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The problem of cavity QED.

The Hamiltonian, $\rightarrow$ ATOMS + FIELD + INTERACTION + DRIVE + ATOMIC DECAY (γ) + CAVITY DECAY (κ)

~~the~~ the density operator Master equation (rotating frame.

$$\dot{\rho} = \underbrace{\mathcal{E}[\hat{a}^{+} - \hat{a}, \rho]}_{\text{DRIVE}} + \underbrace{g[a^{+}\hat{J}_{-} - \hat{a}\hat{J}_{+}, \rho]}_{\text{COUPLING}}$$

$$+ \kappa(\underbrace{2\hat{a}\rho\hat{a}^{+}}_{\text{collapse through cavity}} - \hat{a}^{+}\hat{a}\rho - \rho\hat{a}^{+}\hat{a})$$

the cavity emits.

$$+ \gamma/_2 \sum_{j=1}^{N} (\underbrace{2\hat{\sigma}_{-}^{j}\rho\hat{\sigma}_{+}^{j}}_{\text{collapse through spontaneous emission}} - \hat{\sigma}_{+}^{j}\hat{\sigma}_{-}^{j}\rho - \rho\hat{\sigma}_{+}^{j}\hat{\sigma}_{-}^{j})$$

individual atoms emit.

γ is $\gamma_{\parallel}$

no spatial variation.

The size expansion is the size of the "fluctuation"

$$n_{sat} = \frac{\gamma^2}{\tilde{\gamma}\, g^2} \qquad C_1 = \frac{g^2}{\kappa\gamma}$$

$$\frac{\tilde{\gamma} - \gamma}{\gamma}$$ where $\tilde{\gamma}$ = cavity enhanced spontaneous emission rate

The drive is $\mathcal{E}$

the rate κ

make ~~small~~ expansion of density matrix

in powers of $\frac{\mathcal{E}}{\kappa}$ weak drive.

then

— Density operator factorizes in pure state form in all times,

— the one- and-two quantum amplitudes can be calculated using the symmetrized atomic states

$$\rho(t) = |\chi(t)\rangle\langle\chi(t)|$$

$$|\chi(t)\rangle = |0,0\rangle + A_1(t)|1,0\rangle + A_2(t)|0,1\rangle + A_3(t)|2,0\rangle + A_4(t)|1,1\rangle + A_5(t)|0,2\rangle$$

$|n, m\rangle$ ← excited atoms.

cavity photons

allow up to two excitations!

then you can calculate the equations

of the amplitudes $A_1(t)$ $A_5(t)$

drive.

$$\dot{A}_1 = -\kappa A_1 + \sqrt{N} g A_2 + \mathcal{E} \quad \text{(field)}$$

$$\dot{A}_2 = -(\gamma/2) A_2 - \sqrt{N} g A_1 \quad \text{(polarization)}$$

Drive

$$\overset{\circ}{A}_3 = -2k A_3 + \sqrt{2}\sqrt{N} g A_4 + \sqrt{2} \mathcal{E} A_1$$

Drive

$$\overset{\circ}{A}_4 = -(k + \gamma/2) A_4 - \sqrt{2}\sqrt{N} g A_3 + \sqrt{2}\sqrt{N-1}\, g A_5 + \mathcal{E} A_2$$

$$\overset{\circ}{A}_5 = -\gamma A_5 - \sqrt{2}\sqrt{N-1}\, g A_4$$

please note that an atom is gone from the collective operator

atomic dipole oscillators.

The $\underline{\text{structural}}$ effects in cavity QED
follow from the 8 equations
- cavity enhanced emission

$$\kappa \gg g \gg \gamma/2$$

A_1 adiabatically $\underline{\text{follows}}$ A_2.
then $\overline{\dot{A}_1 = 0}$

$$A_1 = \frac{\mathcal{E}}{\kappa} + \frac{\sqrt{N}g}{\kappa} A_2$$

$$\dot{A}_2 = -\frac{\gamma}{2} A_2 - \sqrt{N}g \left[\frac{\sqrt{N}g}{\kappa} A_2 + \frac{\mathcal{E}}{\kappa} \right]$$

$$\dot{A}_2 = -\frac{\gamma}{2}(1 + 2C) A_2 + \sqrt{N}\, g \left(\frac{\mathcal{E}}{\kappa}\right)$$

when $C = NC_1$ $\qquad C_1 = \frac{g^2}{\kappa\gamma}$

you can see that the cavity enhance decay
rate is ~~no loss~~ now $\quad -\gamma(1+2C)$

$|A_2|^2 \qquad \tilde{\gamma} = \gamma(1+2C_1)$ for a single
atom.

the dipoles go down $\underline{\text{very}}$ fast!
because we are in a cavity

A_1 and A_2 say nothing about quantum fluctuations. We need to have more excitation to find about it, the information will be in A_3, A_4, A_5.

However if it is true that we have a pair of coupled oscillators; then $|x\rangle$ should somehow be a product of coherent states $|A_1\rangle|A_2\rangle$

let us remember the expansion of the ~~coherent~~ state in terms of Fock states.

$$\cancel{|\alpha\rangle = |0\rangle + |1\rangle + \frac{1}{\sqrt{2}}|2\rangle \ldots}$$

$\rangle + \alpha|n\rangle$ $\cdot$ $\cdot^2$

$$A_3 = \frac{A_1^2}{\sqrt{2}} \qquad A_4 = A_1 A_2 \qquad A_5 = \frac{A_2^2}{\sqrt{2}}$$

In order to be consistent we have to modify the equations $\sqrt{N-1} \Rightarrow \sqrt{N}$

then we "close" the loop.

Then the steady state of the system.

$t \to \infty \qquad t \gg \kappa^{-1} \qquad t \gg \left(\frac{\gamma}{2}\right)^{-1}$

$\dot{A}_i = 0$

$$|\psi_{ss}\rangle = |0,0\rangle + \alpha\,|1,0\rangle + \beta\,|0,1\rangle + \frac{\alpha^2}{\sqrt{2}}\,p\,q\,|2,0\rangle + (\alpha\beta)\,q\,|1,1\rangle + \left(\frac{\beta^2}{\sqrt{2}}\right) q\,r\,|0\,2\rangle$$

$\alpha = \left(\frac{\varepsilon}{\kappa}\right)\left(\frac{1}{1+2C}\right)$ ← small parameter! (~~[illegible]~~ page) this is just the steady state

$$\beta = -\frac{\sqrt{N}\,g}{(\gamma/2)}\left(\frac{\varepsilon}{\kappa}\right)\left(\frac{1}{1+2C}\right)$$

$p =$

$$p = 1 - 2C_1'$$

$$q = (1 + 2C)/(1 + 2C - 2C_1')$$

$$r = \sqrt{1 - 1/N}$$

$$C_1' = C_1 \Big/ \left(1 + \frac{\gamma}{2\kappa}\right)$$

if we had a coupled H.O. $p, q, r \approx 1$

r and q are approximately 1.

however p is not by an amount that happens to be very close to the difference between the enhanced and non enhanced spontaneous emission $2C_1' = (\bar{\gamma} - \gamma)/\gamma$ and is independent of $\underline{\underline{\underline{N}}}$ (the number of atoms.

A quantum fluctuation will cause a significant loss of coherence if there is enhancement of the single atom enhanced spontaneous emission rate.

Thus we can measure by the correlation functions of the field → the probability for the simultaneous transmission of two photons by the cavity

big difference.

$$|\langle 00|\hat{a}^2|\chi\rangle|^2 = |\alpha^2 p q|^2$$

for a coherent state it is only $|\alpha^2|^2$

the amplitude of the expansion

this is the equal time correlation function

Let us now look at the collapsed state

$$|\bar{\chi}\rangle = \frac{\hat{a}|\chi\rangle}{\langle 00|\hat{a}|\chi\rangle} \leftarrow \text{normalized.}$$

$$= |00\rangle + \beta q|01\rangle + \alpha p q|10\rangle$$

remember $\langle\alpha\rangle = \frac{\varepsilon}{\kappa}\frac{1}{(1+2C)}$

then we can calculate $h(\tau)$ and $g(\tau)$

$h(\tau) = \alpha \bar{A}_1(\tau)/\alpha^2$ $\bar{A}_1(\tau) \longrightarrow$ $A_1(0)\ \beta q$, $A_2(0)\ \alpha p q.$ } collapsed state

$$h(\tau) = 1 + \frac{\Delta\alpha}{\alpha}\exp\left[-\tfrac{1}{2}(\kappa+\tfrac{\gamma}{2})\tau\right]\left\{\cosh\Omega\tau + \frac{\tfrac{1}{2}(\kappa+\gamma/2)}{\Omega}\sinh\Omega\tau\right\}$$

and $g(\tau) = |\alpha|^2|\bar{A}_1(\tau)|^2/|\alpha|^4$

where $\frac{\Delta\alpha}{\alpha} = -2C_1'\left[\frac{2C}{1+2C-2C_1'}\right]$

$\Omega = \left[\frac{1}{4}(\kappa+\frac{\gamma}{2})^2 - Ng^2\right]^{1/2}$

Now if $N \gg 1$

$$\frac{\Delta\alpha}{\alpha} = -2C_1'$$

↑ negative jump!

Now let us see how the interferences enter for bad cavity $\kappa \gg g \gg \gamma/2$

$$\hat{a} \to \kappa^{-1}(\mathcal{E} + g\hat{J}_-)$$

$$\langle 00|\hat{a}^2|\Psi\rangle = \kappa^{-2}\,{}_a\langle 0|(\mathcal{E}^2 + 2\mathcal{E}g\hat{J}_- + g^2\hat{J}_-^2|\Psi'\rangle$$

$$|\Psi'\rangle = {}_c\langle 0|\Psi\rangle$$

$$= \kappa^{-2}(\mathcal{E}^2 + 2\mathcal{E}\sqrt{N}g\beta + Ng^2\beta^2 qr\rangle$$