UNIVERSITY OF MARYLAND
DEPARTMENT OF PHYSICS
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PHYSICS 732 | **HOMEWORK ASSIGNMENT #3**
DR. H. D. DREW | Due Thursday, March 15, 2005

Read Marder, chapter 19, and 25.5.
Read Ashcroft and Mermin, chapter 29.

Problems

1. Ashcroft and Mermin, chapter 29, #1 .

2. This problem is based on the tutorial notes on carbon nanotubes by Schoenenberger in the references on the Phys732 web page.

a). For graphene show that the corners of the Brillouin zone are $K = \frac{4\pi}{3\sqrt{3}a_0}$ from the zone center and show that the band energies, E(k)=0, at these points.
b). Prove the selection rules (Eq. 32 and 33 in Schoenenberger) for the wavefunctions of grapheme near E=0.
c). Consider a general CNT with a wrapping vector $\vec{w} = n\vec{a}_1 + m\vec{a}_2$. Show that the allowed $k_\perp$ are given by:

$k_{\perp,p} = 2\pi \frac{(m-n)/3 + p}{\pi d}$, where $\pi d = |\vec{w}|$ and p is a integer.

Discuss the corresponding band structure. When is a NT metallic and when is it semiconducting? How large is the band gap?

3. Consider a semiconductor within the $k \bullet P$ approximation in which two bands are very close in energy.
a. Use degenerate $k \bullet P$ perturbation theory to find the energy dispersion of the states in the two bands (B=0) and confirm that the results are consistent with the full $k \bullet P$ effective 2-band Hamiltonian given below:

$$\left[\left\{ \frac{1}{2m} P \bullet \alpha \bullet P + \mu_0 S \bullet g^* \bullet H + V(r) \right\} \frac{E_g}{E + E_g} - E \right] f(r) = 0,$$

where $P = p + \frac{e}{c} A$ is the canonical momentum, α the band bottom effective mass tensor (assume diagonal with $\alpha_{xx} = \alpha_{yy} = \alpha_t$ and $\alpha_{zz} = \alpha_l$) and g^* the effective g tensor given by $g^* = 2\alpha$, and E_g is the energy gap. The far band contributions have been neglected. (Note the similarity to the Dirac Hamitonian)

b. Now assume an applied magnetic field applied along the z axis. Solve the effective Schrodinger equation and find the eigenvalues and eigenfunctions. Sketch the spectrum.

c. Assuming that there is a carrier density, n, which leads to a Fermi energy E_F large compared with the Landau level spacing find the effective cyclotron mass m_c^* and effective g-factor g^* as a function of E_F .

4. Consider a GaAs heterojunction with a mobility of 2,000,000 cm^2/Vs and an electron density of $3x10^{11}$ cm^{-2} and assume only one subband is occupied.

a. For zero magnetic field calculate the Fermi energy, the Fermi velocity, the mean free path, the mean free time, and the uncertainty width of the levels in temperature units.

b. If a magnetic field is applied perpendicular to the plane of the 2DEG determine the magnetic field dependence of the Fermi level.

c. If B =10 T, calculate the l_0, (the magnetic length) and the classical Hall angle. m* = $0.07m_0$, g* = -0.5

5. In semiconductor heterostructures several different band alignments are found. Consider the case of an interface between two semiconductors with the band alignment as shown below where $m_1 < 0$ and $m_2 > 0$ and the chemical potential μ is in the gap as shown. This case leads to bound interface states.

a. Assume the effective mass approximation. Integrate the effective mass Schrodinger equation across the interface to obtain the boundary condition on the wavefunction at the interface.

b. Solve the effective mass Hamiltonian for the states confined to the interface, $f(x, y, z)$, the effective mass wavefunction.

c. Determine the dispersion relation for motion of the interface states in the interface plane.

d. Determine the carrier density as a function of μ.