons, we need to include separate degeneracy factors for the various particles:

$$\begin{aligned} \epsilon &= \sum_i g_i \epsilon_i \\ &= g_* \frac{\pi^2}{30}(k_B T)^4 \ , \end{aligned} \tag{2.4}$$

where $g_* = \left(g_b + \frac{7}{8}g_f\right)$ with g_b and g_f are the degeneracy factors for bosons and fermions, respectively. Each of these degeneracy factors counts the total number of degrees of freedom, summed over the spins, flavors, charge (particle-antiparticle) and colors of particles. When some species are thermally decoupled from others due to the absence of interactions (such as neutrinos at present epoch), they no longer contribute to the degeneracy factor. For example, at temperature above 100 GeV, all particles of the standard model are present. At lower temperatures, the W and Z bosons, top, bottom, and charm quarks freeze out and g_* decreases. Therefore g_* is generally a decreasing function of temperature.

We can now calculate the contribution to the energy density from the quark-gluon plasma as a relativistic free parton gas. For a gluon, there are 2 helicity states and 8 choices of color so we have a total degeneracy of $g_b = 16$. For each quark flavor, there are 3 colors, 2 spin states, and 2 charge states (corresponding to quarks and antiquarks). At temperatures below $k_{\rm B}T \sim 1$ GeV, there are 3 active flavors (up, down and strange) so we expect the fermion degeneracy to be a large number like $g_f \simeq 36$ in this case. Thus we expect for the QGP:

$$\epsilon_{\rm QGP} \simeq 47.5\frac{\pi^2}{30}(k_B T)^4 \ . \tag{2.5}$$

With two quark flavors, the prefactor is $g_* = 37$. (For reference, if one takes into account all standard model particles, $g_* = 106.75$.)

The pressure of the free gas can be calculated just like the case of black-body radiation. For relativistic species,

$$p = \frac{1}{3}\epsilon \ , \tag{2.6}$$

which is the equation of state.

To calculate the entropy of the relativistic gas, we consider the thermodynamics relation, $dE = TdS - pdV$. At constant volume we would have just $dE = TdS$, or $d\epsilon = Tds$ where ϵ (s) is the energy (entropy) per unit volume. Since $\epsilon \propto T^4$, we can easily find that

$$s = \frac{4}{3}\frac{\epsilon}{T} \ . \tag{2.7}$$

For an isolated system of relativistic particles, we expect the total entropy to be conserved.

Now using Eq. 2.4 one can easily see that

$$s \propto g_*(T)T^3 \ , \tag{2.8}$$

where $g_*(T)$ counts the number of active (i.e., non-frozen) degrees of freedom in equilibrium. The total entropy of the active species is given by

$$S \propto sR^3 \propto g_*(T)T^3R^3 \ , \tag{2.9}$$

which is conserved in adiabatic processes.

2.2 The Early Partonic Universe

It has been established, since Hubble's first discovery in the 1920's, that the universe has been expanding for about $\sim$ 10 billion years. The universe as we know it began as a "big bang" where it was much smaller and hotter, and then evolved by expansion and cooling. Our present understanding of the laws of physics allows us to talk about the earliest moment at the so-called Planck time $t_P \sim 10^{-43}$ when the temperature of the universe is at the Planck scale $T \sim M_{\rm pl}$

$$M_{\rm pl} \equiv \sqrt{\frac{\hbar c}{G_N}} \tag{2.10}$$

$$= 1.22 \times 10^{19} \text{ GeV} \ , \tag{2.11}$$

where G_N is Newton's gravitational constant, and $\hbar$ and c are set to 1 unless otherwise specified. However, at this scale, the gravitational interaction is strong, the classical concept of space-time might break down. At times later than the Planck epoch when the universe has cooled below $M_{\rm pl}$, space-time may be described by a classical metric tensor $g^{\mu\nu}$, and the laws of physics as we know them should be applicable.

Since the observed universe is homogeneous and isotropic to a great degree, its expansion can be described by the Robertson-Walker space-time metric,

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] , \tag{2.12}$$

which describes a maximally symmetric 3D space, where $R(t)$ is a scale parameter describing the expansion and k is a curvature parameter with $k = +1, -1, 0$, corresponding to closed, open and flat universe, respectively.

The expansion of the universe after the Planck time is described by Einstein's equation of general relativity, which equates the curvature tensor of the space-time to the energy-momentum tensor $T^{\mu\nu}$. The energy-momentum density comes from both matter and radiation and the vacuum $\Lambda g^{\mu\nu}$ contribution, the infamous "cosmological constant" of Einstein. If the matter expands as ideal gas, the energy-momentum density is

$$T_{\mu\nu} = -p g_{\mu\nu} + (\rho + p) u^\mu u^\nu , \tag{2.13}$$

where p is the pressure and ρ is the energy density, and $u^\mu = (1, 0, 0, 0)$ defines the cosmological comoving frame. The resulting dynamical equation for the scale parameter is

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G_N \rho}{3} - \frac{k}{R^2} + \frac{\Lambda}{3} , \tag{2.14}$$

which is called the Friedmann (or Friedmann-Lemaitre equation). $\dot{R}/R = H$ is the expansion rate (Hubble constant). Another equation needed for studying the expansion comes from energy-momentum conservation,

$$\dot{\rho} = -3H(\rho + p) . \tag{2.15}$$

Together with the equation of state $p = p(\rho)$, the above equations can be solved to yield the evolution of ρ as a function of the scale parameter. There is now strong experimental evidence that we are living in a universe with $k = 0$ and Λ has been negligibly small until recently. Hence, we will focus below on a simplified Friedmann equation for the early universe without the second and third terms on the right-hand side in 2.14.

When the temperature was lower than the Planck scale, the universe was an expanding gas of relativistic particles. These particles include quarks and leptons, the gauge bosons such as photons, gluons, and W and Z bosons, and perhaps more exotic particles like the supersymmetric partners of the standard model particles, heavy right-handed neutrinos, gauge bosons related to grand unification theories, etc. As the temperature cooled below the masses of certain particles (such as the W and Z bosons) they "freeze out" and decay, i.e., they are not longer created by inverse reactions of their decay products due to the lower temperature. Some of these particles with a short life time had disappeared long ago, and some with a long life time may still be with us today in the form of dark matter.

Thus we expect that when the temperature drops below the electroweak scale ($T < 100$ GeV) the early universe will be a hot gas of the standard model particles: quarks, leptons, gluons and photons. Since the system is dominated by the strongly interacting degrees of freedom, quarks and gluons (i.e., partons), it is a good approximation to regard it as a system of quark-gluon plasma. Because of asymptotic freedom, the interaction between quarks and gluons are fairly weak at high-temperature, and it shall be a good approximation to describe the plasma in terms of a non-interacting parton gas.

During this phase of the universe, the energy density ρ is dominated by these relativistic partons and decreases as the universe expands. The evolution of ρ during this time is governed by the fact that we have a gas of relativistic partons. The volume of any piece of the universe increases like R^3, but the energy in every mode decreases as R^{-1} (as the wavelength of the mode expands with the universe). Thus we expect

$$\rho \propto R^{-4} \;, \tag{2.16}$$

and Eq. 2.14 then yields

$$\dot{R} \sim R^{-1} \;, \tag{2.17}$$

which has the solution $R \sim \sqrt{t}$. That is, the size of the universe increases as the square root of time. The energy density then decreases as $\rho \sim t^{-2}$.

If we assume that the number of effective degrees of freedom, g_* is constant during the early evolution of the radiation-dominated universe then the radiation energy density ($\rho \propto T^4$, as in Eq. 2.4) with its variation as R (Eq. 2.16), we find that the temperature varies inversely as the radius parameter $T \propto R^{-1}$ and therefore $T \propto t^{-1/2}$. Note that according to Eq. 2.7 this also implies that the total entropy of the universe is conserved. We then obtain the following relation for the temperature as a function of time:

$$T(t) \simeq \sqrt{\frac{\hbar M_{Pl}}{(g_*)^{1/2} t}} \;. \tag{2.18}$$

If we invert this relation to yield

$$t \simeq \frac{\hbar M_{Pl}}{(g_*)^{1/2} T^2} \tag{2.19}$$

we can construct the timeline for the temperature of the early universe from 10^{-43} sec. through about 10^6 yr. when the radiation dominated phase ends.

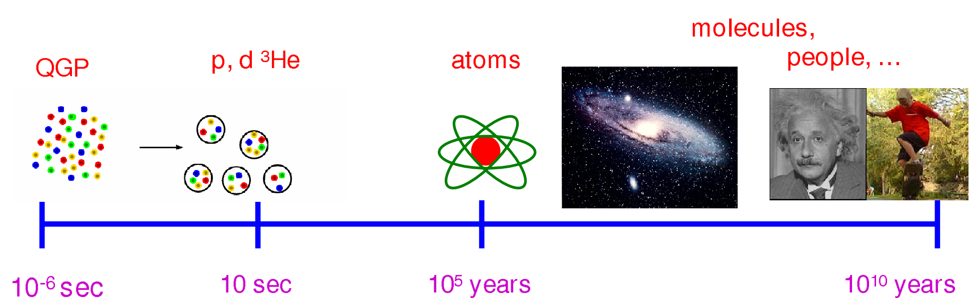

Figure 2.1: History of the universe for temperatures less than $k_B T \sim 100$ GeV.

We have assumed that g_* is constant in obtaining these results. However, we do need to consider the fact that as the temperature drops some particles freeze out, and so $g_*(T)$ then changes. This will modify the expressions 2.18 and 2.18. However, the basic behavior of the expanding universe is qualitatively described by these relations, especially noting that in Eq. 2.18 the dependence of the temperature on g_* is very mild ($T \propto g_*^{-1/4}$).

2.3 The Quark-Gluon Plasma in Perturbative QCD

Until this point we have been treating the quarks and gluons in the QGP as free particles without interactions. Of course, in a high-temperature QGP we expect QCD perturbative theory to be applicable due to asymptotic freedom. One important additional consequence is that chiral symmetry is now a good symmetry, and the chiral condensate must vanish in the plasma

$$\langle \mathrm{QGP}|\overline{\psi}\psi|\mathrm{QGP}\rangle = 0 \ , \tag{2.20}$$

where strictly-speaking the QGP "state" is actually the thermal average over the excited states of the QCD vacuum when the baryon number density is ignored.

Another important feature of the QGP is color deconfinement. In QCD perturbation theory, the quarks and gluons are free particles that can be described by plane waves. Asymptotic freedom will guarantee that high momentum transfer interactions are weak. Small-momentum transfer scattering involves long distance interactions which are screened by the plasma (although this is only strictly true in the color electric sector). As such, the charged quarks and gluons can move freely inside the plasma without being confined to a local region. This remarkable property is radically different from the low energy limit of QCD where all charges are permanently confined to the interior of hadronsa scale about 1 fermi.

Consider a color charge in midst of a color-neutral plasma. The other particles in the plasma will act to screen it, and as a consequence the interaction between color charges is damped exponentially. To calculate the screening length, one can start from a color charge and calculate its induced color fields. The result is a correlation function of gluon fields. This function can be calculated in perturbation theory at high-temperature, and the result for the screening mass is

$$m_D^2 = g^2T^2 \ . \tag{2.21}$$

to leading order in the strong coupling expansion. The so-called Debye screening length is simply $1/m_D$ or $1/gT$, which is very short at high-temperature. When the color charges are screened in a plasma, it has a finite energy and therefore in this sense, the color charges are now liberated.

Unfortunately, the magnetic interaction is only weakly screened; it has a screening mass of order g^2T. Absence of the magnetic screening means that the magnetic sector of the QCD remains non-perturbative even at high-temperature. Fortunately, at high-temperature this non-perturbative part contributes to physical observables only at higher-order in QCD coupling, so the free gas behavior is dominant.

Another important feature of the plasma is the plasma frequency. In a QED plasma, light cannot propagate below the plasma frequency, $\omega_{\mathrm{pl}} = (ne^2/m)^{1/2}$, but will be reflected from the surface, like in a silver-plated mirror. The physics of the QGP is similar: gluons (plasmon) cannot propagate as a free field in the plasma if its energy is too low. In fact, the gluons acquire an effective mass which is effectively the plasma frequency. Perturbative calculations confirms this behavior, and to leading order in perturbation theory the plasma frequency is

$$\omega_{pl} = \frac{1}{3}\sqrt{N_c + N_f/2}(gT) \ . \tag{2.22}$$

where $N_c = 3$ is the number of color and N_f is the number of fermion flavor. The transverse-polarized gluon modes acquire the same mass.

The plasmon and transverse gluon modes are damped in the plasma. One can calculate the damping rate using the so-called hard-thermal loop method in pQCD and the result is gauge-invariant: $\gamma = ag^2 N_C T/(24\pi)$, where numerically a is found to be $a = 6.63538$.

The results we discussed above are the basic leading-order predictions of pQCD. Higher-order contributions can and have been calculated in the literature. Unfortunately perturbative expansions for thermodynamical properties of the plasma converge very slowly. As such, the free plasma picture works only at extremely high-temperature. Even at the temperatures corresponding to 100 GeV the perturbative expansion must be reorganized significantly to get a sensible prediction. We will come back to this point later.

2.4 Transition to the Low-Temperature Phase: Physical Arguments

As we have discussed in the previous chapter, the zero temperature ground state of QCD is strikingly different from the high-temperature QGP: color charges are confined to the interior of individual hadrons and chiral symmetry is broken spontaneously. Therefore, as the plasma cools in the universe, some rapid changes in thermodynamic observables must occur from the high-temperature QGP phase to the low-temperature confining and chiral-symmetry breaking phase, where the quarks and gluons combine to form colorless states of hadronic matter.

It is possible to estimate the transition temperature by comparing the QGP gas pressure with that of hadronic gas. The lightest hadrons are pions, and for $T < 1$ GeV (note that in the following we often use units where $k_B = 1$ and T has units of energy), we might expect a gas of relativistic pions. This is a system with only 3 degrees of freedom, $g = 3$, so the energy density and pressure of the system

$$\rho_\pi = \frac{3\pi^2}{30}T^4, \quad P_\pi = \frac{3\pi^2}{90}T^4, \tag{2.23}$$

This, however, is not the full story. Pions are collective excitations of the non-perturbative QCD vacuum. This true ground state of the QCD vacuum has a lower-energy $-B$ than the perturbative QCD vacuum. (In the MIT bag model of hadrons, this energy is the origin of the quark confinement.) Lorentz invariance requires that the energy-momentum density is of form $T^{\mu\nu} = Bg^{\mu\nu}$. Thus the non-perturbative QCD vacuum has a positive pressure as well. Therefore, the total pressure of the hadronic phase is

$$P_{\rm low} = B + \frac{3\pi^2}{90}T^4 \ , \tag{2.24}$$

On the other hand, from the previous sections, the pressure of the QGP phase with 2 quark flavors is, $P_{\rm QGP} = 37\pi^2 T^4/90$. Equating the two pressures, we find the transition temperature,

$$T_c = (45B/17\pi^2)^{1/4} \sim 180{\rm MeV} \ , \tag{2.25}$$

where we have used the MIT bag constant $B = 200$ MeV as determined by fits to the masses of physical hadrons.

The energy difference (latent heat) between the two phases at the transition temperature is

$$\Delta\rho = \frac{34\pi^2}{30}T^4 + B \ , \tag{2.26}$$

which is on the order of 2 GeV/fm^3.

Another estimate of the transition temperature comes from considering chiral symmetry. At finite but small temperature, the pion gas will dilute the chiral condensate in the zero-temperature vacuum. The quark condensate can be calculated as a response of the system's free energy to the quark mass,

$$\langle\overline{\psi}\psi\rangle_T = \frac{1}{N_f}\frac{\partial F}{\partial m_q} \ , \tag{2.27}$$

where N_f is the number of light quark flavors. The free-energy of the pion gas is

$$F = (N_f^2 - 1)T \int \frac{d^3\vec{p}}{(2\pi)^3} \ln(1 - e^{-E_\pi/T}) \ . \tag{2.28}$$

Thus the pion condensate has the following low-temperature expansion,

$$\langle\overline{\psi}\psi\rangle_T = \langle\overline{\psi}\psi\rangle_0 \left[1 - \frac{N_F^2 - 1}{3N_f}\frac{T^2}{4f_\pi^2} + ...\right] \ , \tag{2.29}$$

where the ellipse indicates higher-order terms in the expansion. If one just keeps the first two terms, the chiral condensate vanishes when

$$T_c = 2f_\pi\sqrt{3N_f}/(N_f^2 - 1) = 200 \text{ MeV} \ , \tag{2.30}$$

which is consistent with the other estimate.

Clearly, the QCD system is strongly interacting around T_c. On the other hand, the above estimates relied on calculations which are valid at temperatures much higher than T_c. To say something rigorous about what happened around T_c, one must resort to lattice QCD, a numerical approach to solve QCD through computer simulation.

2.5 A Brief Tour in Lattice QCD Thermodynamics

At lower temperatures where the coupling constant is larger one must employ non-perturbative methods of calculation. The only known method for solving QCD non-perturbatively is on a space-"time" lattice. Here we present a simple introduction to this method without getting involved in too many technical details.

Consider a QCD system with temperature $T = 1/\beta$ and baryon number zero (this is true to a good approximation in the early universe, as we will discuss later in this chapter). The most important quantity is the partition function,

$$Z = \text{Tr}[\exp(-\beta H)] \ , \tag{2.31}$$

where H is the QCD hamiltonian and trace is over all physical states in Hilbert space. H is a function of 3D quark and gluon fields $\psi(\vec{x})$ and $A^\mu(\vec{x})$, respectively. One can introduce a fourth coordinate x_4 (imaginary time) and the 4D field

$$\phi(\vec{x}, x_4) = e^{Hx_4}\phi(\vec{x})e^{-Hx_4} \ , \tag{2.32}$$

where ϕ collectively labels all QCD fields and x_4 runs between the values 0 and β. Then the partition function can be written as a path integral

$$Z = \mathrm{Tr} \int D[\phi] \exp(-S[\phi]) \ , \tag{2.33}$$

where $D[\phi]$ is a functional integration measure and $S[A]$ is the QCD action defined by the lagrangian in Euclidean time,

$$S[\phi] = \int_0^\beta dx_4 \int d^3\vec{x}\mathcal{L}_{\mathrm{Euc.}} \ . \tag{2.34}$$

Therefore, the partition function is now reduced to a functional integral.

Because of the trace in Z, the gluon potentials A^μ obey the period boundary condition in x_4,

$$A^\mu(x_4, \vec{x}) = A^\mu(x_4 + \beta, \vec{x}) \ , \tag{2.35}$$

whereas the quark fields obey the anti-periodic condition

$$\phi^\mu(x_4, \vec{x}) = -\phi^\mu(x_4 + \beta, \vec{x}) \ . \tag{2.36}$$

The whole thermodynamics formulation is then invariant under SU(3) gauge transformation with $U(x_4, \vec{x})$ satisfying the periodic boundary condition.

The path integral formulation makes explicit that QCD is a quantum field theory with an infinite number of quark and gluon degrees of freedom. To solve it approximately, one first limits the system to a 3D box of dimensions L_x, L_y, L_z, and then replaces the continuous space-"time" variables by a discrete 4D lattice. The quark and gluon degrees of freedom now live on the lattice sites and the bonds in-between the sites, respectively. The number of these d.o.f. is now finite but large (millions to billions in actual simulations).

The integrations over quark fields are of Gaussian type and can be done analytically. Those over the gluon fields can be evaluated using the Monte Carlo sampling method. In this approach, classical gluon configurations are generated on the lattice with the probability distribution corresponding to the Boltzmann factor e^{-S}. The actual physical observables are calculated with hundreds and thousands of these "typical" distributions. As an example, the Debye screening mass can be measured from the space correlation of the gluon fields at equal time. One of the important tools of the lattice calculation is that one can change the parameters of the theory and study how observables change in a world different from the real one.

2.6 Quark Masses and The Nature of the Transition

The nature of the QCD transition from high to low temperature depends on strongly on light-quark masses. For simplicity, let us ignore heavy quarks (c, b, t) and concentrate on up, down and strange light quarks.

One powerful theoretical approach to discuss phase transitions is Laudau-Ginsburg theory. In this framework, one first identifies the order parameter of a transition, which is the fundamental observable which drives the phase transition. An example of order parameter in water-steam transition is the density. One then constructs an effective lagrangian of the order parameter which governs the dynamics of the transition. Here the fundamental symmetry constraints are important to determine the form of the effective lagrangian, from which one can often say a lot about the transition without going into the details of the specific dynamics.

If all three quarks are massless, the low-energy order parameter for chiral transition can be chosen to be Σ, a 3 complex matrix which transforms under chiral $SU(3)_L \times SU(3)_R$ as

$$\Sigma \to U_L \Sigma U_R^\dagger \ , \tag{2.37}$$

One can then construct an effective lagrangian to describe the dynamics of Σ. One of the term that one can write down is $Det\Sigma$ which is trilinear in components of the order parameter. As a consequence, the chiral phase transition described by the theory is in general a first-order phase transition.

When the strange quark is infinitely heavy, and up and down quarks remain massless, the low-energy order parameter can be taken to be a 2×2 unitary matrix. The phase transition in this system is similar to that of an $O(4)$ magnet, a magnetic with four-independent magnetization direction. The Landau-Ginzburg theory for this system leads to a second-order phase transition.

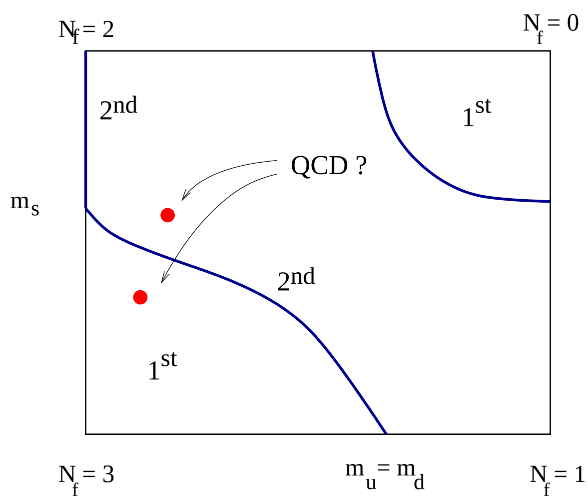

Figure 2.2: Order of phase transition as a function of light quark masses ($m_u = m_d$) vs. the strange quark mass(m_s).

Therefore, if one varies the mass of the strange quark from large to small, the second-order phase transition must end at a tri-critical point, beyond which the transition becomes first order. The tricritical point is characterized by vanishing of the coefficients of the first two terms (quadratic and quartic) in the potential for the order parameter.

The above consideration is not yet sufficient because the up and down strange quark masses do not vanish in the real world.

If up, down and strange quark masses are taken to infinity, we are left with a theory with just pure glue fields. This system at finite temperature has an interesting symmetry called color Z_3 center symmetry. An effective Landau-Ginzburg theory can be constructed to describe the physics of Z_3 symmetry breaking at high-temperature phase. The symmetry argument suggests that the transition is of first order. The strength of the first-order phase transition lowers as the quark mass becomes lighter, and finally the first-order phase transition region is enclosed by a second order phase transition line.

A diagram detailing the nature of QCD phase transitions as a function of quark masses is shown in Fig. 2.2. In the limit of 3 very light quarks, we expect a first order transition. The first order transition region is enclosed by a second order transition line which goes through the tri-critical point. In between the two second-order phase transition lines, one has a broad region of rapid cross-over. The exact locations of these second-order transition lines are not known. Therefore, for a realistic physical situation, where the up and down quark masses are on the order of a few MeV and strange quark mass is about 100 MeV, the transition can either be of a weak first-order or a rapid crossover (2nd order), as shown by two dots in Fig 2.2.

2.7 Physics of the QCD Transition on Lattice

Lattice simulations of QCD thermodynamics have made significant progress in the last decade, due to both rapid rise in computational power and implementation of better algorithms. Some of challenges in achieving a complete realistic simulation include finite lattice size effect, discretization errors, implementing dynamical quarks, and simulations at small quark masses.

From simulations of QCD on a lattice, a transition in thermodynamic observables is clearly seen at a fairly well-defined temperature of about $T_C \simeq 150$ MeV. The energy density undergoes a rapid change near a critical temperature T_C, enhanced by almost an order of magnitude, as indicated in Figure 2.3. Beyond T_C, the energy density is fairly flat as a function of temperature but slightly below that predicted by the free gas model. This rapid change is an indication that the fundamental degrees of freedom are different above and below T_C.

The transition is less dramatic in the equation of state, i.e., the pressure of the system as a function of temperature. At low-temperature, the pressure is very small. As temperature increases, the pressure builds up gradually over a large range of T, from T_c to $2T_c$. When the pressure curve flattens out at high-T, it again undershoots the result of the free gas model. The equation of state is also more sensitive to different quark mass scenarios. These calculations are improved constantly with smaller quark masses and lattice spacing. Improved calculations show that the transition at physical quark masses is a rapid cross over.

An interesting property of this transition emerges from the consideration of chiral symmetry in lattice simulations. Recall that, in the absence of quark masses, the QCD Lagrangian is chirally symmetric, i.e., nature is invariant under separate flavor rotations of right and left-handed quarks. This symmetry is evident in the QGP phase at $T > T_C$ in the lattice simulations. This is quite analogous to the symmetric demagnetized state of a ferromagnet above the Curie temperature. The ferromagnetic property is related to the magnetization $\mathcal{M}$ which vanishes above the Curie temperature. The lattice QCD calculations can provide a measurement of the relevant “order parameter”, the scalar quark density $\langle\bar{\psi}\psi\rangle$, as a function of temperature. This order parameter provides a measure of the effective mass of a quark in the medium. The result is that at $T > T_C$ this effective quark mass becomes small, approaching zero as $T \to \infty$, as one would expect in

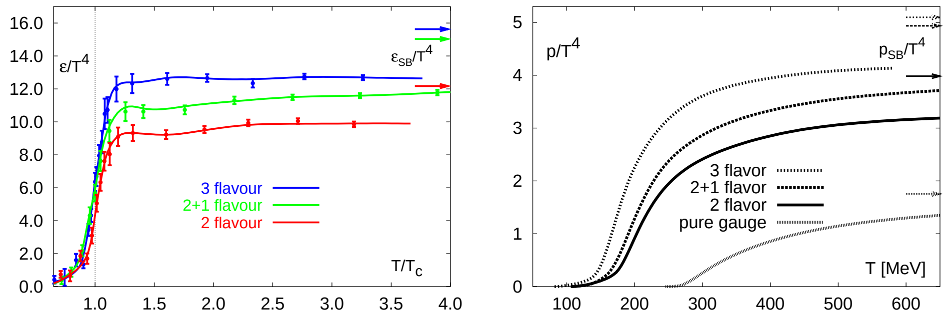

Figure 2.3: The transition from mesonic matter to the QGP phase as suggested by lattice simulations of QCD. The simulations are carried out with 2 or 3 light flavors or 2 light and 1 heavy flavor. The expected limits as given by Eq. 2.5 are shown by arrows on the right side of the figure. (Note that in this figure T actually represents k_B times temperature.)

chirally symmetric QCD. As shown in Figure 2.4, below T_C there is a sharp increase in $\langle\bar{\psi}\psi\rangle$ corresponding to the quarks developing a constituent mass of ~ 300 MeV. This heavy consituent quark is the basis of the quark model of hadrons to be discussed in Chapter XX. This transition to the broken symmetry phase is again analogous to the ferromagnet, which spontaneously breaks rotational symmetry in developing a finite $\mathcal{M}$ below the Curie temperature. Shown in the same figure is the susceptibility of the condensate which shows a peak at the transition temperature.

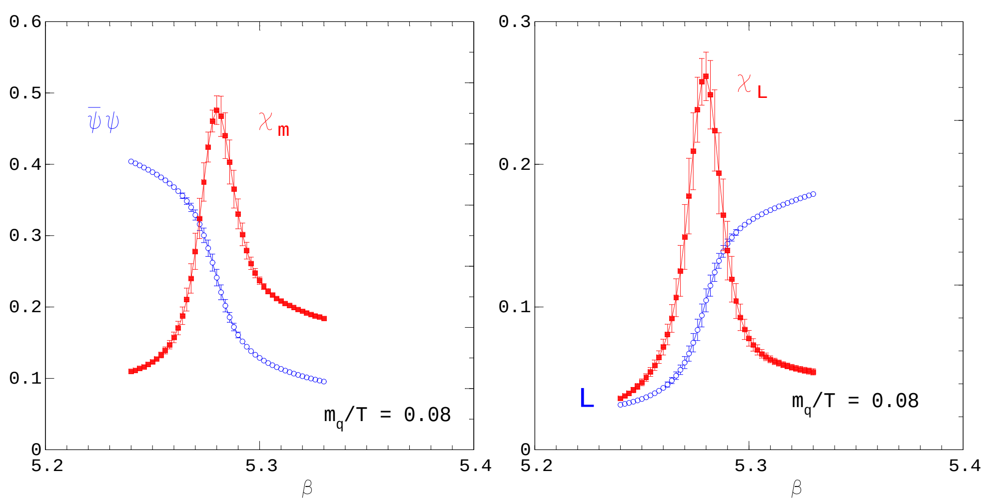

Figure 2.4: The scalar quark density $\langle\bar{\psi}\psi\rangle$ measured in lattice QCD as a function of temperature. At $T < T_C$ chiral symmetry is broken, whereas the chirally symmetric limit is realized at higher temperatures $T > T_C$.

The potential between two quarks is shown for various temperatures in Figure 2.5. One can see that at low temperatures $T < T_C$, the potential continues to rise at larger distances, consistent with the expectation that the quarks will be confined as colorless hadrons. At higher temperatures,

$T > T_C$, one finds that the potential energy at large distances saturates, and it is possible for the quarks to propagate as a free particle. This confirms the picture that above T_C one has a deconfined plasma.

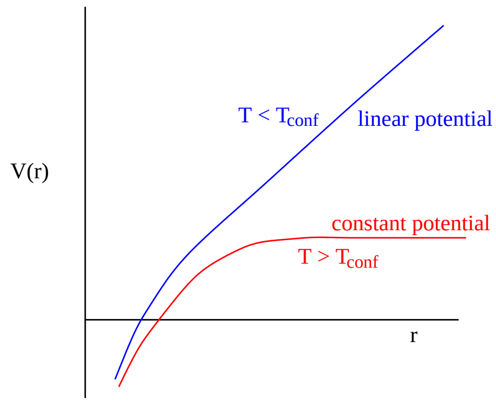

Figure 2.5: The potential energy between two quarks as a function of the separation distance as computed by lattice QCD. The calculation is performed for various temperatures showing that confinement is present at low temperatures $T < T_C$ but not above T_C.

2.8 Evolution of the Universe in Hadronic Phase

The QGP dominated universe will expand and cool until we reach the critical temperature, T_C, to produce a gas of pions along with a few baryons and antibaryons. The number of baryons is clearly suppressed by the Boltzmann factor

$$P \sim e^{-M_B/T} \ , \tag{2.38}$$

which is small but still significant. But the universe we observe is clearly not a pion gas, nor the decayed remnants which would be electrons, positrons and finally just photons. As the universe cools, the pion densities diminish exponentially as a function of temperature. This decrease is achieved by the annihilation

$$\pi^+ + \pi^- \to \gamma + \gamma \ , \tag{2.39}$$

whereas the inverse reaction is difficult because the thermal photons do not have enough energy to produce pions. Similar process happens for baryons and antibaryons, while they continue to annihilate, they cannot be reproduced. If the annihilation rate is rapid enough, the density of baryons follows the thermal Boltzmann distribution. Of course, if the universe had no net baryon number, all baryons would disappear eventually. However, the existence of matter around us shows that the baryon number of the universe is not zero. Therefore, as temperature cools, all anti-baryons are annihilated, all pions are either annihilated and/or decayed, but there is a small tiny baryon density in the form of proton and neutron survives. (The origin of these baryons is the subject of the next section of this chapter. Indeed, these protons and neutrons are the main characters of the remainder of this book.)

What happened to other components of the universe? As temperature lowers, we are left with just leptons and photons. At temperatures below the electron mass, pairs of electrons and positrons are no longer created, so they freeze out and annihilate to produce more photons. We enter a phase of the universe that is dominated by photons and neutrinos: a black-body universe. Of course, due the charge neutrality, a small fraction of electron residue is also present. The energy density will continue to decrease as t^{-2} until much later when, it turns out, the small fraction of baryonic matter becomes a significant factor in the energy density.

We still observe the remnant black-body radiation associated with the early universe. (The neutrinos have not been observed, but are discussed in Chapter 8.) There is presently a uniform distribution of cosmic microwave radiation with a characteristic temperature of $(2.725 \pm 0.001)^\circ$ K and a number density of $410.4{\pm}0.5/\text{cm}^3$. The present baryon to photon ratio is tiny, about $6{\times}10^{-10}$, but the photons have energies of $\sim 2.5 \times 10^{-4}$ eV and the baryons have energy $M_p \sim 1$ GeV. Thus the energy density of the baryons is about 2400 times the energy density of the photons. As the universe cooled from the "radiation dominated" era to the present, the photons were red-shifted due to the expansion. The photons decoupled from the protons and electrons when the latter combined into neutral hydrogen. Since then (temperature $T \sim 0.3$ eV) the photons have been red-shifted by about 10^4, while the protons remained at $M_p \sim 1$ GeV. Thus we now find ourselves in an era where baryonic matter dominates over the cosmic photons. (In addition, we have dark non-baryonic matter and dark energy which remain to be understood - but that is a different story and we will confine our attention here to the baryons and photons.)

2.9 The Origin of Baryonic Matter

Let's now return to the source of the baryonic matter. While the present baryon to photon ratio is indeed tiny, that is the stuff of which we are made. Since the early universe should consist of equal numbers of quarks and antiquarks, eventually they should all annihilate to produce only photons in the end. The observed number of baryons is much greater than can be expected from a random fluctuation associated with the cooling of the enormous number of quarks and gluons in the initial plasma. The annihilation of baryons and antibaryons ceases at a rather low temperature of ~ 20 MeV. The number density of baryons and antibaryons at this temperature is given by the expression (for g_B species of non-relativistic particles)

$$n_B = g_B \left(\frac{M_p T}{2\pi}\right)^{3/2} e^{-\frac{M_p}{T}} . \tag{2.40}$$

The corresponding number density of photons is

$$n_\gamma \simeq \frac{2}{\pi^2} T^3 \tag{2.41}$$

so the ratio is approximately

$$\frac{n_B}{n_\gamma} \sim g_B \left(\frac{M_p}{T}\right)^{3/2} e^{-\frac{M_p}{T}} \tag{2.42}$$

$$\sim 10^{-19} \quad (@T = 20 \text{ MeV}) . \tag{2.43}$$

We would expect the chance excess of baryons over antibaryons to be a very small fraction of the total number of both. Therefore, the observed 6×10^{-10} fraction of baryons relative to the number of photons is not possible to generate by just a statistical fluctuation.

So what is the mechanism that generates the small excess of matter over antimatter? The modern view is that the observed excess of baryonic matter relative to antimatter is likely due to the properties of particles and their interactions during the expansion and cooling of the early universe. This can occur if three criteria, initially discussed by Sakharov, are satisfied:

1. baryon number (B) is violated so that baryons (and antibaryons) can be created,
2. CP is violated so that the rates of baryon and antibaryon production can be different,
3. thermal equilibrium is broken so that forward and reverse reactions become unbalanced.

Studying baryogenesis has been a very active area of theoretical physics for the last few decades. In the standard model, the baryon number can be violated through non-perturbative process via the so-called anomaly. However, to generate enough baryons, one must have a strong first order phase transition which is hard to achieve with the heavy Higgs mass. Moreover, the standard model CP violation through the CMK matrix is too small to generate enough CP asymmetry. Therefore, it appears that baryogensis requires physics beyond the standard model. One of the interesting direction is what happens in a supersymmetric extension of the standard model.

In grand unified models, baryon number is in general violated. One consequence is that protons would be unstable and decay, typically by a process like $p \to e^+\pi^0$. Searches for such decays limit the proton lifetime to $\tau_P > 10^{32}$ years. Consider a heavy particle of mass M_X. Fermi's golden rule can be used to calculate the decay rate as

$$\Gamma = |\langle f|H_X|i\rangle|^2 \frac{dN_f}{dE_f}\,. \tag{2.44}$$

The matrix element will be proportional to α_X/M_X^2 where α_X is the squared coupling constant and the M_X^{-2} is the propagator for the heavy particle exchange. Since the only other energy scale in the problem is the proton mass M_p we can write by dimensional analysis

$$\tau_P \sim \frac{1}{M_P^5}\left(\frac{M_X^2}{\alpha_X}\right)^2\,. \tag{2.45}$$

Using the experimental limit for τ_P and assuming $\alpha_X \sim 0.1$ one obtains a rough limit for the mass of the heavy particle responsible for baryon number violating interactions $M_X > 10^{16}$ GeV. In a typical baryogenesis scenario, one might generate long-lived heavy particles X in the very early universe when $T > M_X$. At $T < M_X$ the X particles freeze out and then much later, when the universe is cooler, they decay into baryons and antibaryons. The presence of CP violation in the decays enables a production of an excess of baryons over antibaryons. However, it is now generally believed that this excess gets washed out at lower temperature by the so-called electroweak sphaleron process.

In more recent years, massive right-handed neutrinos have been considered as good candidates to lead to baryogensis. These particles could have masses slightly smaller than the grand unification scale, and decay asymmetrically (due to CP violation in lepton sector) to produce a net lepton number. This lepton number can be converted in to baryon number through the sphaleron processes. This route of generating baryon asymmetry is called leptogenesis.

2.10 ***Appendix for Chapter 2****

2.10.1 Ginzburg-Landau Theory for Phase Transitions

In a first-order phase transition, the free-energy is continuous and its first-order derivatives are not. In a second-order transition, the first-order derivative of the free energy is continuous and the second-order derivatives are not.

Quite often, phase transitions involve a transition from a symmetrical phase to a less symmetrical one, or vice versa, *i.e.*, a *symmetry-breaking transition.* For instance, in the transition from a paramagnetic to a ferromagnetic system, the rotational symmetry is broken because a spontaneous magnetization defines a unique direction in space. In transition from normal liquid ^{4}He to superfluid liquid ^{4}He, gauge symmetry is broken. Near the critical point in liquid-gas transition, the distinction between liquid and gas disappears above the critical point.

Because of this rather distinct feature of the transitions involving symmetry-breaking, a new macroscopic parameter was introduced by Landau to describe the transition phenomenologically, and is called **order parameter**. The order parameters take zero in a symmetrical phase and non-zero in the unsymmetrical (or less symmetrical) one. The order parameters may be a scalar, vector or tensor, a complex number, or some other quantity, depending on the symmetry of the transition involved. The order parameter changes continuously near the second phase transition point, so the volume or entropy do not change abruptly. For this reason, a second-order phase transition is also called a **continuous transition**. One important difference between the order parameter and other macroscopic variables such as pressure and temperature is that the values of the order parameter are determined by minimizing the thermodynamic potential of a system.

Ginzburg and Landau found a general way to describe symmetry-breaking phase transitions in terms of a free energy functional involving order parameters. For simplicity, let us assume the order parameter is a vector $\vec{\eta}$ and construct a free energy which has a minimum at $\vec{\eta} = 0$ above the transition point ($T > T_C$) and $\vec{\eta} \neq 0$ below it. The free energy which is a scalar function of the order parameter, depending on the scalar-scalar product $\vec{\eta} \cdot \vec{\eta}$. Near the phase transition point where $|\vec{\eta}|$ is small, one can make the following Taylor expansion,

$$\Phi(T, \vec{\eta}) = \Phi_0(T) + \alpha_2(T)|\vec{\eta}|^2 + \alpha_4(T)|\vec{\eta}|^4 + \cdots \tag{1}$$

If we choose $\alpha_2(T) = \alpha_0(T - T_C)$, then when $T > T_C$, $\alpha_2 > 0$ and $\vec{\eta} = 0$ is a local minimum of Φ, when $T < T_C$, $\alpha_2 < 0$ and $\vec{\eta} = 0$ is a local maximum. This can be seen by plotting Φ as a function of $|\vec{\eta}|$ near $\vec{\eta} = 0$. This choice makes the order parameter behave in the way described above. To ensure $\vec{\eta} = 0$ is also a global minimum for $T > T_C$, we take $\alpha_4(T) > 0$ at all T.

If we neglect the high-order terms in (1), the potential and the order parameter of the system evolve with temperature in the following way. At $T > T_C$ the potential is shown in Fig. xx: $\vec{\eta} = 0$ is the minimum and the system is in the symmetrical phase. At $T < T_C$, the potential is shown in Fig. 1b, and there are minima at $|\vec{\eta}| =$ const. with arbitrary phase and a local maximum at $\vec{\eta} = 0$. These can be obtained from

$$\frac{\partial \Phi}{\partial \vec{\eta}} = 2\alpha_2 \vec{\eta} + 4\alpha_4 \, |\vec{\eta}|^2 \, \vec{\eta} = 0 \quad , \tag{2}$$

which gives

$$\vec{\eta} = 0 \quad , \qquad |\vec{\eta}| = \sqrt{\frac{-\alpha_2}{2\alpha_4}} \quad . \tag{3}$$

The second equation tells us that the order parameter changes as $(T_C - T)^\beta$ with $\beta = \frac{1}{2}$ below T_C. β is one of the critical exponents that are introduced to characterize the singular behavior of an observable near the critical point. We will introduce more critical exponents below.

Substituting (3) into (1), we find the free energy

$$\Phi(T) = \begin{cases} \Phi_0(T) & T > T_C \\ \Phi_0(T) - \alpha_2^2 \big/ 4\alpha_4 & T < T_C \end{cases} \tag{4}$$

Thus Φ and its first derivative are continuous across T_C. However the second-order derivative which is related to specific heat,

$$C = -T\frac{\partial^2 \Phi}{\partial T^2} \quad , \tag{5}$$

is discontinuous. It is easy to show

$$C|_{T=T_C^-} \; - \; C|_{T=T_C^+} = T_C \frac{\alpha_0^2}{2\alpha_4} \tag{6}$$

If one defines $C \propto |T - T_C|^{-\alpha}$ where α is another critical exponent, then $\alpha = 0$.

When an external field $\vec{h}$ is applied to the system, the potential is added with a term $\vec{\eta} \cdot \vec{h}V$, where V is the volume of the system. Then $\vec{\eta} \neq 0$ at any temperature, and the second-order phase transition disappears. Eq. (2) becomes

$$2\alpha_2 \vec{\eta} + 4\alpha_4 |\vec{\eta}|^2 \, \vec{\eta} = \vec{h}V \tag{7}$$

Above T_C, $|\vec{\eta}|$ is small, and we have $\vec{\eta} = \vec{h}V/2\alpha_2$. The susceptibility is

$$\chi = \frac{\partial \vec{\eta}}{\partial \vec{h}}\bigg|_{\vec{h}=0} = \frac{V}{2\alpha_2} \sim |T - T_C|^{-\gamma} \tag{8}$$

where the critical exponent γ is 1. Below T_C, $\vec{\eta} = \vec{h}V/(-4\alpha_2)$ and again $\chi \sim |T - T_C|^{-\gamma'}$ with $\gamma' = 1$.

At $T = T_C$ we have the following relation between order parameter and the applied field,

$$\eta = \left(\frac{hV}{4\alpha_4}\right)^{\frac{1}{3}} \sim h^{\frac{1}{\delta}} \quad , \tag{9}$$

where the critical exponent δ is 3.

The critical exponents in Landau theory are independent of the specific values of the parameters that appear in (1). They are completely universal. In other words, the phase transitions in very different physical systems exhibit the same singular behavior. A general theory of phase transition must capture this universality feature.

2.11 Problem Set

1. Calculate the number and energy densities of a relativistic fermion/boson gas. And show that in the standard model $g_* = 106.75$.

2. Calculate the temperature as a function of time during radiation-dominated era of the early Universe, up to the normalization factor.
3. Calculate the free-energy of a free pion gas, from which derive the dependence of the chiral condensate in temperature to first order in T^2/f_π^2.
4. Show that the thermodynamic function of QCD without quarks has a Z_3 symmetry. Construct a Ginzburg-Landau theory for Z_3 phase transition and show it is a first-order transition.
5. Calculate the baryon energy density over the photon energy density in the universe.